Flow of Ideal Fluids

Q1.

(i) If for a flow a stream function ψ exists and satisfies the Laplace equation, then which of the following is the correct statement?

(a) The flow is rotational
(b)The flow is irrotational and incompressible
(c) The flow is rotational and incompressible
(d) The flow is irrotational and compressible

[*Ans.(*b)]

(ii) Circulation is defined as

(a) line integral of velocity along any path
(b) line integral of velocity along a closed path
(c) line integral of tangential component of velocity along a closed path
(d) integral of tangential component of velocity along a path

[*Ans.(*c)]

(iii) When a cylinder is placed in an ideal fluid and the flow is uniform, the pressure coefficient is equal to

(a) $1-8\sin^2\theta$
(b) $1-4\sin^2\theta$
(c) $1-2\sin^2\theta$
(d) $1-\sin^2\theta$

[*Ans.(*b)]

(iv) How could flow past a Rankine oval body be simulated as a combination?

(a) uniform flow and line source
(b) uniform flow and doublet
(c) uniform flow and a source sink pair
(d) uniform flow, doublet and free vortex

[*Ans.(*c)]

(v) How could flow past a circular cylinder be simulated as a combination?

(a) uniform flow and line source
(b) uniform flow and doublet
(c) uniform flow and a source sink pair
(d) uniform flow, doublet and free vortex

[*Ans.(*b)]

Q2.

The velocity potential for a two-dimensional flow field is given by $\phi = x^2 - y^2$. Find the stream function for the flow.

Solution

Thus, the velocity components are found to be

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(x^2 - y^2\right) = 2x$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}\left(x^2 - y^2\right) = -2y$$

Hence, $\frac{\partial u}{\partial x} = 2$

$$\frac{\partial v}{\partial y} = -2$$

Thus, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2 = 0$

Therefore, the above velocity field satisfies the continuity equation for incompressible flow and hence the stream function exists.

From the definition of stream function ψ, we get

$$u = \frac{\partial \psi}{\partial y}$$

or $\psi = \int u dy = \int 2x dy$

or $\psi = 2xy + f(x)$ (1)

Again, $v = \frac{\partial \psi}{\partial x}$

$$\psi = -\int v dx = \int 2y dx$$

or $\psi = 2xy + g(y)$ (2)

Comparing Eqs (1) and (2), we have

$$\psi = 2xy$$

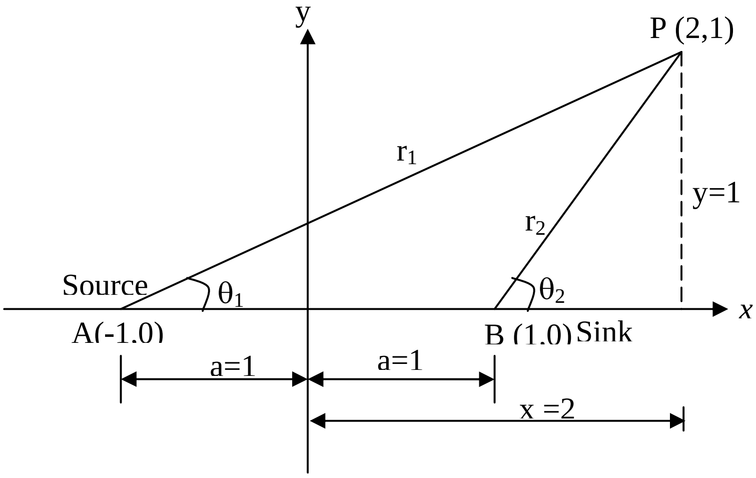
Hence, the stream function for the flow is $\psi = 2xy$

Q3.

A source of strength 5 m^2/s located at (-1,0) is combined with a sink of strength 7 m^2/s located at (1,0). Find the stream function and the velocity potential function at point (2,1).

Solution

The arrangement is shown in the figure below.



From the geometry of the above figure, we have

$$\tan\theta_1 = \frac{y}{x+a} = \frac{1}{2+1} = \frac{1}{3}$$

or $$\theta_1 = \tan^{-1}\left(\frac{1}{3}\right) = 0.322 \text{ rad}$$

$$\tan\theta_2 = \frac{y}{x-a} = \frac{1}{2-1} = 1$$

or $$\theta_2 = \tan^{-1}(1) = 0.785 \text{ rad}$$

The equation of stream function for the combined source and sink pair is given by

$$\psi = \frac{q_1}{2\pi}\theta_1 - \frac{q_2}{2\pi}\theta_2$$

$$= \frac{5}{2\pi}\times 0.322 - \frac{7}{2\pi}\times 0.785 = 0.256 - 0.874 = -0.618 \text{ m}^2/\text{s}$$

From the geometry of the above figure, we have

$$r_1 = \sqrt{(x+a)^2 + y^2} = \sqrt{(2+1)^2 + 1^2} = 3.162 \text{ m}$$

$$r_2 = \sqrt{(x-a)^2 + y^2} = \sqrt{(2-1)^2 + 1^2} = 1.414 \text{ m}$$

The equation of potential function for the combined source and sink pair is given by

$$\phi = \frac{q_1}{2\pi}\ln r_1 - \frac{q_2}{2\pi}\ln r_2$$

$$= \frac{5}{2\pi}\times \ln 3.162 - \frac{7}{2\pi}\times \ln 1.414 = 0.916 - 0.386 = 0.53 \text{ m}^2/\text{s}$$