Fluid Flow Applications

Q1. **Choose the correct answer**

(i) Bernoulli's equation relates
- (a) various forms of mechanical energy
- (b) various forces involved in fluid flow
- (c) torque to change in angular momentum
- (d) various forces with change in momentum

[*Ans.(*a)]

(ii) When is Bernoulli's equation applicable between any two points in a flow field?
- (a) The flow is steady, constant density and rotational
- (b) The flow is unsteady, constant density and irrotational
- (c) The flow is steady, variable density and rotational
- (d) The flow is steady, constant density and irrotational

[*Ans.(*d)]

(iii) A stagnation point is a point in fluid flow where
- (a) total energy is zero
- (b) pressure is zero
- (c) velocity of flow is zero
- (d) total energy is maximum

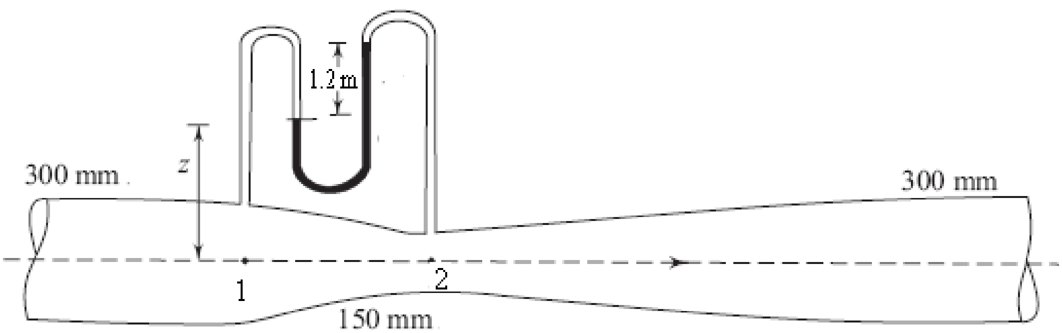
[*Ans.(*c)]

(iv) It is recommended that the diffuser angle of a venturimeter should be kept less than 6° because
- (a) pressure decreases in flow direction and flow separation may occur
- (b) pressure decreases in flow direction and flow may become turbulent
- (c) pressure increases in flow direction and flow separation may occur
- (d) pressure increases in flow direction and flow may become turbulent

[*Ans.(*c)]

Q2.

Water flows through a $300\text{ mm}\times150\text{ mm}$ horizontal venturimeter at the rate of $=0.065\text{ m}^3/\text{s}$ and the differential gauge is deflected 1.2 m, as shown in the figure below. Specific gravity of the manometric liquid is 1.6. Determine the coefficient of discharge of the venturimeter.



Solution

Applying Bernoulli's equation between 1 and 2, along a streamline, we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+0=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+0$$

or $$V_2^2-V_1^2=\frac{2\left(p_1-p_2\right)}{\rho}$$

Applying continuity equation between 1 and 2, one can write

$$A_1V_1=A_2V_2$$

or, $$V_2=\frac{A_1}{A_2}V_1$$

Substituting the expression of V_2 in the above equation, we have

$$V_1=\sqrt{\frac{2\left(p_1-p_2\right)/\rho}{\left(A_1/A_2\right)^2-1}}$$

The actual rate of discharge Q can be written as

$$Q=C_dA_1V_1$$

$$=C_dA_1\sqrt{\frac{2\left(p_1-p_2\right)/\rho}{\left(A_1/A_2\right)^2-1}}$$

where C_d is the coefficient of discharge.

From the principle of hydrostatics applied to the differential gauge, we obtain

$$\frac{p_1}{\rho g}-z=\frac{p_2}{\rho g}-(z+1.2)+1.6\times1.2$$

or $$\frac{p_1-p_2}{\rho g}=0.72 \text{ m}$$

Substituting the respective values in the expression of the discharge, we get

$$0.065=C_d\,\frac{\pi}{4}\left(0.3\right)^2\sqrt{\frac{2\times9.81\times0.72}{16-1}}$$

or $$C_d=0.95$$

Q3.

A vertical venturimeter is fitted with a circular pipe of diameter 30 cm. Diameter of the throat of the venturimeter is 15 cm. The loss of head from the entrance to the throat is $1/6$ times the throat velocity head. The difference in reading of the two limbs of the differential mercury-manometer is 30 cm. Determine the quantity of water flowing through the pipe.

Solution

Applying continuity equation between 1 and 2, we have

$$A_1V_1=A_2V_2$$

or $$V_2=\frac{A_1}{A_2}V_1$$

or $$V_2 = \frac{\frac{\pi}{4}D_1^2}{\frac{\pi}{4}D_1^2}V_1 = \frac{D_1^2}{D_1^2}V_1 = \frac{0.3^2}{0.15^2}V_1 = 4V_1$$

Applying energy equation between 1 and 2, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

or $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{1}{6}\frac{V_2^2}{2g} \qquad \left[\because h_f = \frac{1}{6}\frac{V_2^2}{2g}\right]$$

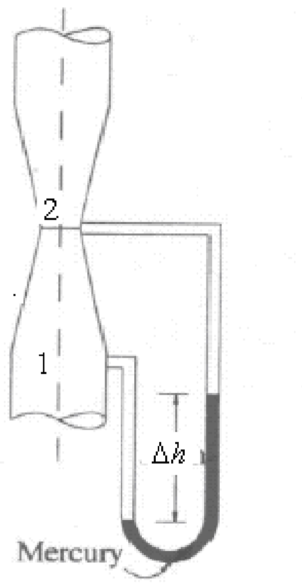
or $$\frac{7}{6}\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 2g\left[\frac{(p_1 - p_2)}{\rho g} + (z_1 - z_2)\right]$$

or $$\frac{7}{6}\frac{(4V_1)^2}{2g} - \frac{V_1^2}{2g} - \frac{V_1^2}{2g} = 2g\left[\frac{(p_1 - p_2)}{\rho g} + (z_1 - z_2)\right] \qquad [\because V_2 = 4V_1]$$

or $$\frac{53V_1^2}{6g} = 2g\left[\frac{(p_1 - p_2)}{\rho g} + (z_1 - z_2)\right]$$

From the principle of hydrostatics applied to the differential manometer (shown in the figure below), we obtain

$$\frac{(p_1 - p_2)}{\rho g} + (z_1 - z_2) = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$$



Substituting the value of piezometric pressure difference in the above equation, we have

or $$\frac{53V_1^2}{6g} = 2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h$$

Substituting the respective values, we obtain

$$V_1 = 9.075 \text{ m/s}$$

The flow rate through the venturimeter is then

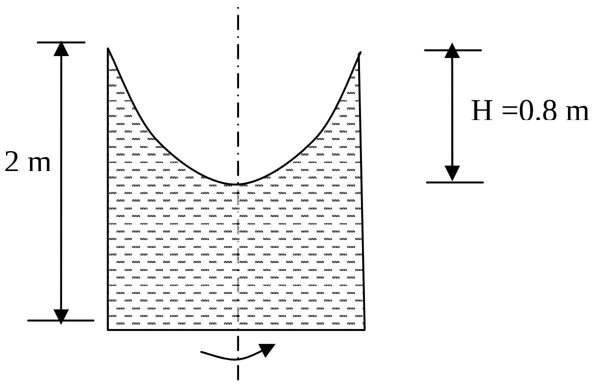
$$Q = A_1V_1 = \frac{\pi}{4}(0.3)^2 \times 9.075 = 0.6415 \text{ m}^3/\text{s}$$

Q4.

An open vertical cylindrical tank 1 m in diameter and 2 m high contains 1.6 m of water. If the tank rotates about its vertical axis, find the maximum rotational speed that can be attained without spilling any water.

Solution

Let ω be the maximum angular velocity of the tank at which there is no spilling of water.



The pressure differential can be expressed as

$$dp = \rho\omega^2 r dr - \rho g dz$$

Consider two points 1(at the centre) and 2 (at the outer wall) lie on the free surface of the liquid. Integrating the above equation between points 1 and 2, we have

$$\int_1^2 dp = \int_1^R \rho\omega^2 r dr - \int_1^2 \rho g dz$$

or $$0 = \frac{\rho\omega^2 R^2}{2} - \rho g H$$

or $$H = \frac{\omega^2 R^2}{2g}$$

Rise of water at the outer wall of the tank = Height of tank-initial height of water in the tank $= 2 - 1.6 = 0.4$ m

Since, the rise of water at ends is equal to the fall of liquid at centre, the fall of liquid at centre =0.4 m

Therefore, $H = 0.8$ m

Substituting the values of H and R in the above equation, we obtain

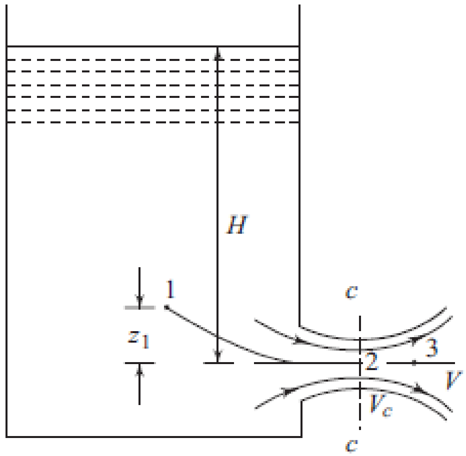
$$\omega = 7.924 \text{ rad/s}$$

Therefore, the maximum rotational speed that can be attained by the tank without spilling any water is

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 7.924}{2\pi} = 75.67 \text{ rpm}$$

Q5.

A convergent-divergent mouthpiece is fitted to the vertical side of a tank containing water as shown in the figure below. There is no loss in the convergent part of the mouthpiece, while the head loss in the divergent part is 0.2 times the velocity head at exit. The diameter at the vena contracta is 25 mm and the head in the tank above the centre-line of the mouthpiece is 2 m. What is the maximum discharge that can be drawn through the outlet and what should be the corresponding diameter at the outlet. Assume that the pressure in the system may be permitted to fall to 2.5 m of water absolute pressure head. Atmospheric pressure head =10.3 m of water.



Solution

Applying Bernoulli's equation between the points 1 (inside tank) and 2 (at throat), along a streamline (refer to the above figure), we have

,
$$\frac{p_{atm}}{\rho g} + (H - z_1) + 0 + z_1 = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + 0$$

or
$$\frac{V_c^2}{2g} = \frac{p_{atm}}{\rho g} + H - \frac{p_c}{\rho g}$$

For a maximum velocity V_c, the pressure p_c will attain its lower limit which is2.5 m of water absolute. Therefore,

$$\frac{V_c^2}{2g} = 10.3 + 2 - 2.5 = 9.8$$

or
$$V_c = \sqrt{2 \times 9.81 \times 9.8} = 13.866 \text{ m/s}$$

Therefore, the maximum possible discharge is

$$Q = a_c V_c = \frac{\pi}{4}(0.025)^2 \times 13.866 = 0.0068 \text{ m}^3\text{/s}$$

Applying Bernoulli's equation between the points 1 and 3 (at exit), along a streamline (refer to the above figure), we have

or,
$$\frac{p_{atm}}{\rho g} + (H - z_1) + 0 + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_3^2}{2g} + 0 + 0.2\frac{V_3^2}{2g}$$

where V_3 is the exit velocity.

From the above equation, we obtain

, $$V_3 = 5.718 \text{ m/s}$$

Let d_3 be the exit diameter of the mouthpiece. Then, the discharge can also be expressed as

$$0.0068 = \frac{\pi}{4} d_3^2 V_3$$

or $$d_3 = 0.039 \text{ m} = 39 \text{ mm}$$