Exercises:

1. Show that the sphere S^2 retracts onto one of its longitudes. If X is the space obtained from S^2 by taking its union with a diameter, there is a surjective group homomorphism $\pi_1(X) \longrightarrow \mathbb{Z}$.
2. Prove that A is a retract of X if and only if every space Y, every continuous map $f : A \longrightarrow Y$ has a continuous extension $\tilde{f} : X \longrightarrow Y$.
3. Show that the fundamental group respects arbitrary products.
4. Construct a retraction from $\{(x, y) : x$ or y is an integer$\}$ onto the boundary of I^2.
5. Show that every homeomorphism of E^2 onto itself must map the boundary to the boundary.
6. Given that there exists a functor T from the category $\mathbf{Top}$ to the category $\mathbf{AbGr}$ such that $T(X)$ is the trivial group for every convex subset X of a Euclidean space and $T(S^n)$ is a non-trivial group, prove that S^n is not a retract of the closed unit ball in $\mathbb{R}^{n+1}$.