**Exercises:**

1. Prove theorem (31.3).
2. Show that for a path connected space $X$, every singleton $\{p\}$ with $p \in X$ is a basis for $H_0(X)$.
3. Complete the proof of theorem (31.5).
4. Show that the set $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k \in \mathbb{R}^n$ is affinely independent if the vectors
$$\mathbf{v}_1 - \mathbf{v}_j, \ldots, \mathbf{v}_{j-1} - \mathbf{v}_j, \mathbf{v}_{j+1} - \mathbf{v}_j, \ldots, \mathbf{v}_{k-1} - \mathbf{v}_k$$
are linearly independent for any $j$ ( $1 \leq j \leq k$).
5. Prove theorem (31.6). Show that the barycentric coordinates are continuous functions of $\mathbf{x}$. All but the $j$-th barycentric coordinates of $\mathbf{v}_j$ vanish. The set of points in (31.2) obtained by setting $t_j = 0$ and varying the other coefficients is called the $j-$th face of the simplex spanned by the given points.
6. Check the continuity of the map $T\sigma$ in theorem (31.7).