# COMPUTATIONAL ERROR

**Lecture 1:** **(a) Definition of Error**

**(b) Types of Error**

**(c) Significant Digits**

**(d) Rounding off Rule**

## 1. Introduction

Numerical methods are to provide practical procedures for obtaining the numerical solutions of problems to a specified degree of accuracy. In numerical analysis, besides the study of the methods, one studies the errors involving in the methods and in the final results.

There are many kinds of error, which we shall discuss in detail later on. At present the meaning of error can be taken as follows; let an approximate value of a number be **x,** whose actual value is **X**. The value difference |X-x| is called the absolute error in X, denoted by $\Delta$ X=|X-x| and $\frac{\Delta X}{|X|}$ is called the relative error in X, which is represented by $\delta$X. Then, the percentage error in X is 100 times its relative error. That is, percentage error in X = $\delta$X×100%.

## 2. Types of error:

In general, the errors in a practical problem may get introduced into four forms as follows:

### I. Initial error/Error of the problem:

These are involved in the statement of problem itself. In fact, the statement of a problem generally gives an idealized model and not the exact picture of the actual phenomena. For example, in the calculation of the value of earth's gravitational force g by simple pendulum, the experiment is based upon certain axioms: such as (i) bob is weight less (ii) the motion of the bob is linear, that is, in a straight line; which are not true, in fact. So the value of the parameter (s) involved can only be determined approximately.

### II. Residual error or truncation error:

This error occurs when mathematical functions like

$$\sin x = x - \frac{X^3}{3!} + \frac{X^5}{5!} \ldots\ldots\ldots\ldots\ldots\ldots\infty \text{ and } e^x = 1 + x + \frac{X^3}{2!} + \frac{X^3}{3!} + \ldots\ldots\ldots\ldots\ldots\infty,$$

whose infinite series expansion exist, are used in the calculations. Because, in-calculating the value of such function for an assigned value of x, only a finite number of terms can be taken, an error get introduced for not considering the remaining terms.

### III. Rounding error/ Round-off error:

When the rational numbers like 1/3; 22/7; 5/9; 8/9 etc, whose decimal representation involve infinite number of digits, are involved in our calculations, we are forced to take only a few number of digits from their decimal expression and thus an error named round-off error gets involved. There are universal rules for rounding a number as rounding rules.

### IV. Inherent error /error of the operation

When performing computations with algebraic operations among approximate numbers, we naturally carry to some extent the errors of the original data into final result. Such errors are called inherent error/error of the operation.

For example, let x=0.3333 and y=3.1416 be two approximate numbers for the exact number 1/3 and $\pi$. Obviously, if we perform an algebraic operation between these two approximate numbers, the error will introduce in the final result accordingly.

## 3. Significant Digit.

In the decimal representation of a number, a digit is said to be significant if it is either a non-zero digit or any zero (s) lying between two non-zero digits are used as a placeholder, to indicate a retrained place. All other zeros used to fix-up the position of the decimal point are not to be counted as significant digit.

The number of significant digits in a number will be counted from the left-most non-zero digit towards right. Thus, the numbers 0.7452 and 0.007452 both have four significant digits. Similarly,

0.00400300 has 6 significant digits

0.4003000 has 7 significant digits

0.30040000 has 8 significant digits

**Rules for Rounding of numbers**

While performing any algebraic operation between two or more numbers written in the decimal number system, it is often required to round-off these numbers, that is, replace each of them having a smaller of significant digits. The rule for doing this as follows:

To round off a number to n significant figures, discard all digits to the right of the $n^{th}$ place; if the discarded number is less than half a unit in the $(n+1)^{th}$ place, leave the $n^{th}$ digit unchanged; if the discarded number is greater than half a unit in the $(n+1)^{th}$ place, add 1 to the $n^{th}$ digit; if the discarded number is exactly half a unit in the $(n+1)^{th}$ place, leave the $n^{th}$ digit unaltered if it is an even number, but increase it by 1 if it is an odd number.

**Correct digit:** In the decimal representation of an approximate number, the $n^{th}$ digit after decimal is said to be correct if the absolute error does not exceed one half unit in the $n^{th}$ place. For example, let an

exact number be A=35.974 and let its approximate value a=36.0

Absolute error $|A-a| = 0.026$

Since $0.026 < 0.05=\frac{1}{2}\times10^{-1} \leq 0.5\times10^{-n}$ for n=1, the approximation 36.0 for 35.974 is correct to one decimal place.

**Example 1.** Round off 37.897456 correct to 5 significant figures.

**Solution:** Discard all digits to the right of the $5^{th}$ place, which is in this case 456. For convenience and better understanding, assume that the discarded number is 0.456 (whatever digits are being discarded, put a decimal before that). Now this number 0.456 is less than half a unit, which is 0.5;

That is $0.456<0.5$ and hence leaving the $5^{th}$ place digit unchanged 37.897456 become 37.897 (correct to 5 significant figures).

**Example 2.** Round off 28.244 795 correct to 5 significant figures.

**Solution:** Discard all the digits to the right of the $5^{th}$ place, which is in this case 795. For convenience and better understanding, assume that the discarded number is 0.795 (whatever digits are being discarded, put a decimal before that). Now this number 0.795 is greater than half a unit, which is 0.5;

That is, $0.795>0.5$ and hence we add 1 to the $5^{th}$ place digit. 28.244795 become 28.245 (correct to 5 significant figures).

**Example 3.** Round off 6.000 559 correct to 4 significant figures.

**Solution:** Discard all the digits to the right of the $4^{th}$ place, which is in this case 559. For convenience and better understanding, assume that the discarded number is 0.559 (whatever digits are being discarded, put a decimal before that). Now this number 0.559 is greater than half a unit, which is 0.5;

That is $0.559>0.5$ and hence we add 1 to the $4^{th}$ place digit. 6.000559 become 6.001 (correct to 4 significant figures).

**Example 4.** Round off 6.002500 correct to 4 significant figures.

**Solution:** Discard all the digits to the right of the $4^{th}$ place, which is in this case 500. Assume that the discarded number is 0.500 (whatever digits are being discarded, put a decimal before that). Now this number 0.500 is exactly equal to half a unit, which is 0.5;

That is $0.500=0.5$ and since the $4^{th}$ place digit is 2 (even), we keep the $4^{th}$ significant digit unaltered. That is 6.002500 become 6.002 (correct to 4 significant figures).

**Example 5.** Round off 5.001500 correct to 4 significant figures.

**Solution:** Discard all the digits to the right of the $n^{th}$ place, which is in this case 500. Assume that the discarded number is 0.500 (whatever digits are being discarded, put a decimal before that). Now this number 0.500 is exactly equal to half a unit, which is 0.5;

That is, $0.500=0.5$ and since the $4^{th}$ place digit is 1(odd), we add 1 to the $4^{th}$ place digit. That is, 5.001500 become 5.002 (correct to 4 significant figures).

# Exercises

1. Round off 0.070000123456 correct to 4 significant figures.
2. Round-off the following numbers correct to 4- significant figures.

   (i) 4.79132, (ii) 23.2975, (iii) 0.000956754, (iv) 0.0082665, (v) 4378.562,

   (vi) 2.000469, (vii) 3.35008, (viii) 87.5555, (ix) 4.000559, (x) 0.0000300085
3. Round-off the following numbers correct to 4- significant figures:

   (i) 3.96312, (ii) 49.00088, (iii) 0.0000656. (iv) 76.000654, (v) 28.555555,

   (vi) 5.00109600, (vii) 538.29059, (viii) 0.0129456, (ix) 1.45008, (x) 8.999999
4. If f(x) = 5tan(x) – 9x, find the percentage error in f(x) for x=$\frac{\pi}{4}$, if the error in

   x is 0.003.

5. Find the sum of the following approximate numbers, if the numbers

   are correct to the last digit:

   a) 8.37642, 6.876, 2.896555, 0.22359
   b) 0.00024356, 19.42, 8.987, 0.35793, 173682, 3.73928

6. a) Find the absolute error ($E_A$), relative error ($E_R$) and percentage error ($E_P$) of the numbers whose true value ($V_T$) and approximate value ($V_A$) are given:
   i) $V_T$ = 5.86593, $V_A$=5.866;
   ii) $V_T$ = 2.55555, $V_A$ = 2.556;
   iii) $V_T$ = 9.75600, $V_A$=9.757

   b) Given $E_A$= $0.5\times10^{-3}$, $E_R$=$0.37\times10^{-5}$, find $V_T$.
   c) If $V_A$=0.468, $E_P$=6%, find $V_A$.