# Kinematics of Fluid

Q1. Choose the correct answer

(i) A streamline is a line

(a) which is along the path of a particle
(b) drawn normal to the velocity vector at any point
(c) such that the streamlines divide the passage into equal number of parts
(d) on which tangent drawn at any point gives the direction of velocity

[*Ans.(*d)]

(ii) Streamline, pathline and streakline are identical when

(a) the flow is uniform
(b) the flow is steady
(c) the flow velocities do not change steadily with time
(d) the flow is neither steady nor uniform.

[*Ans.(*b)]

(iii) The material acceleration is zero for a

(a) steady flow
(b) uniform flow
(c) steady and uniform flow
(d) unsteady and non-uniform flow

[*Ans.(*c)]

(iv) In a two dimensional flow in x-y plane, if $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$ (*u* and *v* are the velocity components in the *x* and *y* directions respectively) then the fluid element will undergo

(a) translation only
(b) translation and rotation
(c) translation and deformation
(d) rotation and deformation

[*Ans.(*c)]

Q2.

The velocity field in a fluid flow is given by

$$\vec{V} = x^2 t\hat{i} + 2xyt\hat{j} + 2yzt\hat{k}$$

Evaluate the acceleration of a fluid particle at (2,-1, 1) at $t = 1$ s.

***Solution***

Acceleration is given by

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V}$$

Here, $\vec{V} = x^2 t\hat{i} + 2xyt\hat{j} + 2yzt\hat{k}$

Hence, $\frac{\partial \vec{V}}{\partial t} = x^2\hat{i} + 2xy\hat{j} + 2yz\hat{k}$

$$\left(\vec{V}.\nabla\right)\vec{V} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\vec{V}$$

$$= \left( u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \right)\left[ x^2 t\hat{i} + 2xyt\hat{j} + 2yzt\hat{k} \right]$$
$$= u\left(2xt\hat{i} + 2yt\hat{j}\right) + v\left(2xt\hat{j} + 2zt\hat{k}\right) + w\left(2yt\hat{k}\right)$$
$$= x^2 t\left(2xt\hat{i} + 2yt\hat{j}\right) + 2xyt\left(2xt\hat{j} + 2zt\hat{k}\right) + 2yzt\left(2yt\hat{k}\right)$$
$$= 2x^3t^2\hat{i} + 6x^2yt^2\hat{j} + \left(4xyzt^2 + 4y^2zt^2\right)\hat{k}$$

Finally, the acceleration field can be expressed as

$$\vec{a} = x^2\hat{i} + 2xy\hat{j} + 2yz\hat{k} + 2x^3t^2\hat{i} + 6x^2yt^2\hat{j} + \left(4xyzt^2 + 4y^2zt^2\right)\hat{k}$$
$$= \left(x^2 + 2x^3t^2\right)\hat{i} + \left(2xy + 6x^2yt^2\right)\hat{j} + \left(2yz + 4xyzt^2 + 4y^2zt^2\right)\hat{k}$$

The acceleration vector at the point (2,-1, 1) and at time $t = 1$ s can be found by substituting the values of *x*,*y*,*z* and *t* in the above expression as

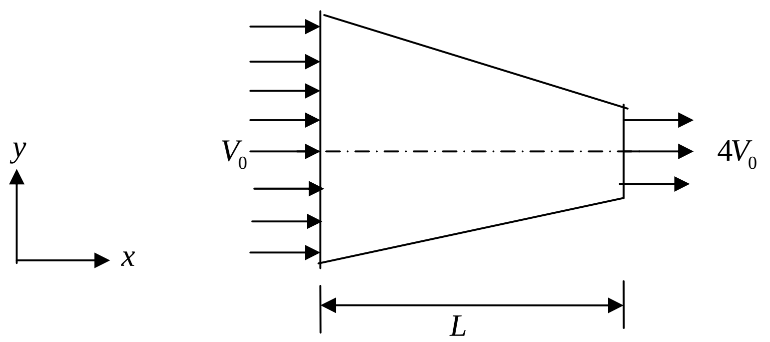
$$\vec{a} = \left[\left(2^2\right) - 2\left(2^3\right)\left(1^2\right)\right]\hat{i} + \left[2(2)(-1) + 6\left(2^2\right)(-1)\left(1^2\right)\right]\hat{j}$$
$$+ \left[2(-1)(1) + 4(2)(-1)(1)\left(1^2\right) + 4(-1)^2(1)\left(1^2\right)\right]\hat{k}$$
$$= -12\hat{i} - 28\hat{j} - 14\hat{k}$$

Q3.

Fluid flows steadily through a converging nozzle of length *L*. Flow can be approximated as one-dimensional such that the axial velocity varies linearly from entrance to exit. The velocities at entrance and exit are $V_0$ and $4V_0$ respectively. Find out an expression of the acceleration of a particle flowing through the nozzle.

***Solution***

The converging nozzle is shown in the figure below.




Since the axial velocity (*u*) varies linearly, let us consider

$$u = Ax + B$$

where *A*, and *B* are constants and their values are to be determined from the boundary conditions as given below.

The appropriate boundary conditions are (refer to the above figure)

At *x*=*0*, $u = V_0$, and

At *x*=*L*, $u = 4V_0$

Applying the boundary conditions, the constants are found to be

$$A = \frac{3V_0}{L} \text{ and } B = V_0$$

Therefore, the velocity field can be expressed as

$$u = \frac{3V_0}{L}x + V_0 = V_0\left(1 + \frac{3x}{L}\right)$$

Hence, $$\frac{\partial u}{\partial x} = \frac{3V_0}{L}$$

For steady, one-dimensional flow, acceleration can be written as

$$a = u\frac{\partial u}{\partial x}$$

For the given velocity field, the acceleration can be expressed as

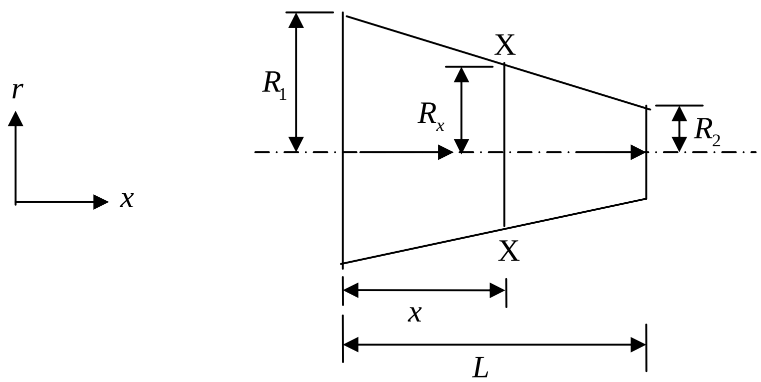
$$a = u\frac{\partial u}{\partial x} = V_0\left(1 + \frac{3x}{L}\right) \times \frac{3V_0}{L} = \frac{3V_0^2}{L}\left(1 + \frac{3x}{L}\right)$$

Q4.

Fluid flows at a constant rate of $Q$ through a convergent pipe of length L having inlet and outlet radii of $R_1$ and $R_2$ respectively. Assuming that the velocity to be axial and uniform at any cross section, find out the acceleration at the exit.

**Solution**

Consider a section XX, at a distance $x$ from the inlet as shown in the figure below.




Radius of the pipe at section XX is given by

$$R_x = R_1 + \frac{R_2 - R_1}{L}x$$

Velocity at section XX can be written as

$$u = \frac{Q}{\pi R_x^2} = \frac{Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^2}$$

$$\frac{\partial u}{\partial x} = -\frac{2Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^3}\frac{R_2 - R_1}{L}$$

The acceleration field can be expressed as

$$a_x = u\frac{\partial u}{\partial x}$$

$$= \frac{Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^2} \times \frac{-2Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^3}\frac{R_2 - R_1}{L}$$

Acceleration at the exit is then

$$a_x\big|_{x=L} = \frac{Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}L\right)^2} \times \frac{-2Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}L\right)^3}\frac{R_2 - R_1}{L}$$

$$= \frac{2Q^2\left(R_1 - R_2\right)}{\pi^2 L R_2^5}$$

Q5.

A three-dimensional velocity field is given by $u = -x$, $v = 2y$, $w = 5 - z$. Find the equation of streamline through (2,2,1).

**Solution**

The equation of a streamline in three-dimensional flow is

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Here, $u = -x$

$v = 2y$

$w = 5 - z$

Streamline in the *xy*-plane is given by

$$\frac{dx}{u} = \frac{dy}{v}$$

or $$\frac{dx}{-x} = \frac{dy}{2y}$$

or $$\frac{1}{2}\ln y = -\ln x + \ln\ C_1$$

or $$y^{\frac{1}{2}}x = C_1$$

Equation of streamline passing through point (2,2,1) is

$$y^{\frac{1}{2}}x = 2\sqrt{2}$$

or $$x^2 y = 8$$

Streamline in the *yz*-plane is given by

$$\frac{dy}{v} = \frac{dz}{w}$$

or $$\frac{dy}{2y} = \frac{dz}{5-z}$$

or $$\frac{1}{2}\ln y = -\ln(5-z) + \ln C_2$$

or $$y^{\frac{1}{2}}(5-z) = C_2$$

Equation of streamline passing through point (2,2,1) is

$$y^{\frac{1}{2}}(5-z) = 4\sqrt{2}$$

or $$y(5-z)^2 = 32$$

Q6.

A three-dimensional velocity field is given by

$$u(x,y,z) = Ax + 2By + C$$

$$v(x,y,z) = Ay + D$$

$$w(x,y,z) = -2Az + E$$

where $A$, $B$, $C$, $D$, $E$ are constants.

Find the components of

(a) the strain rates for the above velocity field

(b) the rotational velocity, and

(c) the vorticity

**Solution**

(a) Rate of linear strain along *x* direction is

$$\dot{\varepsilon}_x = \frac{\partial u}{\partial x} = A$$

Rate of linear strain along *y* direction is

$$\dot{\varepsilon}_y = \frac{\partial v}{\partial y} = A$$

Rate of linear strain along *x* direction is

$$\dot{\varepsilon}_z = \frac{\partial w}{\partial z} = -2A$$

Rate of volumetric strain is

$$\dot{\varepsilon}_{\text{vol}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = A + A - 2A = 0$$

The shear strain rates are found to be

$$\dot{\gamma}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 2B + 0 = 2B$$

$$\dot{\gamma}_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 + 0 = 0$$

$$\dot{\gamma}_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 + 0 = 0$$

(b) The components of the rotational velocity are as follows

$$\dot{\omega}_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 2A) = -A$$

$$\dot{\omega}_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}(0 - 0) = 0$$

$$\dot{\omega}_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}(0 - 0) = 0$$

(c) The components of the vorticity are as follows

$$\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 2A = -2A$$

$$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 - 0 = 0$$

$$\Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0 - 0 = 0$$

Q7.

The velocity field in a fluid medium is given by $\vec{V} = 3xy^2\hat{i} + 2xy\hat{j} + (2zy + 3t)\hat{k}$. Determine the rotational velocity vector at the point $(1, 2, 1)$ and at time $t = 3$.

**Solution**

Given $u = 3xy^2$, $v = 2xy$ and $w = (2zy + 3t)$

For a two-dimensional flow, rotation is given by

$$\dot{\omega}_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\dot{\omega}_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\dot{\omega}_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

For the given velocity field

$$\frac{\partial u}{\partial y} = 6xy, \ \frac{\partial v}{\partial x} = 2y, \ \frac{\partial w}{\partial y} = 2z, \ \frac{\partial v}{\partial z} = 0, \ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial x} = 0$$

Thus,

$$\dot{\omega}_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(2y - 6xy) = y - 3xy$$

$$\dot{\omega}_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}(2z - 0) = z$$

$$\dot{\omega}_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}(0 - 0) = 0$$

At point $(1, 2, 1)$ and time $t = 3$, we have

$$\dot{\omega}_z = 2 - 3 \times 1 \times 2 = -4$$

$$\dot{\omega}_x = 1$$
$$\dot{\omega}_y = 0$$

The rotational velocity vector at the point $(1,2,1)$ and at time $t = 3$ is

$$\vec{\omega} = -4\hat{i} + \hat{j}$$