Exercises:

1. Show that the map defined by (34.1) is the restriction to Δ_p of an affine map. Note: An affine map is the composition of a linear map and a translation.

2. Suppose $T : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{m+1}$ is an affine map such that $T(\Delta_n) \subset \Delta_m$, then $T_\sharp$ maps the subgroup $A_p(\Delta_n)$ into $A_p(\Delta_m)$ and is a chain map from the complex $\{A_p(\Delta_n)\}$ to $\{A_p(\Delta_m)\}$. Further prove the following:

(i) If $\mathbf{b} \in \Delta_n$ and $\sigma \in A_p(\Delta_n)$ then . $T_\sharp(K_{\mathbf{b}}\sigma) = K_{T\mathbf{b}}(T_\sharp\sigma)$

(ii) If **b** is the barycenter of σ then **b** is the barycenter of $T_\sharp\sigma$.

What happens if we consider a *degenerate* two simplex where the points $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are not affinely independent? Discuss the case of the two simplex $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_2]$.

3. Examine what happens if the term referred to as junk in equation (34.7) is retained.

4. Complete the details of the proof of theorem (34.4).

5. Show that $\mathcal{B}^k$ is chain homotopic to the identity map. What is the chain homotopy?

6. Suppose that the maps g and h in the exact sequence

$$A \longrightarrow B \xrightarrow{g} C \xrightarrow{h} D \longrightarrow E$$

are replaced by the composites

$$\tilde{g} : B \xrightarrow{g} C \xrightarrow{\lambda} X, \qquad \tilde{h} : X \xrightarrow{\lambda^{-1}} C \xrightarrow{h} D$$

the result is again an exact sequence.

7. Fill in the details in the proof of theorem (34.8). See exercise 6 of lecture 29.