Application of Viscous Flow through Pipes

Q1. Choose the correct answer

(i) For pipes arranged in series

(a) the flow may be different in different pipes
(b) the head loss per unit length must be more in a larger pipe
(c) the head loss must be the same in all pipes
(d) the flow rate must be the same in all pipes

[*Ans.(*d)]

(ii) In parallel pipe systems

(a) the pipes must be placed geometrically parallel to each other
(b) the head loss per unit length must be same for all pipes
(c) the head loss across each of the parallel pipes must be the same
(d) the flow must be the same in all pipes

[*Ans.(*c)]

Q2.

Water at 20°C flows through a 500 m long cast-iron pipe of 50 mm diameter at 0.005 m^3/s. Determine the loss of head in friction. Also calculate the pumping power required to maintain the flow. Average surface roughness for cast-iron pipe is 0.25 mm. Kinematic viscosity of water at 20°C is $10^{-6}\ m^2/s$.

Solution

Average velocity of flow is

$$V = \frac{0.005}{\frac{\pi}{4}(0.05)^2} = 2.55 \text{ m/s}$$

Therefore, Reynolds number is

$$\text{Re} = \frac{VD}{\upsilon} = \frac{2.55 \times 0.05}{10^{-6}} = 1.275 \times 10^5$$

Relative roughness is $\frac{\varepsilon}{D} = \frac{0.25}{50} = 0.005$

From Moody's diagram, friction factor $f = 0.0095$

The loss of head due to friction is given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Substituting the respective values, we obtain

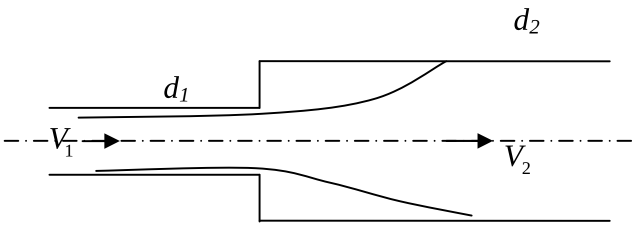
$$h_f = 0.0095 \frac{500}{0.05} \frac{2.55^2}{2 \times 9.81} = 31.48 \text{ m}$$

Q3.

There is a sudden increase in the diameter of a pipe from d_1 to d_2. What would be the ratio d_2/d_1 if the minor loss is independent of the direction of flow? Assume coefficient of contraction $C_c = 0.6$.

Solution

Consider flow of a constant density fluid through a pipe as shown in the figure below.



Let the velocities corresponding to diameters d_1 and d_2 be V_1 and V_2 respectively.
From continuity equation, we have

$$Q = A_1V_1 = A_2V_2$$

or
$$V_2 = \frac{A_1}{A_2}V_1 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}\times V_1 = \frac{d_1^2}{d_2^2}V_1$$

Loss of head due to sudden enlargement is given by

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{d_1^2}{d_2^2}V_1\right)^2}{2g} = \left(1 - \frac{d_1^2}{d_2^2}\right)^2 \frac{V_1^2}{2g}$$

If the direction of flow is reversed, there will be a sudden contraction from d_2 to d_1. Then the loss of head due to sudden contraction is given by

$$h_c = \frac{V_1^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{V_1^2}{2g}\left[\frac{1}{0.6} - 1\right]^2 = 0.444\frac{V_1^2}{2g}$$

When the minor loss is independent of the direction of flow, the loss of head due to sudden enlargement should be equal to the loss of head due to sudden contraction. Therefore, we have

$$\left(1 - \frac{d_1^2}{d_2^2}\right)^2 \frac{V_1^2}{2g} = \frac{V_1^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

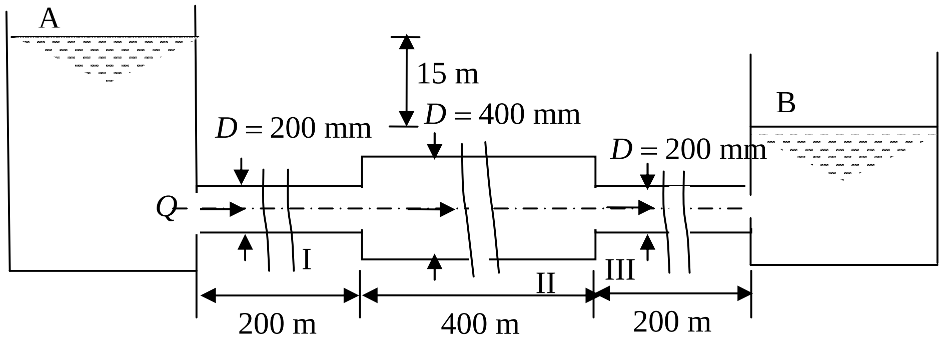
or
$$\left(1 - \frac{d_1^2}{d_2^2}\right) = \frac{1}{C_c} - 1$$

or
$$\frac{d_1}{d_2} = \left(2 - \frac{1}{C_c}\right)^{1/2} = \left(2 - \frac{1}{0.6}\right)^{1/2} = \left(\frac{1}{3}\right)^{1/2}$$

or
$$\frac{d_2}{d_1} = \sqrt{3}$$

Q4.

Three pipes of 200 mm, 400 mm and 200 mm diameters and having lengths of 200 m, 400 m and 200 m, respectively are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference in water levels is 15 m, as shown in the figure below. If the friction factor f, for all the pipes is same and equal to 0.02, determine the discharge through the compound pipe neglecting first the minor losses and then including them. Take coefficient of contraction as 0.6.



Solution

Applying energy equation between sections A and B gives

$$\frac{p_{atm}}{\rho g}+0+15=\frac{p_{atm}}{\rho g}+0+0+h_f$$

or $\quad h_f = 15$ m

Let Q be the volumetric rate of discharge through the pipelines. Then,

The velocity of flow in pipe I $\quad = \dfrac{4Q}{\pi(0.2)^2}=31.83Q$

The velocity of flow in pipe II $\quad = \dfrac{4Q}{\pi(0.4)^2}=7.96Q$

The velocity of flow in pipe III $\quad = \dfrac{4Q}{\pi(0.2)^2}=31.83Q$

When minor losses are not considered, the loss of head h_f, in the course of flow from A to B constitutes of the friction losses in three pipes only, and can be written as

$$h_f = \left[0.02\times\frac{200}{0.2}\times\frac{(31.83)^2}{2g}+0.02\times\frac{400}{0.4}\times\frac{(7.96)^2}{2g}+0.02\times\frac{200}{0.2}\times\frac{(31.83)^2}{2g}\right]Q^2$$

or $\quad h_f = 2130.13Q^2$

which gives $\quad Q = 0.084\ \text{m}^3/\text{s}$

When minor losses are not considered, one can write

$$h_f = 15 = 0.5\times\frac{(31.83Q)^2}{2g}+0.02\times\frac{200}{0.2}\times\frac{(31.83Q)^2}{2g}+\frac{(31.83Q-7.96Q)^2}{2g}$$

$$+0.02\times\frac{400}{0.4}\times\frac{(7.96Q)^2}{2g}+\left(\frac{1}{0.6}-1\right)^2\times\frac{(7.96Q)^2}{2g}+0.02\times\frac{200}{0.2}\times\frac{(31.83Q)^2}{2g}+\frac{(31.83Q)^2}{2g}$$

or $\quad 15 = 2238.06Q^2$

which gives $Q = 0.082\ \text{m}^3/\text{s}$