Module 2 : General Topology

Lecture 2 : Preliminaries from general topology

Exercises:

1. Prove that a topological space is compact if and only if it satisfies the following condition known as the *finite intersection property*. For every family $\{F_\alpha\}$ of closed sets with $\cap_\alpha F_\alpha = \emptyset$, there is a finite sub-collection whose intersection is empty
2. Show that $f : [0,1] \longrightarrow [0,1]$ is continuous if and only if its graph is a compact subset of I^2.
3. Examine whether the exponential map from $\mathbb{C}$ onto $\mathbb{C} - \{0\}$ is proper. What about the exponential map as a map from $\mathbb{R}$ onto $(0, \infty)$?
4. (Gluing Lemma) Suppose that $\left\{U_\alpha\right\}_{\alpha\in\Lambda}$ is a family of open subsets of a topological space and for each $\alpha \in \Lambda$ we are given a continuous function $f_\alpha : U_\alpha \longrightarrow Y$. Assume that whenever $f_\alpha(x) = f_\beta(x)$ whenever $x \in U_\alpha \cap U_\beta$. Show that there exists a unique continuous function $f : \bigcup_{\alpha\in\Lambda} U_\alpha \longrightarrow Y$ such that $f(x) = f_\alpha(x)$ for all $x \in U_\alpha$ and for all $\alpha \in \Lambda$. Show that the result holds if all the U_α are closed sets and Λ is a finite set.
5. How would you show rigorously that the closed unit disc in the plane is homeomorphic to the closed triangular region determined by three non-collinear points? You are allowed to use results from complex analysis, provided you state them clearly.
6. Prove that any two closed triangular planar regions (as described in the previous exercise) are homeomorphic. Show that any such closed triangular region is homeomorphic to I^2.
7. Suppose that Z is a Hausdorff space and $f, g : Z \longrightarrow X$ are continuous functions then the set $\{z \in Z / f_1(z) = f_2(z)\}$ is closed in Z.
8. Show that the space obtained by rotating the circle $(x-2)^2 + y^2 = 1$ about the $y-$axis is homeomorphic to $S^1 \times S^1$.