Exercises:

1. Check that the map ϕ constructed in the proof of theorem 11.3 is continuous and is indeed a homotopy. Work out the proof of theorem 11.5.

2. Show that the boundary ∂M of the Möbius band M is not a deformation retract of M by taking a base point x_0 on the boundary and computing explicitly the group homomorphism
$$i_* : \pi_1(\partial M, x_0) \longrightarrow \pi_1(M, x_0).$$

3. Show that the boundary of the Möbius band is not even a retract of the Möbius band.

4. Fill in the details on the continuity of the map G in the example preceding corollary 11.9.

5. Show that the space $\mathbb{R}^3 - \{(x, y, z)/x^2 + y^2 = 1,\ z = 0\}$ deformation retracts to a sphere with a diameter attached to it.

6. Let X be the union of S^2 and one of its diameters D, $Y = S^2 \vee S^1$ and Z be the union of S^2 with a punctured half disc contained in a half with edge along D. Show that X and Y are both deformation retracts of Z and so they have the same homotopy type.