EXERCISE

1. For the initial value problem $\frac{dy}{dx} = 3x + y^2, x_0 = 0, y_0 = 1$, find first three approximations by Picard's method for $x = 0.1$.

   Ans: $y^{(1)} = 1.11500$; $y^{(2)} = 1.12640$; $y^{(3)} = 1.12721$

2. Use Picard's method to find third approximation to solve

   $\frac{dy}{dx} = 1 + xy$, with $x_0 = 2$, $y_0 = 0$.

   Ans: $\frac{x^5}{15} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{22}{15}$

3. Using Euler's modified method, find a solution of the equation

   $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial condition $y = 1$ at $x = 0$ for the range $0 \le x \le 0.6$ in steps of 0.2. Carry all calculations to 4D only.

   Ans: $y^{ccc}_{(0.2)} = 1.2309$; $y^{ccc}_{(0.4)} = 1.5253$; $y^{ccc}_{(0.6)} = 1.8861$

4. Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$. Obtain the Taylor series for $y(x)$ and compute $y(0.1)$corret to four decimal places.

   Ans: $y(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} +$----------; $y(0.1) = 1.1053$

5. Given the differential equation

   $\frac{dy}{dx} = \frac{1}{x^2 + y}$ with $y(4) = 4$

   Obtain $y(4.1)$ and $y(4.2)$ by Taylor's series method to 4D.

   Ans: 4.0050, 4.0098

6. Use the Runge-Kutta fourth order method to find the value of y when $x = 1$ given that $y = 1$ when $x = 0$ and that $\frac{dy}{dx} = \frac{y - x}{y + x}$

   Ans: 1.4983

7. Using Runge-Kutta method, solve $y' = xy$ for $x = 1.4$ initially $x = 1$, y = 2 by taking $h = 0.2$.

   Ans: 2.99486

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