ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-32 STEFAN FLOW MODEL

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1. Inert mass transfer without heat transfer
2. Inert mass transfer with heat transfer
3. Mass transfer with heat transfer and simple chemical reaction (SCR)
4. Mass transfer with heat transfer and arbitrary chemical reaction (ACR)

Reminder of Gov Eqns - L32($\frac{1}{20}$)

In the Stefan flow model, under steady state

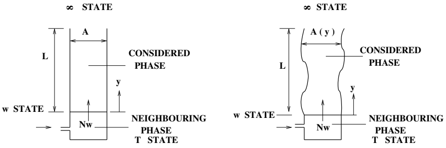
$$\frac{1}{A}\,\frac{d}{dy}\,\left[N_{\Psi,y}\,A\right] = \frac{1}{A}\,\frac{d}{dy}\,\left[\rho_m\,v\,A\,\Psi - \Gamma_\Psi\,A\,\frac{d\,\Psi}{dy}\right] = S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
ω_k	$\rho_m\,D$	R_k
η_α	$\rho_m\,D$	0
h_m	k_m/cp_m	- $A^{-1}\,d(\sum\,Am''_{y,k}\,h_k)/dy$

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m\,D\,(d\omega_k/dy)$

Inert MT without HT - 1 - L32($\frac{2}{20}$)

1. Consider evaporation of pure water through a stagnant column of air
2. Both water and air are at same temperature. Hence, no HT
3. Air does not dissolve in water
4. Steady state prevails . That is, water is supplied at the bottom at the evaporation rate . So, L is constant.
5. There are two species only. Air (a) - water vapour (v)



(a) CONSTANT AREA MODEL (b) VARIABLE AREA MODEL

Two Governing equations are

$$\frac{d}{dy}\,[N_{a,y}\,A] = \frac{d}{dy}\,[N_{v,y}\,A] = 0$$

or,
$\dot{m}_w = A_w\,N_w = A\,(N_{v,y} + N_{a,y}) =$ const. But, in stagnant air,
$\dot{m}_{a,w} = A\,N_{a,y} = A_w\,N_{a,w} = 0.$
Also, $\omega_a + \omega_v = 1$.

Inert MT without HT - 2 - L32($\frac{3}{20}$)

1. Therefore,

$$
\begin{aligned}
\dot{m}_w &= \dot{m}_v = A\,(\rho_m\,V\,\omega_v - \rho_m\,D\,\frac{d\,\omega_v}{dy}) = A\,\rho_m\,V \\
\text{or} \qquad & \dot{m}_w\,\omega_v - \rho_m\,D\,A\,\frac{d\,\omega_v}{dy} = \dot{m}_w \\
\text{or} \qquad & -\frac{d\,\omega_v}{1-\omega_v} = (\frac{\dot{m}_w}{\rho_m\,D})\,\frac{dy}{A}
\end{aligned}
$$

2. If A = A_w = const , then $N_w = \dot{m}_w/A_w$ and integration from y = 0 to y = L gives

$$
N_w = (\frac{\rho_m\,D}{L})\,\ln(\frac{1-\omega_{v,\infty}}{1-\omega_{v,w}}) = g_m^*\,\ln(1+B_m) \;\; \text{where}
$$

$$
B_m = \frac{\omega_{v,\infty}-\omega_{v,w}}{\omega_{v,w}-1} \;\; \text{and} \;\; g_m^* = \frac{\rho_m\,D}{L}\;(\frac{\text{kg}}{m^2-\text{s}})
$$

Inert MT without HT - 3 - L32($\frac{4}{20}$)

1. For a spherical droplet evaporation in stagnant surroundings, A = $4\pi r^2$. Then,

$$-\frac{d\,\omega_v}{1-\omega_v} = \left(\frac{\dot{m}_w}{\rho_m D}\right)\frac{dr}{4\,\pi\,r^2}$$

2. Integration from $r = r_w$ (droplet radius) to r = ∞ gives

$$\ln\left(\frac{1-\omega_{v,\infty}}{1-\omega_{v,w}}\right) = \frac{\dot{m}_w}{4\,\pi\rho_m\,D\,r_w}$$

3. Hence

$$N_w = \frac{\dot{m}_w}{4\,\pi\,r_w^2} = \left(\frac{\rho_m\,D}{r_w}\right)\ln\left(\frac{1-\omega_{v,\infty}}{1-\omega_{v,w}}\right) = g_m^*\ln\left(1+B_m\right) \text{ where}$$

$$B_m = \frac{\omega_{v,\infty}-\omega_{v,w}}{\omega_{v,w}-1} \text{ and } g_m^* = \frac{\rho_m\,D}{r_w}\;\left(\frac{\text{kg}}{m^2-s}\right)$$

Comments - L32($\frac{5}{20}$)

1. Both results show that in diffusion mass transfer $N_w = g_m^* \ln(1 + B_m)$
2. But, as $B_m \rightarrow 0$, $\ln(1 + B_m) \rightarrow B_m$ for both $\pm B_m$. Thus, the linear relation $N_w = g \times B_m$ holds only for very small B_m or N_w.
3. Negative B_m implies Condensation
4. Therefore, in general, we may write

   $$N_w = g \times B_m \quad \text{with} \quad \frac{g}{g_m^*} = \frac{\ln(1 + B_m)}{B_m}$$

   where g_m^* is value of g when $|B_m| \rightarrow 0$
5. This result has significance even in Conv MT

Inert MT with HT - 1 - L32($\frac{6}{20}$)

1. Let the air in the previous example be at $T_\infty > T_w$ where T_w is the temperature of the water surface. Then, under steady state, besides species conservation, energy eqn is

$$\frac{d(N_{v,y}\, h_m A)}{dy} = \frac{d}{dy}\,\left[A(k_m\,\frac{dT}{dy} + \rho_m\, D\,\left\{\frac{d\omega_v}{dy}\,h_v + \frac{d\omega_a}{dy}\,h_a\right\})\right]$$

   where $h_m = \omega_v\, h_v + (1-\omega_v)\, h_a$, $h_v = c_{p,v}\,(T - T_{ref}) + \lambda_{ref}$, $h_a = c_{p,a}\,(T - T_{ref})$ and $c_{p,m} = \omega_v\, c_{p,v} + (1-\omega_v)\, c_{p,a}$. λ_{ref} is latent heat at T_{ref} .

2. Further,

$$k_m\,\frac{dT}{dy} = \rho_m\,\alpha_m\,c_{p,m}\,\frac{dT}{dy} = \rho_m\,\alpha_m\,\left[\omega_v\,\frac{d\,h_v}{dy} + \omega_a\,\frac{d\,h_a}{dy}\right]$$

Inert MT with HT - 2 - L32($\frac{7}{20}$)

Substitution gives

$$\begin{aligned}\frac{d(N_{v,y}\,h_m A)}{dy} &= \frac{d}{dy}\left[\rho_m\,A\,\alpha_m\,(\omega_v\,\frac{d\,h_v}{dy}+\omega_a\,\frac{d\,h_a}{dy})\right]\\ &+ \frac{d}{dy}\left[\rho_m\,A\,D\,(\frac{d\omega_v}{dy}\,h_v+\frac{d\omega_a}{dy}\,h_a)\right]\end{aligned}$$

We now define Schmidt No (Sc) $\equiv \nu/D$ and Lewis No (Le) $\equiv$ Pr/Sc = D/α . For gaseous mixtures , Le $\simeq$ 1 assumption (or, D = α) is routinely made. Then

$$\begin{aligned}\frac{d(N_{v,y}\,h_m A)}{dy} &= \frac{d}{dy}\left[\Gamma_{m,h}\,A\,\left\{\frac{d}{dy}\,(\omega_v\,h_v+\omega_a\,h_a)\right\}\right]\\ &= \frac{d}{dy}\left[\Gamma_{m,h}\,A\,\frac{d\,h_m}{dy}\right]\end{aligned}$$

where $\Gamma_{m,h}=\rho_m\,D=\rho_m\,\alpha_m$

Inert MT with HT - 3 - L32($\frac{8}{20}$)

1. Now, from species conservation, $N_{v,y} = N_w$. Hence, the last result can also be written as

$$\frac{d}{dy}\left[A\left\{N_w\left(h_m - h_{m,T}\right) - \Gamma_{m,h}\frac{d}{dy}\left(h_m - h_{m,T}\right)\right\}\right] = 0$$

   where $h_{m,T} = c_{p,l}\left(T_T - T_{ref}\right)$ is the specific enthalpy of the make-up water deep inside the neighbouring phase. $c_{p,l}$ is liquid specific heat. This is again an eqn in conserved property $(h_m - h_{m,T})$

2. Integration as before gives

$$N_w = g^*_{mh}\ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh}\ln\left(1 + B_h\right) \text{ where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}} \text{ and } g^*_{mh} = \frac{\Gamma_{mh}}{r_w} \text{ or } \frac{\Gamma_{mh}}{L}$$

Comments - 1 - L32($\frac{9}{20}$)

1. Because Le = 1, $\Gamma_{mh} = \Gamma_m = \Gamma_h$. Hence,

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}} = B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

2. This relation provides the important link between $\omega_{v,w}$ and T_w because $h_{m,w} = h_{v,w}\,\omega_{v,w} + h_{a,w}\,(1 - \omega_{v,w})$. Hence,taking $T_{ref} = 0$,

$$h_{m,w} = c_{p,a}\,T_w + [(c_{p,v} - c_{p,a})\,T_w + \lambda_{ref}]\,\omega_{v,w}$$

3. Hence, for given T_∞ and T_T, the $B_m = B_h$ relation will iteratively give $\omega_{v,w} \sim T_w$. If Le = 1 assumption is correct, this relation must be same as the saturation equilibrium relationship corresponding to RH = 100 % of the psychrometric chart .

Comments - 2 - L32($\frac{10}{20}$)

For air-water vapour mixture, saturation condition is correlated as

$$\begin{aligned}\omega_{v,w} &\simeq 3.416\times10^{-3} + (2.7308\times10^{-4})\,T_w + (1.372\times10^{-5})\,T_w^2\\ &+ (8.2516\times10^{-8})\,T_w^3 - (6.9092\times10^{-9})\,T_w^4\\ &+ (3.5313\times10^{-10})\,T_w^5 - (3.7037\times10^{-12})\,T_w^6\\ &+ (6.1923\times10^{-15})\,T_w^7 + (9.9349\times10^{-17})\,T_w^8\end{aligned}$$

where $-20 < T_w\,(C) < 100$.

For the evaporating fuel, $\omega_{v,w} \sim T_w$ relation must be determined from Clausius-Clapeyron equation. Thus

$$\begin{aligned}\omega_{v,w} &= \left(\frac{p_{sat}\,(T_w)}{p_{tot}}\right)\times\left(\frac{M_v}{M_{mix}}\right) = x_{v,w}\left(\frac{M_v}{M_{mix}}\right)\\ x_{v,w} &= \exp\left\{-\frac{h_{fg}}{R_g}\left(\frac{1}{T_w}-\frac{1}{T_{bp}}\right)\right\} \rightarrow T_{bp} \equiv \text{(Boiling Point)}\end{aligned}$$

MT with HT and SCR - 1 - L32($\frac{11}{20}$)

1. Consider highly volatile liquid fuel that burns in the considered phase according to Simple Chemical Reaction (SCR)

   $$1 \text{ kg of fuel} + r_{st} \text{ kg of } O_2 = (1 + r_{st}) \text{ kg of products}$$

   where r_{st} is stoichiometric ratio for the fuel.

2. We have 3 species, fuel, oxygen and products. Hence,

$$\begin{aligned}
\frac{d}{dy}\left[A\left(N_w\,\omega_{fu} - \rho_m\,D\,\frac{d\,\omega_{fu}}{dy}\right)\right] &= -\,|\,R_{fu}\,|\,A \\
\frac{d}{dy}\left[A\left(N_w\,\omega_{O_2} - \rho_m\,D\,\frac{d\,\omega_{O_2}}{dy}\right)\right] &= -\,|\,R_{O_2}\,|\,A \\
\frac{d}{dy}\left[A\left(N_w\,\omega_{pr} - \rho_m\,D\,\frac{d\,\omega_{pr}}{dy}\right)\right] &= R_{pr}\,A
\end{aligned}$$

MT with HT and SCR - 2 - L32($\frac{12}{20}$)

1. Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
2. Thus, SCR implies $R_{O_2} = r_{st}\, R_{fu}$, $R_{pr} = -\,(1 + r_{st})\, R_{fu}$ and $\dot{m}''_{O_2} = r_{st}\, \dot{m}''_{fu}$, $\dot{m}''_{pr} = -\,(1 + r_{st})\, \dot{m}''_{fu}$
3. Further dividing 2nd eqn by r_{st} and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\,(N_w\,\Phi - \Gamma_m\,\frac{d\,\Phi}{dy})\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}: \; N_w = g_m^*\,\ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g_m^*\,\ln\,(1 + B_m) \;\; \text{where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g_m^* = \frac{\Gamma_m}{r_w} \;\text{or}\; \frac{\Gamma_m}{L}$$

MT with HT and SCR - 3 - L32($\frac{13}{20}$)

1. The energy eqn will read as

$$\frac{d}{dy}\left[A\,(N_w\,h_m - k_m\,\frac{d\,T}{dy})\right] = -\,\frac{d\,A\,\sum_k\,(\dot{m}''_{y,k}\,h_k)}{dy} = (\mathrm{RHS})$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

2. Hence, making use of definitions of Φ

$$\begin{aligned}
h_m &= \sum \omega_k\,h_k = \sum \omega_k\,h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\
&= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\,h^0_{f,O_2} + \omega_{pr}\,h^0_{f,pr} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,h^0_{f,fu} + \omega_{fu}\,r_{st}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,\left\{h^0_{f,fu} + r_{st}\,h^0_{f,O_2} - (1+r_{st})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref})
\end{aligned}$$

MT with HT and SCR - 4 - L32($\frac{14}{20}$)

We now consider the RHS. Then

$$-\sum_k (\dot{m}''_{y,k}\, h_k) \;=\; (h^0_{fu} + c_{p,fu}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{fu}}{dy} +$$

$$(h^0_{O_2} + c_{p,O_2}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{O_2}}{dy} \;+\; (h^0_{pr} + c_{p,pr}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{pr}}{dy}$$

If we now assume that $c_{p,k} = c_{pm}$ (equal specific heats) and use stoichiometric relations $\omega_{O_2} = r_{st}\,\omega_{fu}$ and $\omega_{pr} = -(1 + r_{st})\,\omega_{fu}$

$$-\sum_k (\dot{m}''_{y,k}\, h_k) = \Delta h_c\, \rho_m\, D\, \frac{d\,\omega_{fu}}{dy}$$

because $c_{pm}\,\Delta T\;\sum_k\; D\, d\,\omega_k/dy = 0$. Also, from the previous slide

$$c_{pm}\frac{dT}{dy} = \frac{d\,h_m}{dy} - \Delta h_c\,\frac{d\,\omega_{fu}}{dy}$$

MT with HT and SCR - 5 - L32($\frac{15}{20}$)

Hence, the energy Eqn will read as

$$\frac{d}{dy}\left[A\left\{N_w\,h_m - \frac{k_m}{c_{pm}}\,(\frac{d\,h_m}{dy} - \Delta h_c\,\frac{d\,\omega_{fu}}{dy}) - \Delta h_c\,\rho_m\,D\,\frac{d\omega_{fu}}{dy}\right\}\right] = 0$$

Noting that $k_m/c_{pm} = \rho_m\,\alpha_m = \Gamma_h$ and assuming $\alpha_m = D$ (or Le = 1) , the above eqn can be rearranged so that the Burning flux can be calculated from

$$\frac{d}{dy}\ \left[A\,(N_w\,h_m - \Gamma_h\,\frac{d\,h_m}{dy})\right] = 0$$

$$N_w \;=\; g^*_{mh}\ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh}\ln\,(1 + B_h)\ \text{where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}}\ \text{and}\ g^*_{mh} = \frac{\Gamma_{mh}}{r_w}\ \text{or}\ \frac{\Gamma_{mh}}{L}$$

Comments - L32($\frac{16}{20}$)

1. In general, for an SCR ,
    1. $h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c$, $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T$
    2. $h_{fu} = c_{pm}\,\Delta T$, $h_{O_2} = c_{pm}\,\Delta T + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$ and $h_{pr} = c_{pm}\,\Delta T$
    3. $h_{fu} = c_{pm}\,\Delta T$, $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T - \frac{\omega_{pr}}{1+r_{st}}\,\Delta h_c$
2. For a liquid fuel burning in air, we choose 2nd type

$$h_m = \sum_k \omega_k\,h_k = c_{p,m}\,(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$$

. where $T_{ref} = T_w$. Then, B_h can be calculated from

$$B_h = \frac{c_{pm}\,(T_\infty - T_w) + \Delta h_c\,(\omega_{O_2,\infty} - \omega_{O_2,w})/r_{st}}{\Delta h_c\,\omega_{O_2,w}/r_{st} - c_{p,l}\,(T_T - T_w)}$$

where $\omega_{O_2,T} = 0$ and T_T is known or knowable

3. If $T_w = T_{bp}$ (boiling point) then $\omega_{O_2,w} = 0$
4. If $T_w \neq T_{bp}$ then $\omega_{O_2,w} \sim T_w$ is iteratively determined from $B_m = B_h$.

MT with HT and ACR - 1 - L32($\frac{17}{20}$)

1. Here, the governing eqn is

$$\frac{d}{dy}\left[A\,(N_w\,\eta_\alpha - \Gamma_h\,\frac{d\,\eta_\alpha}{dy})\right] = 0$$

   where η_α is a conserved property with α = C, H, N or O.

2. This eqn is often used for solids combustion or ablation .
3. Thus, consider burning of Graphite (C^*) at high temperature ($T_w \sim$ 1950 K). Then the reaction mechanism is

   R1 : $C^* + 0.5\,O_2 = CO\ \ (K_p = \infty)$,
   R2 : $C^* + CO_2 = 2\,CO\ \ (K_p = 4000)$,
   R3 : $C^* + H_2O = CO + H_2\ \ (K_p = 1230)$,
   R4 : $C^* + 2\,H_2 = CH_4\ \ (K_p = 1/790)$,
   R5 : $CO_2 = CO + 0.5\,O_2\ \ (K_p = 1/1800)$,
   R6 : $H_2O = H_2 + 0.5\,O_2\ \ (K_p = 1/40,000)$

MT with HT and ACR - 2 - L32($\frac{18}{20}$)

1. Let the ∞ state comprise complete products CO_2, H_2O and N_2 only
2. Noting the equilibrium constants K_p for each reaction, it can be shown that in the considered phase, CH_4 cannot survive in appreciable magnitudes . Hence, it will comprise CO_2, H_2, CO and H_2O only.
3. Similarly, in the w-state, only CO and H_2 will survive
4. Since, species change in different stets, it is best to define

$$\begin{aligned}
\eta_C &= \omega_{C*} + \frac{12}{44}\,\omega_{CO_2} + \frac{12}{28}\,\omega_{CO}\ , \\
\eta_H &= \omega_{H_2} + \frac{2}{18}\,\omega_{H_2O}\ , \\
\eta_O &= \frac{32}{44}\,\omega_{CO_2} + \frac{16}{28}\,\omega_{CO} + \frac{16}{18}\,\omega_{H_2O}
\end{aligned}$$

5. Thus, we have 3 eqns in η_C, η_O and η_H.

MT with HT and ACR - 3 - L32($\frac{19}{20}$)

1. We need to solve for any one element because the conserved property solution are same for each η_α. Difficulty arises in applying BCs
2. Since mass-fractions of CO and H_2 are not known in the w-state, we define a composite variable

$$\Phi \equiv \eta_C - \frac{3}{4}\,\eta_O = \omega_C - \frac{3}{11}\,\omega_{CO_2} - \frac{2}{3}\,\omega_{H_2O}$$

3. Then,

$$\begin{aligned}
\Phi_w &= (\eta_C - \frac{3}{4}\,\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0 \\
\Phi_\infty &= 0 - \frac{3}{11}\,\omega_{CO_2,\infty} - \frac{2}{3}\,\omega_{H_2O,\infty} \\
N_w &= g_m^* \ln\left(1 + B_m\right) \ \rightarrow \ B_m = \frac{3}{11}\,\omega_{CO_2,\infty} + \frac{2}{3}\,\omega_{H_2O,\infty}
\end{aligned}$$

Summary - L32($\frac{20}{20}$)

1. Thus, in all types of mass transfer, we have shown that MT is governed by conserved property Eqn

$$\frac{d}{dy}\left[A\,(N_w\,\Psi - \Gamma\,\frac{d\,\Psi}{dy})\right] = 0 \text{ where } A = \text{ const or } = 4\,\pi\,r^2$$

$$N_w = g \times B \quad \text{with} \quad \frac{g}{g^*} = \frac{\ln\,(1+B)}{B} \quad \rightarrow \quad B = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}$$

where $g^* = (\Gamma/r_w \text{ or } \Gamma/L)$ is value of g when $|B| \rightarrow 0$

2. In Inert MT without HT , $\Psi = \omega_v$ and $\Gamma = \rho_m\,D$
3. In Inert MT with HT , $\Psi = \omega_v$ and h_m and $\Gamma_{mh} = \rho_m\,D = \rho_m\,\alpha_m$ with Le = 1
4. In MT with SCR , $\Psi =$ appropriate Φ and h_m and $\Gamma_{mh} = \rho_m\,D = \rho_m\,\alpha_m$ with Le = 1 and equal $c_{p,k} = c_{pm}$
5. In MT with ACR , $\Psi =$ appropriate Φ and $\Gamma_m = \rho_m\,D$