# ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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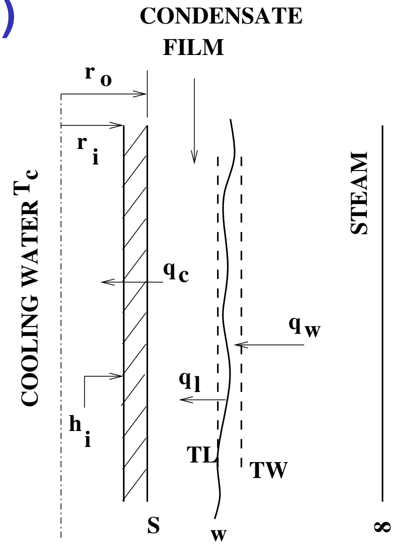
LECTURE-40 CONV M T - REYNOLDS FLOW MODEL - 2

# LECTURE-40 CONV M T - REYNOLDS FLOW MODEL - 2

1. Condensation
2. Transpiration Cooling
3. Volatile Fuel Burning
4. Drying
5. Solid Dissolution in Liquid

# Condensation - L40($\frac{1}{15}$)

**Prob:** Consider condensation of steam at 1 atm on the outside of a Copper tube ( 2.5 cm ID and 2.9 cm OD ). The tube carries cooling water at $50^0$C
Calculate steam condensation rate when ( a ) steam is pure and saturated and ( b ) steam is mixed with 20 % air by mass .
Assume condensate film thickness $\delta$ = 0.125 mm,
$k_{cu}$ = 300 W/m-K,
$k_{water}$ = 0.68 W/m-K
$h_{cof,i}$ = 4620 W/$m^2$-K,
$\lambda_{ref}$ = 2257 kJ/kg and $T_{ref} = T_w$




$h_{cof,o}$ = 115 W/$m^2$-K
( single phase )

# Theory of Condensation - L40($\frac{2}{15}$)

Here

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l / N_w} = \frac{N_w}{g}$$

If we take $T_{ref} = T_w$ then $h_{TL}$ = 0, $h_{m,w} = \lambda_{ref} \times \omega_{v,w}$ and $h_{m,\infty} = c_{pm} \, (T_\infty - T_w) + \lambda_{ref} \, \omega_{v,\infty}$. Substitution gives

$$N_w = \frac{q_l}{c_{pm} \, (T_\infty - T_w)/B - \lambda_{ref}} = g \times B$$

But, from Heat Transfer Theory, $q_l = h_{cond} \, (T_w - T_s)$ and for pure steam, B = ( 1 - $\omega_{v,w}$ ) / ( $\omega_{v,w}$ - 1 ) = - 1. Hence,

$$N_w = \frac{- \, h_{cond} \, (T_w - T_s)}{c_{pm} \, (T_w - T_\infty) + \lambda_{ref}} = - \, g$$

Thus, our mass transfer formula accords with the heat transfer formula with $h_{cond}$ = condensation heat transfer coef.

# Soln - 1 - L40($\frac{3}{15}$)

**Soln: part ( a )** In our problem, $T_s$ ( outside tube wall temp ) is not known but cooling water temperature $T_c$ is known. Therefore, we invoke the notion of total heat transfer coef ( U ) and write $(T_w - T_c) = q_l/U$ where

$$\frac{1}{U} = \frac{1}{h_{cof,i}} + \frac{r_i}{k_{cu}} \ln\left(1 + \frac{r_o - r_i}{r_i}\right) + \frac{r_i}{k_l} \ln\left(1 + \frac{\delta}{r_o}\right)$$

Substitution gives U = 2663 W/$m^2$-K . For pure steam, at p = 1 atm, $T_w = T_\infty = T_{sat} = 100^0$C and $- N_w$ = g = $h_{cof,o}/cp_v = 115/(1.88 \times 10^3)$ = 0.06117 kg/$m^2$ - s . But, from our model

$$- N_w = \frac{2663\,(100 - 50)}{1880\,(100 - 100) + 2257 \times 10^3} = 0.059 \;\; \mathrm{kg}/m^2 - s$$

The two results are very close. Negative sign indicates condensation.

# Soln - 2 - L40($\frac{4}{15}$)

**Soln: Part ( b )** For 20% air in steam, $\omega_{v,\infty}$ = 0.8 and $q_l = U\,(T_w - 50)$ but $T_w$ is not known . Thus,

$$N_w = \frac{2663\,(T_w - 50)}{c_{p,m}\,(100 - \; T_w)/B - \lambda_{ref}} = (\frac{h_{cof,o}}{c_{pm}}) \times \ln{(1 + B)}$$

where B, $\lambda_{ref}$ and $c_{pm}$ are functions of $T_w$. Hence, we need trial-and-error solution.

| $T_w$ | $\omega_{v,w}$ | B | $c_{pm}$ | $\lambda_{ref}$ | LHS | RHS |
|---|---|---|---|---|---|---|
| 90 | 0.5933 | -0.5082 | 1614 | 2283.2e3 | -0.046 | -0.050 |
| 91 | 0.627 | -0.4637 | 1629 | 2280.6e3 | -0.0472 | -0.044 |
| 90.5 | 0.6094 | -0.488 | 1621 | 2282.0e3 | -0.0475 | -0.047 |

We accept the last soln $N_w \simeq -0.0473$ kg/$m^2$-s (Ans b)

Compared to pure steam, in the presence of air, B, $N_w$ and $T_w$ are reduced.

# Transpiration Cooling - L40($\frac{5}{15}$)

**Prob:** A porous metal surface is swept by air at 540$^0$C. Since the metal oxidises at 425$^0$C, it is decided to keep the surface temperature down to 370$^0$C by blowing gases through the pores. For this purpose, 3 candidate gases at 35$^0$C are considered: ( a ) Air, ( b ) He and ( c ) $H_2$. Calculate supply rate of each gas assuming operating g = 370 kg/$m^2$-hr.

**Soln: Part ( a )** In case of air, assuming const sp heat

$$B = \frac{h_\infty - h_w}{h_w - h_T} = \frac{c_p\,(T_\infty - T_w)}{c_p\,(T_w - T_T)} = \frac{540 - 370}{370 - 35} = 0.5074$$

Hence,$N_{w,a} = g \times B = 187.75$ kg/$m^2$-hr ( Ans ) .

# Soln ( Contd ) - 1 - L40($\frac{6}{15}$)

**Soln: Part ( b )** In this case, ( $c_{p,He}$ = 5.25 kJ/kg-K and $c_{p,a,\infty}$ = 1.1 kJ/kg-K ) and taking $T_{ref} = T_w$

$$B = \frac{c_{p,a}\left(T_\infty - T_{ref}\right) - c_{p,m}\left(T_w - T_{ref}\right)}{c_{p,m}\left(T_w - T_{ref}\right) - c_{p,He}\left(T_T - T_{ref}\right)}$$

$$= \frac{c_{p,a}\left(T_\infty - T_{ref}\right)}{-c_{p,He}\left(T_T - T_{ref}\right)}$$

$$= \frac{1.1\,(540 - 370)}{-5.25\,(35 - 370)} = 0.1063$$

Hence, $N_{w,He} = g \times B =$ 39.34 kg/$m^2$-hr ( Ans ) .

# Soln ( Contd ) - 2 - L40($\frac{7}{15}$)

**Soln: Part ( c )** In this case ( $c_{p,H_2}$ = 14.5 kJ/kg-K ), we assume an ( SCR $H_2 + 0.5\ O_2 = H_2O$ ) giving $r_{st}$ = 16/2 = 8 with $\Delta H_c$ = 118.4 MJ/kg. Then, taking $h = c_{p,m}\,(T - T_{ref}) + (\Delta H_c / r_{st})\,\omega_{O_2}$ and $T_{ref} = T_w$ , we have

$$B = \frac{c_{p,a,\infty}\,(T_\infty - T_w) + (\Delta H_c / r_{st})\,\omega_{O_2,\infty}}{-\,c_{p,H_2}\,(T_T - T_w)}$$

$$= \frac{1.1\,(540 - 370) + (118.4 \times 10^3 / 8)\,0.232}{-14.5\,(35 - 370)} = 0.745$$

Hence, $N_{w,H_2} = g \times B = 275.8$ kg/$m^2$-hr ( Ans ) .

Thus, $N_{w,H_2} > N_{w,air} > N_{w,He}$.

# Missile Cooling - L40($\frac{8}{15}$)

**Prob:** Consider axi-symmetric stagnation point of a missile traveling at 5500 m /s through air where static temperature is $\simeq$ 0 K. It is desired to maintain the surface temperature at $1200^0$C by transpiration of $H_2$ at $38^0$C. Evaluate B and $N_w$ . Given: $g^*$ = 0.467 kg/$m^2$-s.

**Soln:** Here, we account for KE contribution and define $h_m = c_{p,m}\left(T - T_{ref}\right) + \left(\Delta H_c / r_{st}\right)\omega_{O_2} + V^2/2000$ kJ / kg. Taking $T_{ref} = T_w$ so that $h_{m,w} = 0$,

$$B = \frac{c_{p,a,\infty}\left(T_\infty - T_w\right) + \left(\Delta H_c / r_{st}\right)\omega_{O_2,\infty} + V_\infty^2/2000}{-c_{p,H_2}\left(T_T - T_w\right)}$$

$$= \frac{1.1\,(0 - 1473) + (118.4 \times 10^3/8)\,0.232 + 5500^2/2000}{-14.5\,(38 - 1200)}$$

$$= 1.0053 \quad \rightarrow \quad N_w = g^* \ln\left(1 + B\right) = 0.325\,\frac{kg}{m^2 - s} \quad \text{(Ans)}$$

# Burning of a Volatile Fuel - L40($\frac{9}{15}$)

**Prob:** In a Diesel engine, liquid fuel ( $C_{12}H_{26}$, $\Delta H_c$ = 44 MJ/kg, sp. gr. = 0.854, $h_{fg}$ = 358 kJ/kg and $T_{bp} = 425^0$C ) is injected in the form of small droplets. After ignition delay, part of the fuel vapourises and burns abruptly and the remainder burns as fast as the fuel vapourises. Estimate burning time of a 5 $\mu$m droplet. Given: Cylinder Temp = 800$^0$C, $k_{fu}$ = 0.0463 W/m-K.

**Soln:** From stoichiometry, $r_{st} = 18.5 \times 32/170$ = 3.482. We define $h_m = c_{p,m}\,(T - T_{ref}) + (\Delta H_c/r_{st})\,\omega_{O_2}$. Assuming droplet at $T_{ref} = T_{bp} = T_w = T_T$, so that $h_{m,w} = h_{TL} = q_l = 0$, $\omega_{O_2,w} = 0$ and $q_w = N_w\,(h_{TW} - h_{TL}) = N_w\,h_{fg}$ we have

$$B = \frac{h_{m,\infty} - 0}{0 - 0 + h_{fg}} = \frac{c_{p,\infty}\,(T_\infty - T_{bp}) + (\Delta H_c/r_{st})\,\omega_{O_2,\infty}}{h_{fg}}$$

$$= \frac{1.15\,(800 - 425) + (44 \times 10^3/3.482)\,0.232}{358} = 9.394$$

# Soln ( Contd. ) - L40($\frac{10}{15}$)

Since B is large, the instantaneous burning rate and burning time are given by

$$\dot{m} = (\frac{\Gamma_h}{r_w})\, 4\,\pi\, r_w^2 \ln{(1+B)} \quad \rightarrow \quad t_{burn} = \frac{\rho_l\, D_{wi}^2}{8\,\Gamma_h \ln{(1+B)}}$$

At 800$^0$C, $k_a$ = 0.075 W/m-k. Therefore,
$k_m = 0.4 \times k_{fu} + 0.6 \times k_a$ = 0.06353 W/m-K.
Taking $c_{pm}$ = 1.2 kJ/kg-K, $\Gamma_h = 0.06353/1200 = 5.29 \times 10^{-5}$ kg/m-s. Also, $\rho_l =$ sp. gr. $\times$ 1000 = 853 kg/$m^3$. Hence

$$t_{burn} = \frac{853 \times (5 \times 10^{-6})^2}{8 \times 5.29 \times 10^{-5} \ln{(1+9.394)}} = 2.15 \times 10^{-5} \ \ \text{s}$$

or, $t_{burn}$ = 0.0215 ms ( Ans )

# Drying - L40($\frac{11}{15}$)

**Prob:** In a laundry dryer, dry air is available at 1 bar and 20$^0$C. The air is mixed with superheated steam at 1 bar and 250$^0$C. Examine effect of mixing in the range $0 \leq \omega_{v,\infty} \leq 0.5$ . Assume g is unchanged with change in $\omega_{v,\infty}$.

**Soln:** For superheated steam, $h_{v,\infty}$ = 2974.3 kJ/kg. There will be adiabatic conditions at the drying surface ( $q_l$ = 0 ). Hence

$$
\begin{aligned}
B &= \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL}} \\
h_{m,\infty} &= 1.005 \times 20 \times (1 - \omega_{v,\infty}) + 2974.3\, \omega_{v,\infty} \\
h_{m,w} &= 1.005\, T_w + \{(1.88 - 1.001)\, T_w + 2503\}\, \omega_{v,w} \\
h_{TL} &= 4.187 \times T_w
\end{aligned}
$$

where $T_w$ and $\omega_{v,w}$ are related by equilibrium relation given in lecture 37. Iterative solutions on next slide.

# Soln ( Contd. ) - 1 - L40($\frac{12}{15}$)

| $\omega_{v,\infty}$ | $T^0_w C$ | $\omega_{v,w}$ | $B_m = B_h$ |
|---|---|---|---|
| 0.0 | 6.07 | 0.0059 | 0.0056 |
| 0.01 | 17.571 | 0.0127 | 0.00274 |
| 0.03 | 32.189 | 0.0302 | 0.00018 |
| 0.05 | - | - | $\rightarrow 0$ |
| 0.08 | 50.267 | 0.0808 | 0.0009 |
| 0.10 | 54.745 | 0.102 | 0.00225 |
| 0.20 | 69.18 | 0.210 | 0.0122 |
| 0.30 | 77.7 | 0.317 | 0.0224 |
| 0.40 | 83.485 | 0.422 | 0.0379 |
| 0.50 | 87.85 | 0.525 | 0.0517 |

The drying rate is non-linear at small fraction $\omega_{v,\infty}$. It is difficult to balance $B_m$ and $B_h$ at $\omega_{v,\infty} = 0.05$ because $B \rightarrow 0$. Compared to dry air, drying rate improves monotonically for $0.2 \leq \omega_{v,\infty} \leq 0.5$.

# Dissolution of Solid - L40($\frac{13}{15}$)

**Prob:** A thin plate ( 15 cm $\times$ 15 cm ) of solid salt is dragged through sea-water ( edgewise ) at 20$^0$C with a velocity of 5 m/s. Sea water has salt concentration of 3 % by weight. Saturated salt solution in water has concentration of 30 gms / 100 gms of water at 20$^0$C. ( a ) Assuming transition criterion of Fraser & Milne, determine if transition will occur and ( b ) estimate the rate at which salt goes into solution Take Sc = 745, $\nu_{water} = 10^{-6}\ m^2/s$, Salt sp gr. = 2.163.

**Soln: Part ( a )** For m = 0, $\delta_2^* = 0.645\ Re_x^{-0.5}$ and from FM, transition criterion is $Re_{\delta_2} = 163 + \exp{(6.91)} = 1165.2$. Combining, we get $Re_{x,tr} = 3.08 \times 10^6$ or with $U_\infty$ = 5 m/s, $x_{tr} = 61.6\ cm > 15\ cm$.

Hence transition will not occur. ( Ans )

# Soln ( Contd ) - L40($\frac{14}{15}$)

**Soln: Part ( b )** In this problem, $\omega_\infty = 0.03$, $\omega_w = 36/136 = 0.2647$ and $\omega_T = 1.0$.
Hence, B = ( 0.03 - 0.2647 ) / ( 0.2647 -1 ) = 0.3192

Now, $Re_{plate} = 5 \times 0.15/10^{-6} = 7.5 \times 10^5$. Therefore,
Sh = $g^* \, L/(\rho_m \, D) = 0.664 \, Re_L^{0.5} \, Sc^{0.33} = 5099.3$.
But, $\rho_m = \rho_{water} \, (1 - \omega_{mean}) + \omega_{mean} \, \rho_{salt}$ where
$\omega_{mean} = 0.5 \, (0.03 + 0.2647) = 0.147$. Hence, $\rho_m$ = 1169.2.

Therefore, $g^*$ = 192 kg/$m^2$-hr
Hence, $N_w = g^*$ ln $(1 + B)$ = 53.2 kg/$m^2$-hr and
Mass loss from 2 sides = $53.2 \times 2 \times 0.15^2 = 2.394$ kg/hr ( Ans )

# Summary - L40($\frac{15}{15}$)

1. This completes duscussion of Convective Mass Transfer
2. We have shown that the algebraic Reynolds flow model with property corrections is a good proxy for the Boundary Layer model because mass transfer coefficient is evaluated from $h_{cof,v_w=0}$ for the corresponding heat transfer situation. This feature obviates the need for solving complete set of differential BL equations.
3. The 1D Stefan flow model provides reliable solutions in diffusion mass transfer. The 1D Couette flow model , though very approximate, provides means for estimating effect of property variations in a boundary layer.
4. In the remaining lectures, we shall consider 2 special topics of ( a ) Natural Convection BLs, and ( b ) Laminar and Turbulent Diffusion Flames