Exercises:

1. Show that the sphere S^3 is isomorphic (as a topological group) to $SU(2,\mathbb{C})$.
2. Show that the center of the group of non-zero quaternions is the set of non-zero real numbers. In the light of this explain why ker $D\psi(1)$ in lemma (22.6) is non-trivial.
3. Explain why the map ϕ defined in theorem (22.8) is bijective.
4. Verify the properties of the map T_A in the proof of theorem (22.10). Also fill in the details concerning the properties of the map ϕ (except for the claims made concerning its derivative).
5. Use exercise 4 to find a generator of $\pi_1(SO(3,\mathbb{R}))$. Let

   $i : SO(2,\mathbb{R}) \longrightarrow SO(3,\mathbb{R})$ be given by

   $$A \mapsto \begin{pmatrix} A & 0 \\ 0 & 1 \end{pmatrix}, \quad A \in SO(2,\mathbb{R}).$$

   Show that $i_* : \pi_1(SO(2,\mathbb{R})) \longrightarrow \pi_1(SO(3,\mathbb{R}))$ is surjective.