ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-35 BOUNDARY LAYER FLOW MODEL

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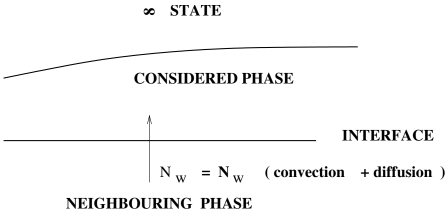
1. Definitions
2. Governing Equations
3. Conserved property eqns for all types of mass transfer
4. Boundary Conditions
    1. Mass Conservation Principle
    2. Energy Conservation Principle
5. $N_w = g \times B$ for small and large mass transfer rates

BL Flow Model - L35($\frac{1}{14}$)

1. Conv MT takes place due to concentration gradients of the transferred species
2. Since Reynolds flow model mimics the real flow, Interface mass transfer flux N_w (kg/m^2-s) from

$$N_w = g\,B$$

3. N_w and g have same units



N_w is positive when mass transfer takes place from the *neighbouring phase* into the *considered phase* across the interface & *vice versa*

Governing Equations - L35($\frac{2}{14}$)

Assuming Steady-state mass transfer

$$\frac{\partial(\rho_m\, u\, \Psi)}{\partial x} + \frac{\partial(\rho_m\, v\, \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ	
1	0	0	Bulk Mass
u	$\mu_{m,eff}$	- $dp\ /\ dx$	Momentum
ω_k	$\rho_m\ D_{eff}$	R_k	Species transfer
η_α	$\rho_m\ D_{eff}$	0	Element transfer
h_m	$k_{m,eff}/cp_m$	$-\partial(\sum m''_{y,k}\ h_k)/\partial y$	Energy

where $m''_{y,k} = -\ \rho_m\ D_{eff}\ \partial\omega_k/\partial y$. Sources Dp/Dt, $\dot{Q}_{rad}$, $\dot{Q}_{others}$ and $\mu_{eff}\ (\partial u/\partial y)^2$ are ignored in the energy equation. All equations are coupled requiring numerical solutions . Simplifications of ω_k and h_m equations are possible under certain assumptions so that they are rendered to conserved property equations .

Conserved Property Eqn - L35($\frac{3}{14}$)

$$\frac{\partial(\rho_m\, u\, \Psi)}{\partial x} + \frac{\partial(\rho_m\, v\, \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right]$$

$$N_w = g \times B \quad \text{with} \quad B = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}$$

1. In Inert MT without HT , $\Psi = \omega_v$ and $\Gamma = \rho_m\, D$
2. In Inert MT with HT , $\Psi = \omega_v$ and h_m and $\Gamma_{mh} = \rho_m\, D = \rho_m\, \alpha_m$ with Le = 1
3. In MT with SCR , $\Psi =$ appropriate Φ and h_m and $\Gamma_{mh} = \rho_m\, D = \rho_m\, \alpha_m$ with Le = 1 and equal $c_{p,k} = c_{pm}$
4. In MT with ACR , $\Psi =$ appropriate Φ and $\Gamma_m = \rho_m\, D$

In each case, we need Boundary Conditions at y = 0 in w-state.

BCs - Mass Conservation - L35($\frac{4}{14}$)

1. For Inert Mass Transfer , consider mass conservation between T- and w-states. Then

$$N_w\,\omega_{k,T} \;=\; N_w\,\omega_{k,w} - \rho_m\, D\, \frac{\partial \omega_k}{\partial y}|_w$$

$$N_w \;=\; \frac{\rho_m\, D\,(\partial \omega_k/\partial y)_w}{\omega_{k,w} - \omega_{k,T}}$$

2. For Conserved property Φ

$$N_w\,\Phi_T \;=\; N_w\,\Phi_w - \rho_m\, D\, \frac{\partial \Phi}{\partial y}|_w$$

$$N_w \;=\; \frac{\rho_m\, D\,(\partial \Phi/\partial y)_w}{\Phi_w - \Phi_T}$$

where $\Phi = \omega_{fu} - \omega_{O_2}/r_{st} = \omega_{fu} + \omega_{pr}/(1 + r_{st})$ for an SCR or $\Phi = \sum a_\alpha\, \eta_\alpha$ and a_α are suitable chosen coefficients for an ACR

BCs - Energy Conservation - 1 - L35($\frac{5}{14}$)

1. Consider control volume between T- and w- states. Then

$$N_w\, h_{m,T} = N_w\, h_{m,w} + \Big(\sum_k -\rho_m\, D\, \frac{\partial \omega_k}{\partial y}\Big|_w\, h_k\Big) - q_w \quad \text{where}$$

$$q_w = k_m\, \frac{\partial T}{\partial y}\Big|_w = c_{pm}\, \Gamma_h\, \frac{\partial T}{\partial y}\Big|_w \quad \text{hence}$$

$$N_w = \frac{\left(\sum_k \Gamma_m\, (\partial \omega_k / \partial y)_w\, h_k\right) + c_{pm}\, \Gamma_h\, (\partial T / \partial y)_w}{h_{m,w} - h_{m,T}}$$

2. This is the general energy conservation principle. The final form of the Numerator will depend on mass transfer application.

BCs - Energy Conservation - 2 - L35($\frac{6}{14}$)

1. For Inert MT with HT , Le = 1 gives $\Gamma_h = \Gamma_m$. Hence

$$c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k\,\omega_k\,c_{p,k})\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k\,\omega_k\,\frac{\partial h_k}{\partial y})_w$$

2. Hence,

$$N_w = \frac{\Gamma_m\,(\sum_k\,(\partial\omega_k/\partial y)_w\,h_k) + \Gamma_h\,(\sum_k\,\omega_k\,\partial h_k/\partial y)_w}{h_{m,w} - h_{m,T}}$$

$$= \frac{\Gamma_{mh}\,(\sum_k\,\{\partial(\omega_k\,h_k)/\partial y\}_w)}{h_{m,w} - h_{m,T}}$$

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

BCs - Energy Conservation - 3 - L35($\frac{7}{14}$)

1. For MT with HT and SCR , taking Le =1, $c_{p,k} = c_{pm}$ and $\Delta T = (T - T_{ref})$, we have

$$h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c \quad \text{and} \quad h_{O_2} = h_{pr} = c_{pm}\,\Delta T$$

2. Hence,

$$\begin{aligned}
(\sum_k \Gamma_m\,(\frac{\partial \omega_k}{\partial y})_w\,h_k) &= \Gamma_m\,(\frac{\partial \omega_{fu}}{\partial y})_w\,\Delta h_c \quad \text{because} \\
c_{pm}\,\Delta T\,\Gamma_h\,\sum_k (\frac{\partial \omega_k}{\partial y})_w &= 0 \\
\text{and} \quad c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w &= \Gamma_h\,(\frac{\partial h_m}{\partial y})_w - \Gamma_h\,\Delta h_c\,(\frac{\partial \omega_{fu}}{\partial y})_w
\end{aligned}$$

3. Hence, substitution with $\Gamma_m = \Gamma_h = \Gamma_{mh}$ gives

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

BCs - Energy Conservation - 4 - L35($\frac{8}{14}$)

1. Finally, for single component convective mass transfer

$$
\begin{aligned}
(\sum_k \Gamma_m \, (\frac{\partial \omega_k}{\partial y})_w \, h_k) &= 0 \quad \text{because} \quad \omega_k = 1 \\
\text{and} \quad c_{pm} \, \Gamma_h \, (\frac{\partial T}{\partial y})_w &= \Gamma_h \, (\frac{\partial h_m}{\partial y})_w
\end{aligned}
$$

2. Hence

$$N_w = \frac{\Gamma_h \, (\partial h_m / \partial y)_w}{h_{m,w} - h_{m,T}}$$

3. If specific heats in all states are equal

$$N_w = \frac{\Gamma_h \, (\partial T / \partial y)_w}{T_w - T_T}$$

Comments - 1 - L35($\frac{9}{14}$)

1. Thus in all cases of mass transfer, mass and energy conservation principles give identical formula for N_w
2. Combining with Reynolds flow model which claims to mimic the real boundary layer flow model, we have

$$N_w = \frac{\Gamma_\Psi \, (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_T} = g \times \left[\frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T} \right] = (\rho_m \, V)_w$$

3. Hence,

$$N_w \propto (\Psi_\infty - \Psi_w) \propto \Gamma_\Psi \, (\frac{\partial \Psi}{\partial y})_w \propto V_w$$

4. This shows that even when Γ is uniform, the Ψ-eqn is non-linear because velocity field (u and v) is a function of V_w and $(\Psi_\infty - \Psi_w)$. This is akin to Natural Convection in which u and v are functions of $(T_\infty - T_w)$.

Comments - 2 - L35($\frac{10}{14}$)

1. In Natural convection, the momentum and energy eqns are coupled through buoyancy source in the momentum eqn . In contrast, in Mass transfer, momentum, energy and species eqns are coupled through boundary conditions .
2. The coupling between momentum and Ψ-eqns can be ignored when $N_w \propto V_w \rightarrow 0$. Thus

$$g^* \equiv (\frac{N_w}{B_\Psi})_{N_w \rightarrow 0} = \frac{-\Gamma_\Psi \, (\partial\Psi/\partial y)_w}{\Psi_w - \Psi_\infty}$$

   where g^* now depends only on the Ψ - profiles. This definition is analogous to that used to define heat transfer coefficient.
3. When N_w is large, coupling is strong and $N_w = g \times B_\Psi$. Hence, g must be a function of B_Ψ and $g^* \equiv g_{B_\Psi \rightarrow 0}$

Comments - 3 - L35($\frac{11}{14}$)

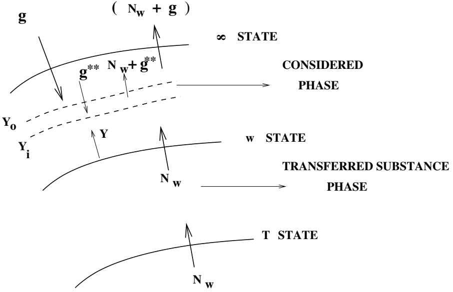
1. By analysing Experimental data on mass transfer with and without combustion, Spalding[1] showed that within experimental scatter ,

$$\frac{g}{g^*} = \frac{N_w/B}{(N_w/B)_{N_w \to 0}} = F(B) \ \ \text{only}$$

2. This eqn shows that (g / g^*) is not influenced by Re, Pr or Sc numbers . The validity of this assertion will be examined later.
3. Thus, all that is required is the value of g^* (evaluated from $h_{cof,V_w=0}/c_{pm}$) and F (B) to obtain g.

[1] Spalding D B Convective Mass Transfer, Edward Arnold Ltd, London (1963)

Form of F (B) - 1 - L35($\frac{12}{14}$)



1. Using computer solutions of the BL eqns as well as experimental data, Spalding showed that

$$\frac{g}{g^*} = F(B) \simeq \frac{\ln(1+B)}{B}$$

   This relationship was also predicted by the Stefan and Couette flow models.

2. It can be derived for Reynolds flow model as well (see figure)

Consider 2 surfaces y_0 and y_i in the considered phase. Let local Reynolds flux g^{**} cross the y_o surface carrying properties at y_o . Similarly, let Reynolds flux $g^{**} + N_w$ cross the y_o surface carrying properties at y_i .

Form of F (B) - 2 - L35($\frac{13}{14}$)

1. The physical idea behind introduction of g^{**} is that real flow processes like heat conduction, mass diffusion, turbulence etc do behave like the Reynolds flow but on a much smaller scale $\Delta y = (y_o - y_i) \rightarrow 0$.
2. Thus, writing mass conservation over y_0- and T-states

$$N_w \Psi_T + g^{**} \Psi_{y_o} = (g^{**} + N_w) \Psi_{y_i} \rightarrow \frac{N_w}{g^{**}} = \frac{\Psi_{y_o} - \Psi_{y_i}}{\Psi_{y_i} - \Psi_T} = \frac{d\ \Psi_y}{\Psi_{y_i} - \Psi_T}$$

3. Considering a large number of Δy between ∞- and w-states

$$N_w \sum_w^{\infty} \frac{1}{g^{**}} = \int_0^{\infty} \frac{d\ \Psi_y}{\Psi_{y_i} - \Psi_T} = \ln \left[1 + \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right] = \ln(1 + B_\Psi)$$

Form of F (B) - 3 - L35($\frac{14}{14}$)

1. Thus, as $B_\Psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = B_\Psi / N_w$
2. Therefore, comparison with the observation of slide 11 shows that as $B_\Psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = (g^*)^{-1}$. Hence,

$$\begin{aligned} N_w &= g \times B_\Psi = g^* \ln(1 + B_\Psi) \text{ and} \\ \frac{g}{g^*} &= F(B) = \frac{\ln(1 + B_\Psi)}{B_\Psi} \end{aligned}$$

3. This formula can be used for large mass transfer rates obtained in liquid-fuel burning and transpiration cooling. Small mass transfer rates are encountered in Combustion of solid fuel or evaporative cooling/air-conditioning.
4. The validity of the formula will be checked in the next lecture.