

Q 7.1 Flywheel weighing 30kg was allowed to swing as a pendulum about a knife-edge at inner side of the rim. If the measured period of oscillations was 1.22s, determine the moment of inertia of flywheel about its geometric axis.

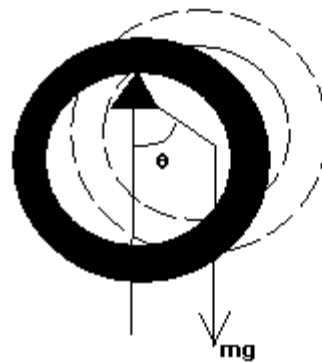


Flywheel

Given: mass of flywheel  $m=30\text{kg}$

Time period  $T=1.22\text{s}$

Let the centre of mass be deviated by an angle  $\theta$ , very small. Writing torque equations about pivot point



$$j_p \ddot{\theta} + mgr \sin \theta = 0$$

$$j_p = j_{cm} + mr^2$$

Icm of flywheel =  $mr^2$  (hollow disk)

Substituting in the equation above

$$j_p = 2mr^2$$

therefore

$$2mr^2 \ddot{\theta} + mgr \sin \theta = 0$$

since  $\theta$  is very less,  $\sin \theta \approx \theta$

$$2mr^2 \ddot{\theta} + mgr \theta = 0$$

$$\ddot{\theta} + \frac{g}{2r} \theta = 0$$

also

$$\frac{2\pi}{T} = \sqrt{\frac{g}{2r}}$$

solving we get

=

$$r = \frac{gT^2}{8\pi^2} = \frac{9.81 \cdot 1.22^2}{8 \cdot 3.1416^2} = 0.185m$$

Moment of inertia about  $I_{cm} = mr^2 = 30 \cdot 0.185^2$

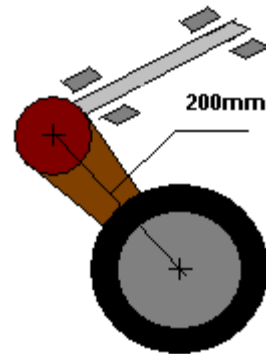
Q.7.2 A steel shaft 1.5 m long and 20mm in diameter is used as a torsion spring for the wheels of a light automobile. Determine the natural frequency of the system if the weight of wheel and tyre assembly and its radius of gyration about its axle is 50mm. Discuss the difference in natural frequency with wheel locked and when unlocked to the arm.

Given : length of shaft = 1.5m

Diameter = 30mm

Mass of wheel and tyre assembly = 25kg

Radius of gyration about axle = 50mm



Wheel Tyre assembly

Assume pure

Case I

When wheel tyre assembly is locked at a distance of 0.2m

$I = mr^2$  moment of inertia about shaft axis

$$J = 25 \times (0.2)^2 = 25 \times 0.04 = 1 \text{ kgm}^2$$

$$K_t = \frac{GI_p}{l} = \frac{0.8 \times 10^{11} \times \frac{\pi}{32} (0.02)^4}{1.5}$$

$$\text{Polar moment} = \left[ \frac{\pi}{32} d^4 \right] = \frac{\pi}{32} \times (0.02)^4$$

$K_t = 837.76 \text{ Nm/rad}$

$$\omega = \sqrt{\frac{K_t}{J}} = \sqrt{\frac{837.76}{1}} = 28.9 \text{ c/s}$$

Case II

displacement during motion  $|\bar{y}| = \gamma\theta$

$|\bar{y}| = y_{cm}$  of the wheel

$$y_{cm} = l\phi = \gamma\theta$$

Now the total energy is constant

$$\text{KE of wheel about its axis} = \frac{1}{2} I_{cm} \dot{\theta}^2$$

$$\text{KE of wheel about the shaft} = \frac{1}{2} I_{ws} \dot{\theta}^2$$

$$\text{PE in wheel} = \frac{1}{2} K_t \phi^2$$

$$TE = \frac{1}{2} I_{cm} \dot{\theta}^2 + \frac{1}{2} I_{ws} \dot{\theta}^2 + \frac{1}{2} K_t \phi^2$$

$$\theta = \frac{\ell}{\gamma} \phi$$

$$\Rightarrow \frac{1}{2} I_m \left( \frac{\ell}{\gamma} \right)^2 \dot{\phi}^2 + \frac{1}{2} I_{ws} \dot{\phi}^2 + \frac{1}{2} K_t \phi^2$$

differentiate w.r.t. time

$$\left[ I_m \left( \frac{\ell}{\gamma} \right)^2 + I_{ws} \right] \ddot{\phi} + K_t \phi = 0$$

$$\omega_n = \sqrt{\frac{K_t}{I_m \left( \frac{\ell}{\gamma} \right)^2 + I_{ws}}}$$