ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-34 REYNOLDS FLOW MODEL

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Definitions - 1 - L34($\frac{1}{14}$)

1. This model postulates ficticious mass flux (g) called Reynolds flux in the ∞ - state
2. Model develops algebraic relations that are related to real transport phenomena by reference to (g)
3. Model develops $N_w = g \times B$ relation.
4. The objective is to relate g to heat transfer coefficient $h_{N_w \to 0}$
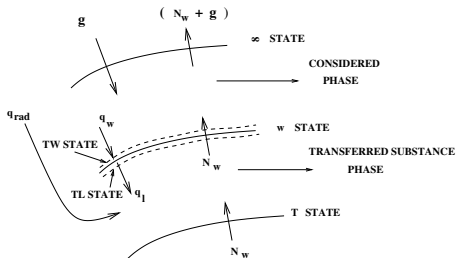
1. TW - state is just inside the Considered phase
2. TL - state is just inside the Neighbouring phase
3. In the T - state, temperature and compositions are uniform

Definitions - 2 - L34($\frac{2}{14}$)

1. **Mass Transfer Flux** is sum of all species transferred across the interface. Thus

$$N_w = \sum_j N_{j,w} = \sum_k N_{k,T}$$

where species j and k need not be the same .

2. q_w is positive when flowing towards the interface from the considered phase.

3. q_l is positive when flowing away from the interface in the Neighbouring phase

4. Therefore, $(q_w - q_l) = N_w (h_{TW} - h_{TL})$ where h_{TW} and h_{TL} are enthalpies of the transferred substance at the TW and TL states . When $(q_w - q_l) \neq 0$, phase-change occurs .

5. q_{rad} from Considered phase to the interface is positive. It is accounted in the Neighbouring phase for convenience.

The Hypothesis - L34($\frac{3}{14}$)

1. In different applications with or without chemical reaction, complex processes occur in the considered phase
2. To avoid solution of any differential equations , it is assumed that the complex processes can be effectively simulated by
    1. A ficticious flux (g) through the $\infty - \infty$ plane towards the interface carrying with it properties of the ∞ - state .
    2. A ficticious flux (N_w + g) through the $\infty - \infty$ plane away from the interface carrying with it properties of the w - state .
3. The magnitude of (g) is not affected by presence of gradients of ω_j or T or turbulence, radiation etc in the considered phase
4. The g - flux is also not affected by direction of N_w
5. The g-flux is supposed to produce same effect at the interface that the real flow is likely to produce

An Illustration
Evaporative Cooling - L34($\frac{4}{14}$)

Consider a solid surface (T_s) to be protected from hot gases ($T_\infty > T_s$) and ($\omega_{v,\infty} < \omega_{v,w}$) by a thin water film . The Reynolds Hypothesis states that a g-flux of hot gases towards the interface together with ($N_w + g$) flux of cooler moist air away from the interface will produce the effect: $N_w > 0$, $q_w > 0$ and $q_l < 0$.

The real flow will also produce precisely these effects.
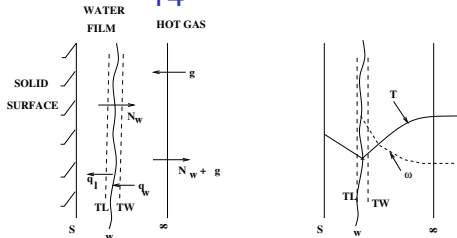
In this sense, the g-flux is considered capable of responding to the mass-fraction and temperature gradients as well as turbulence effects in the considered phase as shown in the right figure.

Momentum Transfer - L34($\frac{5}{14}$)

1. Consider control volume between ∞- and w- states. Then

   Rate of Momentum Flux In $= g\,U_\infty$

   Rate of Momentum Flux Out $= \tau_w + (N_w + g)\,u_w = \tau_w$

2. Hence,

$$\frac{g\,U_\infty}{\tau_w} = 1 = \frac{N_w}{\rho\,V_w} \quad \text{or}$$

$$N_w = g \times \frac{\rho\,v_w\,U_\infty}{\tau_w} = g \times \frac{v_w/U_\infty}{\tau_w/(\rho\,U_\infty^2)}$$

$$N_w = g \times B_f \quad \rightarrow \quad B_f = \frac{v_w/U_\infty}{(C_{f,x}/2)}$$

3. Further $g = \tau_w/U_\infty = \rho\,U_\infty\,(C_{f,x}/2)$

Single Phase Conv HT - 1 - L34($\frac{6}{14}$)

1. Here, CV between T- and ∞-states is considered. Then

$$\text{Rate of Heat Flux In} = g\, h_\infty + N_w\, h_T$$
$$\text{Rate of Heat Flux Out} = (N_w + g)\, h_w$$

2. Equating and rearranging and taking $T_{ref} = 0$

$$N_w = g \times B_h \quad \rightarrow \quad B_h = \frac{h_\infty - h_w}{h_w - h_T} = \frac{(c_p T)_\infty - (c_p T)_w}{(c_p T)_w - (c_p T)_T}$$

where $N_w = \rho_w\, V_w$

3. If specific heats are equal

$$B_h = \frac{T_\infty - T_w}{T_w - T_T}$$

Single Phase Conv HT - 2 - L34($\frac{7}{14}$)

1. Further, CV between ∞- and w-states is considered. Then

$$\text{Rate of Heat Flux In} \; = \; g \, h_\infty + N_w \, h_w$$
$$\text{Rate of Heat Flux Out} \; = \; (N_w + g) \, h_w + q_w$$

2. Equating and rearranging $g = q_w/(h_\infty - h_w)$

3. If specific heats are equal

$$g = \frac{q_w}{c_p \, (T_\infty - T_w)} = \frac{h_{cof, v_w}}{c_p}$$

Thus, g is related to h_{cof, v_w}.

Inert MT without HT - L34($\frac{8}{14}$)

1. Here, in all states, temperatures are equal ($T_T = T_w = T_\infty$)
2. Consider CV between T- and ∞ - states. Then

$$\text{Rate of mass Flux In} = g\,\omega_{v,\infty} + N_w\,\omega_{v,T}$$
$$\text{Rate of Mass Flux Out} = (N_w + g)\,\omega_{v,w}$$

3. Equating and rearranging

$$N_w = g \times B_m \quad \rightarrow \quad B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}}$$

where for a pure liquid in the T-state, $\omega_{v,T} = 1$.
$\omega_{v,w}$ is evaluated from equilibrium at T_w

Inert MT with HT - 1 - L34($\frac{9}{14}$)

1. Here, let $T_\infty \neq T_w \neq T_T$. Then
2. Consider, CV between ∞- and T-states is considered. Then

$$\text{Rate of Heat Flux In} \quad = \quad g\, h_{m,\infty} + N_w\, h_{m,T}$$
$$\text{Rate of Heat Flux Out} \quad = \quad (N_w + g)\, h_{m,w}$$
$$\text{Equating}: \quad N_w = g \times B_{mh} \quad \text{where} \quad B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}}$$

$$h_m = \omega_v\, h_v + (1 - \omega_v)\, h_a$$
$$h_v = c_{p,v}\,(T - T_{ref}) + \lambda_{ref}, \quad h_a = c_{p,a}\,(T - T_{ref})$$

3. Making Le = 1 assumption, $B_{mh} = B_m$ from which $\omega_w \sim h_w\,(T_w)$ relationship is iteratively calculated.

Inert MT with HT - 2 - L34($\frac{10}{14}$)

1. Now, consider CV between ∞- and w-states. Then

$$q_w + (g + N_w)\, h_{m,w} \quad = \quad g\, h_{m,\infty} + N_w\, h_{m,TW}$$

$$N_w = g \times B_{mh} \quad \text{where} \quad B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TW} + q_w/N_w}$$

2. But, $(q_w - q_l) = N_w\,(h_{TW} - h_{TL})$. Hence

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TL} + q_L/N_w}$$

If $T_w = T_{bp}$, $q_L = 0$ and $h_{m,w} - h_{m,TL} = h_{fg}$.

3. If radiation is included then $N_w\, h_{m,T} + q_L + q_{rad} = N_w\, h_{TL}$. Hence

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T} - q_{rad}/N_w}$$

MT with HT and SCR - 1 - L34($\frac{11}{14}$)

1. Here, we have 3 species: fu, O_2 and Pr with stoichiometric coefficient r_{st} = kg of O_2 / kg of fuel.

2. Consider CV between ∞- and T-states. Then

   For fuel $\quad g\,\omega_{fu,\infty} + \omega_{fu,T}\,N_w - \omega_{fu_w}\,(g + N_w) = -\,|R_{fu}|$

   For O_2 $\quad g\,\omega_{O_2,\infty} + \omega_{O_2,T}\,N_w - \omega_{O_2,w}\,(g + N_w) = -\,|R_{O_2}|$

3. But, $R_{O_2} = r_{st}\,R_{fu}$. Hence, dividing the 2nd Eqn by r_{st} and subtracting from 1st Eqn, we have

$$N_w = g \times B_\Phi \quad \rightarrow \quad B_\Phi = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \rightarrow \quad \Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}}$$

4. By the same reasoning, it is also possible to define

$$\Phi = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} = \frac{\omega_{pr}}{1 + r_{st}} + \frac{\omega_{O_2}}{r_{st}}$$

1. Invoking the energy conservation principle and assuming $c_{p,k} = c_{pm}$, we have shown that

$$h_m = \sum_k \omega_k h_k = c_{pm}(T - T_{ref}) + \omega_{fu}\Delta h_c = c_{pm}(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\Delta h_c$$

2. Consider CV between ∞- and w-states. Then
$$g\, h_{m,\infty} + N_w\, h_{m,TW} = (g + N_w)\, h_{m,w} + q_w$$

3. Taking the 2nd definition for h_m, we have $N_w = g \times B_{mh}$ where taking $T_{ref} = T_w$

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TW} + q_w/N_w}$$

$$B_{mh} = \frac{c_{pm}\,(T_\infty - T_w) + \Delta h_c\,(\omega_{O_2,\infty} - \omega_{O_2,w})/\,r_{st}}{\Delta h_c\,(\omega_{O_2,w} - \omega_{O_2,TW})/\,r_{st} + q_w/N_w}$$

1. For a volatile liquid fuel or transpiration cooling by a combustible gas (e. g. H_2) , $\omega_{O_2,w} = 0$ and since the transferred substance does not contain O_2, $\omega_{O_2,TW} = 0$. Hence,

$$B_{mh} = \frac{c_{pm}\left(T_\infty - T_w\right) + \Delta h_c\, \omega_{O_2,\infty}/\, r_{st}}{q_w/N_w}$$

2. Now, for a liquid fuel at its boiling point ($T_w = T_{bp}$), $q_w - q_l = N_w\left(h_{m,TW} - h_{m,TL}\right) = N_w\, \lambda_{fu}$. Hence,

$$B_{mh} = \frac{c_{pm}\left(T_\infty - T_{bp}\right) + \Delta h_c\, \omega_{O_2,\infty}/\, r_{st}}{\lambda_{fu} + q_L/N_w}$$

3. For an atomised tiny liquid droplet, if $T_T \simeq T_{bp}$, $q_L = 0$

MT with HT and ACR - L34($\frac{13}{14}$)

1. When it is difficult to ascertain mass-fractions of compounds in different states, it is preferred to use conserved property $\eta_\alpha = \sum_k \eta_{\alpha,k} \, \omega_k$ where $\eta_{\alpha,k} = M_\alpha / M_k$.

2. Consider CV between ∞- and T-states. Element balance gives

$$g \, \eta_{\alpha,\infty} \;+\; N_w \, \eta_{\alpha,T} = (g + N_w) \, \eta_{\alpha,w} \;\; \text{or}$$
$$N_w \;=\; g \times B_m \;\; \rightarrow \;\; B_m = \frac{\eta_{\alpha,\infty} - \eta_{\alpha,w}}{\eta_{\alpha,w} - \eta_{\alpha,T}}$$

3. It is usually convenient to combine different η_α into a new conserved property variable Φ as was shown in lecture 32 (slide 18)

Summary - L34($\frac{14}{14}$)

1. Thus, in all cases, we have shown that

$$N_w = g \times B \quad \rightarrow \quad B = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}$$

where $\Psi = \omega_v, \eta_\alpha, h_m$ and Φ.

2. Any linear combinations of Ψ are also solutions

3. The result $N_w = g \times B$ from the Reynolds flow model is different from the result $N_w = g^* \ln(1 + B)$ for 1D Stefan flow model (diffusion) and Couette flow model (convection).

4. The result from the model correctly identifies B_f in momentum transfer and shows that $g = h_{cof,v_w}/c_p$ in single phase Conv HT.

5. The Reynolds flow model result will be employed to provide interface boundary conditions to the Boundary Layer flow model in the next lecture.