NPTEL web course on Complex Analysis

A. Swaminathan
I.I.T. Roorkee, India

and

V.K. Katiyar
I.I.T. Roorkee, India

Module: 1: **Introduction**
Lecture: 1: **Number System**

Introduction

Number system

Natural Numbers

$$\mathbb{N} := \{1, 2, 3, \ldots\}.$$

- Closed under addition (Addition of two natural numbers is again a natural number).
- $n + 0 = n$ for all $n \in \mathbb{N}$.

Natural Numbers

$$\mathbb{N} := \{1, 2, 3, \ldots\}.$$

- Closed under addition (Addition of two natural numbers is again a natural number).
- $n + 0 = n$ for all $n \in \mathbb{N}$.

Constraints

Does not solve the equations of the form $x + n = 0$ when $n \in \mathbb{N}$.

Integers

$$\mathbb{Z} := \{0, \pm 1, \pm 2, \ldots\}.$$

- Closed under addition and multiplication
- Closed under subtraction as well; i.e., it has an additive inverse.

Integers

$$\mathbb{Z} := \{0, \pm 1, \pm 2, \ldots\}.$$

- Closed under addition and multiplication
- Closed under subtraction as well; i.e., it has an additive inverse.

Constraints

Not closed under division.

Rational Numbers

$$\mathbb{Q} := \{x : x = p/q, p, q \in \mathbb{Z}, q \neq 0\}.$$

- Assumed that p and q have the greatest common divisor as 1.
- Satisfies the algebraic properties such as commutative law of addition and multiplication.
- Satisfies associative law of addition and multiplication and distributive law.
- Solves the equations of the form $\alpha x + \beta = 0$, where $\alpha, \beta \in \mathbb{Q}$, $\alpha \neq 0$.

Solution of quadratic equations

Consider a quadratic equation of the form

$$x^2 - \alpha = 0, \qquad \alpha > 0.$$

Then $x = \pm\sqrt{\alpha}$.

- If α is a perfect square of the form $\alpha = \gamma^2$, then $x = \pm\gamma$.
- If otherwise, these solutions $\pm\sqrt{\alpha}$ belongs to a set called irrational numbers.

Solution of quadratic equations

Consider a quadratic equation of the form

$$x^2 - \alpha = 0, \qquad \alpha > 0.$$

Then $x = \pm\sqrt{\alpha}$.

- If α is a perfect square of the form $\alpha = \gamma^2$, then $x = \pm\gamma$.
- If otherwise, these solutions $\pm\sqrt{\alpha}$ belongs to a set called irrational numbers.

Irrational Numbers

$$e, \pi, \sqrt{\alpha}, \sqrt[3]{\alpha}, \sqrt[4]{\alpha}, \ldots, \qquad \text{if} \quad \alpha \text{ - prime},$$

are also irrational numbers.

Real Numbers

- Notation: $\mathbb{R}$.
- Geometrically the points on the real line are considered to be elements of $\mathbb{R}$.
- The set $\mathbb{R}$ is an ordered set, in the sense that, for any two distinct elements a and b in $\mathbb{R}$, there exist a relation that either $a < b$ or $b < a$.
- Between any two rational numbers there exist at least one irrational number and between any two irrational numbers there exist at least one rational number.

Algebra of Complex number system

Complex Number system

Motivation

- We extend the Real number system to a more general one called Complex number System denoted by $\mathbb{C}$ by defining the number

$$i = \sqrt{-1},$$

i.e., i is a root of the equation $x^2 + 1 = 0$ to solve the equation of the form $x^2 + 1 = 0$ which is the simplest form of $x^2 + \alpha = 0$, $\alpha > 0$.

Complex Number system

Complex number

Definition

A **Complex number** is an expression of the form $a + ib$, where a and b are real numbers. Two complex numbers $a + ib$ and $c + id$ are said to be equal ($a + ib = c + id$) if and only if a=c and b=d.

Complex Number system

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.

Complex Number system

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.
- The element $a + ib$ is also denoted by the ordered pair (a, b), where a is called the real part and b is called the imaginary part.

Complex Number system

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.
- The element $a + ib$ is also denoted by the ordered pair (a, b), where a is called the real part and b is called the imaginary part.
- The set of all points $x + i0$ is called the real axis and the set of all points $o + iy$ is called the imaginary axis.

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.
- The element $a + ib$ is also denoted by the ordered pair (a, b), where a is called the real part and b is called the imaginary part.
- The set of all points $x + i0$ is called the real axis and the set of all points $o + iy$ is called the imaginary axis. For the complex number $z = x + iy$, $x, y \in \mathbb{R}$, y is the imaginary part and iy is not the imaginary part.

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.
- The element $a + ib$ is also denoted by the ordered pair (a, b), where a is called the real part and b is called the imaginary part.
- The set of all points $x + i0$ is called the real axis and the set of all points $o + iy$ is called the imaginary axis. For the complex number $z = x + iy$, $x, y \in \mathbb{R}$, y is the imaginary part and iy is not the imaginary part.
- The point $0 + i0$ which is common to both real and imaginary axis is called the origin.

Complex Number system

Complex Plane

- Elements in the complex number system are observed as points on a plane. This plane is called the Complex plane.
- The element $a + ib$ is also denoted by the ordered pair (a, b), where a is called the real part and b is called the imaginary part.
- The set of all points $x + i0$ is called the real axis and the set of all points $o + iy$ is called the imaginary axis. For the complex number $z = x + iy$, $x, y \in \mathbb{R}$, y is the imaginary part and iy is not the imaginary part.
- The point $0 + i0$ which is common to both real and imaginary axis is called the origin.
- The members of the complex plane are usually denoted by the symbol z, where $z = x + iy$.

Complex Number system

Complex Plane