

Vibrations of Structures

Module III: Vibrations of Beams

Exercises

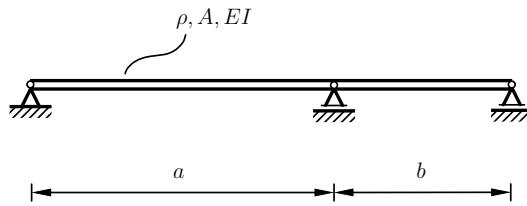


Figure 1: Exercise 1

1. A uniform Euler-Bernoulli beam of length $a + b$, flexural stiffness EI and linear density ρA is supported, as shown in Fig. 1. Plot the variation of the first few eigenfrequencies for variation of b in the range $(0, a)$.

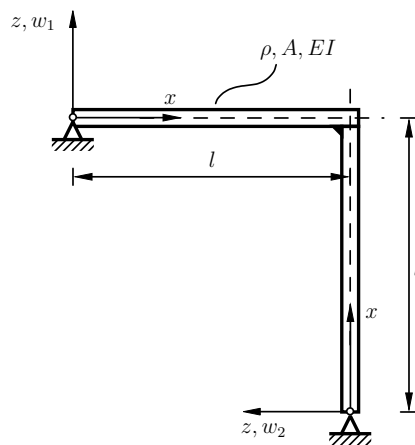


Figure 2: Exercise 2

2. The frame shown in Fig. 2 is made by welding two uniform homogeneous Euler-Bernoulli beams infinitely stiff in tension. Determine the first few eigenfrequencies and the corresponding modes of vibration of the frame.

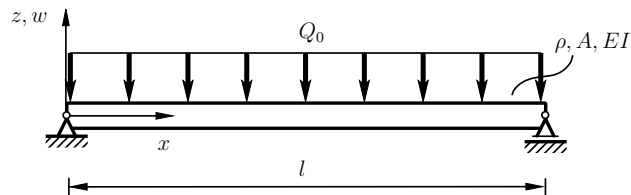


Figure 3: Exercise 3

3. A simply-supported uniform beam is loaded by a constant distributed force $q(x, t) = Q_0$, as shown in Fig. 3. Determine the motion of the beam when the force is suddenly removed at $t = 0$.

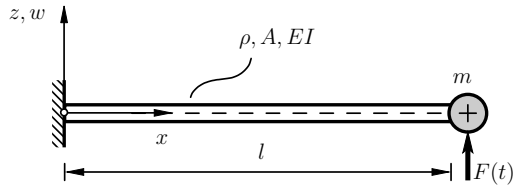


Figure 4: Exercise 4

4. Determine the steady-state response of a uniform cantilever beam with end-mass when excited by a force $F(t) = F_0 \cos \Omega t$, as shown in Fig. 4.

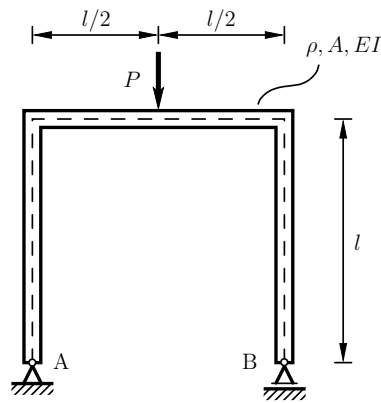


Figure 5: Exercise 5

5. A frame shown in Fig. 5 is made of three uniform beams each having flexural rigidity EI , linear density ρA . A constant force P acts at the center of the horizontal beam as shown. Estimate the first eigenfrequency of the frame $\omega_1(P)$ as a function of the force P . Re-calculate the first eigenfrequency if the frame is pinned at the support B.

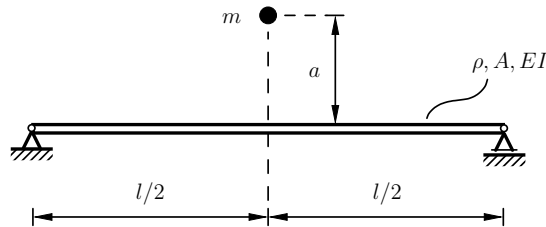


Figure 6: Exercise 6

6. A particle of mass m falls from height h on the center of a simply-supported uniform beam, as shown in Fig. 6. Determine the reaction forces at the particle-beam interface and at the supports as a function of time. When does the particle leave the beam?

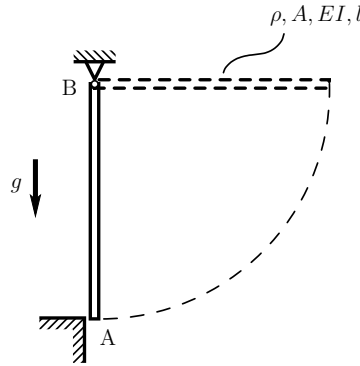


Figure 7: Exercise 7

7. A uniform beam, pinned at one end, is released from rest from a horizontal position, as shown in Fig. 7. The free-end A of the beam hits a rigid edge when the beam reaches a vertical position. Determine the reaction forces at the ends A and B as functions time.
8. A simply-supported beam of rectangular cross-section has a constant width b , while the height varies from $h/2$ to h over its length l . The material has density ρ , and Young's modulus E . Estimate the first two modal frequencies and modes of vibration.

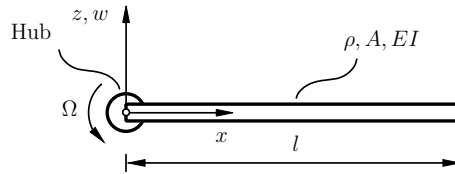


Figure 8: Exercise 9

9. A uniform Euler-Bernoulli beam is fixed to a small hub rotating at a constant angular speed Ω in the vertical plane, as shown in Fig. 8, has a Lagrangian

$$\mathcal{L} = \frac{1}{2} \int_0^l [\rho A [(\Omega x + w_t)^2 + \Omega^2 w^2] - T w_{,x}^2 - EI w_{,xx}^2] dx,$$

where $dT/dx = \rho A \Omega^2 x$. Derive the equation of motion for transverse vibrations of the beam. Using the Ritz method, estimate the first few eigenfrequencies of the beam, and plot their variation with Ω .

10. A uniform belt under constant tension T passes at a constant speed v between two roller-stands separated by a distance l , as shown in Fig. 9. Modeling the belt as an Euler-Bernoulli

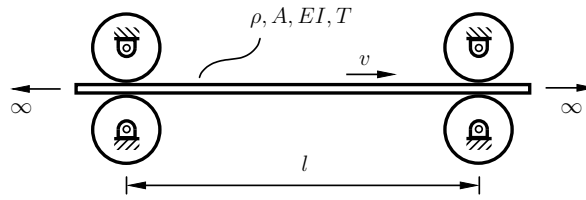


Figure 9: Exercise 10

beam, derive the equation of motion. Assuming clamped boundary condition at the rollers, estimate the first critical speed of the belt.