Module 3 : Fundamental groups & its basic properties

Lecture 14 : Test - II

1. Suppose X is a metric space and A is a retract of X. Show that A is closed in X. Is the space homeomorphic to the letter Y a deformation retract of a space homeomorphic to E^2 ?

2. Show that if X has the fixed point property and A is a retract of X then A also has the fixed point property.

3. Find the degree of the following maps $f : S^1 \longrightarrow S^1$ given by:

   (i) $f(z) = \exp(z- \text{ Re } z)$. (ii) $f(z) = \bar{z}^2 z^3$.

4. Show that S^1 is not homeomorphic to any subset of $\mathbb{R}$. Can S^2 be homeomorphic to a subset of $\mathbb{R}^2$?

5. Determine $\pi_1(\mathbb{R}P^2 - \{p\})$ where p is any point of $\mathbb{R}P^2$.

6. For the map $f : S^1 \longrightarrow S^1 \times S^1$ given by $f(z) = (z^p, z^q)$, where p and q are positive integers, find the induced group homomorphism $f_* : \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}$.