

**Exercises:**

1. Prove that a homeomorphism  $E^n$  onto itself maps each boundary point of  $E^n$  to a boundary point.
2. Determine the homology groups of the Klein's bottle.
3. Determine the homology groups of the double torus.
4. Establish the isomorphism  $H_0(U \cap V) \rightarrow H_0(U) \oplus H_0(V)$  in the proof of theorem (35.4)
5. Let  $C_k$  be the disjoint union of  $k$  copies of  $S^1$  in  $\mathbb{R}^3$ . Determine the homology groups of the complement  $\mathbb{R}^3 - C_k$ .
6. Determine the homology groups of  $\mathbb{R}P^3$ . Try computing the homology groups of  $\mathbb{R}P^4$ .
7. Determine the homology groups of  $S^m \vee S^m$ . Use exercise 4 of lecture 25. to calculate the homology groups of  $S^2 \times S^4$ .