Exercises:

1. Fill in the details in the computation of the fundamental group of the projective plane, Klein's bottle and the torus done in the lecture by providing a careful proof of equations (26.8), (26.10) and (26.11). Hint: Use polar coordinates. Continuously shrink the path β to the point x_0.

2. Show that the fundamental group of the wedge of n copies of S^1 is the free group on n generators. Calculate the fundamental group of the truncated grid
$$\{(x, y) \in \mathbb{R}^2 / x \in \mathbb{Z} \text{ or } y \in \mathbb{Z},\ 0 \leq x \leq n,\ 0 \leq y \leq n\}.$$

3. Determine the generators of double torus by expressing it as a union of open sets each of which is a torus from which a tiny closed disc has been removed.

4. Let C be the union of the two *unlinked* circles
$$(x-2)^2 + y^2 = 1,\ z = 0,$$
$$(x+2)^2 + y^2 = 1,\ z = 0.$$
in $\mathbb{R}^3$. Show that $\pi_1(\mathbb{R}^3 - C)$ is the free group on two generators.

5. Calculate the fundamental groups of the following spaces

   (i) $\mathbb{R}^4$ minus a line.
   (ii) $\mathbb{R}^4$ minus a two dimensional linear subspace.
   (iii) $\mathbb{R}^4$ minus two parallel lines.
   (iv) $\mathbb{R}^4$ minus two intersecting lines.
   (v) $\mathbb{R}^3$ minus the coordinate axes
   (vi) $\mathbb{C}^2 - \{(z_1, z_2) / z_1 z_2 = 0\}$
   (vii) $\mathbb{R}^3$ minus finitely many points.