Exercises:

1. Explicitly construct a homotopy between the loop $\gamma(t) = (\cos 2\pi t, \sin 2\pi t, 0)$ on the sphere S^2 and the constant loop based at $(1, 0, 0)$. Note that an explicit formula is being demanded here.
2. Show that a loop in X based at a point $x_0 \in X$ may be regarded as a continuous map $f : S^1 \longrightarrow X$ such that $f(1) = x_0$. Show that if f is homotopic to the constant loop ε_{x_0} then f extends as a continuous map from the closed unit disc to X.
3. Show that if γ is a path starting at x_0 and γ^{-1} is the inverse path then prove by imitating the proof of the reparametrization theorem (that is by taking convex combination of two functions) that $\gamma * \gamma^{-1}$ is homotopic to the constant loop ϵ_{x_0}.
4. Prove theorems (7.2) and theorem (7.6) using Tietze's extension theorem.
5. Suppose $\phi : [0, 1] \longrightarrow [0, 1]$ is a continuous function such that $\phi(0) = \phi(1) = 0$ and γ is a closed loop in X based at $x_0 \in X$. Is it true that $\gamma \circ \phi$ is homotopic to the constant loop ε_{x_0}? 6. Show that the group isomorphism in theorem (7.8) is natural namely, if $f : X \longrightarrow Y$ is continuous and $x_1, x_2 \in X$ then

$$h_{[f\circ\sigma]} \circ f'_* = h_{[\sigma]} \circ f''_*$$

where, $y_1 = f(x_1)$, $y_2 = f(x_2)$ and σ is a path joining x_1 and x_2. The maps f'_* and f''_* are the maps induced by f on the fundamental groups. This information is better described by saying that the following diagram *commutes:*

$$\begin{array}{ccc} \pi_1(X, x_1) & \xrightarrow{f'_*} & \pi_1(Y, y_1) \\ h_{[\sigma]}\downarrow & & h_{[f\circ\sigma]}\downarrow \\ \pi_1(X, x_2) & \xrightarrow{f''_*} & \pi_1(Y, y_2) \end{array}$$