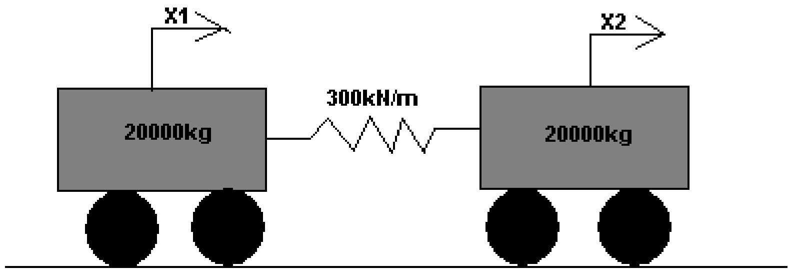
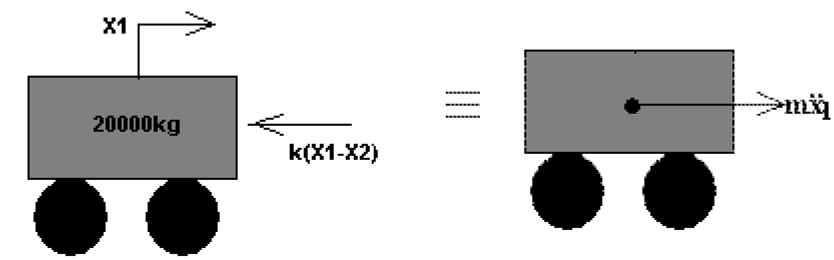
10.1 An electrical train made up of two cars , each weighing 20000kg is connected by couplings of stiffness equal to 300kN/m. Dteremine the natural frequency of the system.

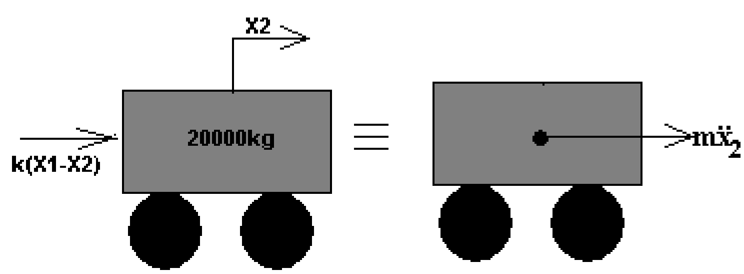


FBD of mass 1



$$m\ddot{x}_1 + k(x_1 - x_2) = 0$$

FBD of mass 2



$$m\ddot{x}_2 + k(x_1 - x_2) = 0$$

writing the equations in matrix form

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} k & -k \\ -k & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$x_1 = X_1 \sin \omega t, \ddot{x}_1 = -\omega^2 X_1 \sin \omega t$$
$$x_2 = X_2 \sin \omega t, \ddot{x}_2 = -\omega^2 X_2 \sin \omega t$$

substituting these in above equation, we get,

$$-\omega^2\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} k & -k \\ -k & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

taking inverse we get

$$\omega^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{k}{m} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

characteristic equation

$$\begin{vmatrix} \frac{k}{m}-\lambda & -\frac{k}{m} \\ -\frac{k}{m} & \frac{k}{m}-\lambda \end{vmatrix} = 0$$

Solving the above equation we get

$$\lambda = 0, \frac{2k}{m}$$

ie ω_n =0, 5.47c/s

The first frequency is zero implying that the two masses move together resulting in a rigid body like motion which does not cause any stretch/compression in the spring.