Exercises:

1. Show that if R' and R'' are two reflections (each with respect to a coordinate plane) then they are conjugate by a homeomorphism. Deduce that both R' and R'' have degree -1.

2. Show that if a continuous map $f : S^n \longrightarrow S^n$ misses a point of S^n then f is homotopic to the constant map and so has degree zero.

3. Show that if n is odd then the antipodal map of S^n is homotopic to the identity map. Hint: Do it first for the case and show that the homotopy may be achieved via a continuous rotation. The general case follows along similar lines by working with pairs of coordinates.

4. Show that $\mathbb{R}P^{2n}$ has the fixed point property.

5. Let $\eta : S^{2n} \longrightarrow \mathbb{R}P^{2n}$ be the covering projection. Show that $H_{2n}(\eta)$ is the zero map.

6. Show that the map (36.5) is a homeomorphism and (36.6) defines a continuous map. More generally given a continuous map $f : X \longrightarrow Y$ show that the composite

$$X \times [0,1] \xrightarrow{f \times \mathrm{id}} Y \times [0,1] \longrightarrow \Sigma Y$$

   induces a map $\Sigma f : \Sigma X \longrightarrow \Sigma Y$. Imitate the computation in theorem [//] of lecture [//] to show that $H_{n+1}(\Sigma X) = H_n(X)$ when $n \geq 1$. What happens when $n = 0$?

7. Prove theorem (36.11). Note that the map $f : S^1 \longrightarrow S^1$ given by $f(z) = z^m$ has degree m.

8. Determine the degree of a polynomial as a map from S^2 to itself. Reprove the fundamental theorem of algebra.