

Module 4 : Theory of Covering Spaces

Lecture 21 : Test - III

1. Show that a homeomorphism of E^2 onto itself must preserve the boundary. That is it must map a boundary point to a boundary point.

2. Is it true that $\mathbb{R}P^3$ minus a point deformation retracts to a space homeomorphic to $\mathbb{R}P^2$?

3. Let G be the infinite grid

$$G = \{(x, y) \in \mathbb{R}^2 / x \in \mathbb{Z} \text{ or } y \in \mathbb{Z}\}.$$

Consider the covering map from G onto the figure eight loop $(S^1 \times \{1\}) \cup (\{1\} \times S^1)$ given by

$$p(x, y) = (\exp(2\pi ix), \exp(2\pi iy)).$$

Determine the deck transformations of this covering. Is this a regular covering?

4. Given topological spaces X and Y , a map $p : X \rightarrow Y$ is said to be a local homeomorphism if each $x_0 \in X$ has a neighborhood N_{x_0} such that the restriction map

$$f|_{N_{x_0}} : N_{x_0} \rightarrow f(N_{x_0})$$

is a homeomorphism. Show that a local homeomorphism which is a proper map is a covering projection.

5. Show that the map $f : \mathbb{C} - \{0, 1, -1\} \rightarrow \mathbb{C} - \{\pm 2\}$ is a local homeomorphism.

Is this map a covering projection? If so what is the group of deck transformations?