Principles of Similarity

Q1. Choose the correct answer

(i) If there are n physical quantities and m fundamental dimensions describing a particular process, the number of independent non-dimensional parameters describing the process is

(a) $m+n$

(b) $n-m$

(c) $m\times n$

(d) m/n

[*Ans.(*b)]

(ii) The repeating variables in a dimensional analysis should

(a) be equal in number to that of the fundamental dimensions involved in the problem variables

(b) include the dependent variable

(c) have at least one variable containing all the fundamental dimensions

(d) collectively contain all the fundamental dimensions

[*Ans.*(a) and *(*d)]

(iii) The correct dimensionless group formed with the variables ρ (density), N (rotational speed), D (diameter) and μ (viscosity)is

(a) $\frac{\rho ND}{\mu}$

(b) $\frac{\rho ND^2}{\mu}$

(c) $\frac{ND}{\rho\mu}$

(d) $\frac{ND^2}{\rho\mu}$

[*Ans.(*b)]

(iv) In a similitude with gravity force, where equality of Froude number exists, the acceleration ratio a_r becomes

(a) L_r^2

(b) $L_r^{5/2}$

(c) 1

(d) $L_r^{3/2}$

where L_r is the geometrical scale factor.

[*Ans.(*c)]

Q2.

For rotodynamic fluid machines of a given shape, and handling an incompressible fluid, the relevant variables involved are D (the rotor diameter), Q (the volume flow rate through the machine), N (the rotational speed of the machine), gH (the difference of head across the machine, i.e., energy per unit mass), ρ (the density of fluid), μ (the dynamic viscosity of the fluid) and P (the power transferred between fluid and rotor). Show with the help of Buckingham's pi theorem that the relationship between the variables can be expressed by a functional form of the pertinent dimensionless parameters as

$$f\left(\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{P}{\rho N^3 D^5}, \frac{\mu}{\rho N D^2}\right) = 0$$

Solution

The problem is described by 7 variables as

$$F\left(P, N, D, \rho, Q, gH, \mu\right) = 0$$

These variables are expressed by 3 fundamental dimensions M, L, and T. Therefore, the number of π terms $= 7 - 3 = 4$

Using D, ρ and N as repeating variables, π terms can be written as

$$\pi_1 = D^{a_1}\rho^{b_1}N^{c_1}Q \qquad (1)$$

$$\pi_2 = D^{a_2}\rho^{b_2}N^{c_2}gH \qquad (2)$$

$$\pi_3 = D^{a_3}\rho^{b_3}N^{c_3}P \qquad (3)$$

$$\pi_4 = D^{a_4}\rho^{b_4}N^{c_4}\mu \qquad (4)$$

Substituting the variables of Eqs (1-4) in terms of their fundamental dimensions M, L, and T, we get

$$\mathrm{M^0L^0T^0} = \left(\mathrm{L}\right)^{a_1}\left(\mathrm{ML^{-3}}\right)^{b_1}\left(\mathrm{T^{-1}}\right)^{c_1}\mathrm{L^3T^{-1}} \qquad (5)$$

$$\mathrm{M^0L^0T^0} = \left(\mathrm{L}\right)^{a_2}\left(\mathrm{ML^{-3}}\right)^{b_2}\left(\mathrm{T^{-1}}\right)^{c_2}\mathrm{L^2T^{-2}} \qquad (6)$$

$$\mathrm{M^0L^0T^0} = \left(\mathrm{L}\right)^{a_3}\left(\mathrm{ML^{-3}}\right)^{b_3}\left(\mathrm{T^{-1}}\right)^{c_3}\mathrm{ML^2T^{-3}} \qquad (7)$$

$$\mathrm{M^0L^0T^0} = \left(\mathrm{L}\right)^{a_4}\left(\mathrm{ML^{-3}}\right)^{b_4}\left(\mathrm{T^{-1}}\right)^{c_4}\mathrm{ML^{-1}T^{-1}} \qquad (8)$$

Equating the exponents of M, L and T from Eq.(5), we have

$$b_1 = 0$$

$$a_1 + 3 = 0$$

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$$-c_1 - 1 = 0$$

which give $a_1 = -3,\ b_1 = 0,\ c_1 = -1$

Substituting these values into Eq. (1), we have

$$\pi_1 = \frac{Q}{ND^3}$$

Similarly, from Eq. (6), we get

$$b_2 = 0$$

$$a_2 - 3b_2 + 2 = 0$$

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$$-c_2 - 2 = 0$$

which give $a_2 = -2,\ b_2 = 0,\ c_2 = -2$

Substituting these values into Eq. (2), we have

$$\pi_2 = \frac{gH}{N^2D^2}$$

Equating the exponents of M, L and T from Eq.(7), we have

$$b_3 + 1 = 0$$
$$a_3 - 3b_3 + 2 = 0$$
$$-c_3 - 3 = 0$$

which give $a_3 = -5,\ b_3 = -1,\ c_3 = -3$

Substituting these values into Eq. (3), we have

$$\pi_3 = \frac{P}{\rho N^3 D^5}$$

Similarly, from Eq. (8), we get

$$b_4 + 1 = 0$$
$$a_4 - 3b_4 - 1 = 0$$
$$-c_4 - 1 = 0$$

which give $a_4 = -2,\ b_4 = -1,\ c_4 = -1$

Substituting these values into Eq. (4), we have

$$\pi_4 = \frac{\mu}{\rho N D^2}$$

Therefore, the problem can be expressed in terms of independent dimensionless parameters as

$$f\left(\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{P}{\rho N^3 D^5}, \frac{\mu}{\rho N D^2}\right) = 0$$

Q3.

A model of reservoir is completely drained in 5 minutes by means of a sluice gate. If the model is built to a scale of 1: 400, what time will be required to drain the prototype?

Solution

For dynamic similarity, Froude number should be same for model and prototype. From the equality of Froude number, one can write

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

where V_m = Velocity of fluid in model, L_m = linear dimension of the model, and V_p, and L_p are the corresponding values of velocity, and linear dimension in the prototype.

or $$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

Now, the time scale for the model and the prototype can be expressed as

$$\frac{T_m}{T_p} = \frac{\left(\frac{L}{V}\right)_m}{\left(\frac{L}{V}\right)_p} = \frac{L_m}{L_p} \times \frac{V_p}{V_m}$$

$$= \frac{L_m}{L_p} \times \sqrt{\frac{L_p}{L_m}}$$

or $$T_p = T_m \sqrt{\frac{L_p}{L_m}}$$

Substituting $T_m = 5$ min and $L_p / L_m = 400$ in the above equation, we get

$$T_p = 5 \times \sqrt{400} = 100 \text{ min}$$