

Exercises:

1. Show that the $p + q - 1$ chain on the right hand side of (33.4) is a cycle.
2. Check that $\sigma \times \tau$ as defined by equation (33.5) satisfies (33.1).
3. Show that the product in theorem (33.1) defines a bilinear map
$$H_p(X) \times H_q(Y) \longrightarrow H_{p+q}(X \times Y).$$
4. Determine explicitly the two/three chain z satisfying (33.4) when
 - (i) $p = 1$ and $q = 1$.
 - (ii) $p = 1$ and $q = 2$.

Hint: In the proof of lemma (32.2), we chopped the square into two triangles. When Π_X we need to chop a prism into three pieces and map Δ_3 affinely onto each of them.

5. Use the map Π_X of the previous lecture to calculate the generators of $H_1(S^1 \times S^1)$.
6. Use equation (33.1) to determine the image of the pair of generating one cycles of the previous exercise under the map $H_1(S^1) \times H_1(S^1) \longrightarrow H_2(S^1 \times S^1)$.