EXERCISE

1. Use Simpson's $\frac{1}{3}$rd rule dividing the range into ten equal parts, to show that

$$\int_0^1 \frac{\log_e (1+x^2)}{(1+x^2)} = 0.1730$$

2. Calculate an approximate value of $\int_0^{\pi/2} \sin x \, dx \, by$

   (i) Traperoidal rule

   (ii) Simpson's $\frac{1}{3}$rd rule, using 11 ordinates

Ans: $(i) 0.9981 \; (ii) 1.0006$

3. A river is 80 feet wide. The depth d (in feet) of the river at a distance x from one bank is given by the following table:

x:	0	10	20	30	40	50	60	70	80
d:	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross- section of the river using Simpson's $\frac{1}{3}$rd rule.

Ans:710 sq feet.

4. Compute the values of integral $I = \int_0^1 \frac{dx}{1+x^2}$ by using the Traperoidal rule with $h =$ 0.5,0.25 and 0.125. Then, obtain a better estimate by using Romberg's method. Compare your result with the true value.

Ans: $0.77500, 0.78279, 0.78475, 0.7854$

5. Determine the maximum error in evaluating the integral

$$\mathrm{I} = \int_0^{\pi} \cos \; x \; dx$$

by both Trapezoidal and Simpson's $\frac{1}{3}$rd rules using four subintervals.

Ans: $0.0202, 0.000173$

6. Using Simpson's $\frac{3}{8}$th rule, evaluate

$$\mathrm{I} = \int_0^1 \frac{1}{1+x} dx$$

with $h = 1/6$. Evaluate the integral by using Trapezoidal rule as well as Simpson's $\frac{1}{3}$rd , Simpson's $\frac{3}{8}$th rule and compare the results with exact value

Exact: 0.69315; $T.R.$ 0.69488; $\frac{1}{3}$rd $S.R.$; 0.69317; $\frac{3}{8}$th S. R.; 0.69320

7. Use three-point Gauss-Legendre formula to evaluate the integral $\int_0^{\pi} \sin x\, dx$. Compare this result with that obtained by Simpson's rule using seven points.

Ans: 1.00002

8. Compute the integral $I = \int_{1.0}^{1.5} \sqrt{x}\, dx$, corret to 5D using Simpson's $\frac{1}{3}$rd Simpson's rule by finding h sufficiently large. Compare the result with two- terms Gaussian formula

Ans: 0.55808

9. Use Simpson's $\frac{3}{8}$th rule to evaluate $\int_0^{0.3} e^x dx$ where $h = 0.05$ and compare with the exact value. **Ans:** 0.34986

10. Evaluate the following integrals using Gauss three-points quadrature formula.

$(i) \int_{0.2}^{1.5} e^{-x^2} dx$ $(ii) \int_0^{\pi} sinx^2 dx$

$(iii) \int_{-4}^{4} \frac{1}{1+x^2} dx$ $(iv) \int_0^{\pi/2} \frac{1}{\sqrt{1-0.25sin^2x)}} dx$

Ans: $(i)0.65860$ $(ii) - 1.85036$ $(iii)3.97484$ $(iv)1.6857$

11. Using Romberg's method, compute the integral

$I = \int_1^2 e^x\, dx$, by taking $h = 0.5, 0.25, 0.125$ upto the order $O(h^6)$.

Ans: 4.67077

12. Evaluate the following integrals using Romberg's approach with Simpson's $\frac{1}{3}$rd rule correct to $O(h^6)$ by taking suitable h:

$i) \int_0^1 \frac{1}{1+x^2} dx$ $(ii) \int_4^{5.2} \log_e x\ dx$

Ans: 0.78540, 0.79383

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