

Module 2 Probability and Statistics

BASIC CONCEPTS

Multiple Choice

Identify the choice that best completes the statement or answers the question.

- _____ 1. The standard deviation of a standard normal distribution
- is always equal to zero
 - is always equal to one
 - can be any positive value
 - can be any value
- _____ 2. X is a normally distributed random variable with a mean of 8 and a standard deviation of 4. The probability that X is between 1.48 and 15.56 is
- 0.0222
 - 0.4190
 - 0.5222
 - 0.9190

Exhibit 6-1

The assembly time for a product is uniformly distributed between 6 to 10 minutes.

- _____ 3. Refer to Exhibit 6-1. The probability of assembling the product in less than 6 minutes is
- zero
 - 0.50
 - 0.15
 - 1

Exhibit 6-2

The weight of football players is normally distributed with a mean of 200 pounds and a standard deviation of 25 pounds.

- _____ 4. Refer to Exhibit 6-2. The probability of a player weighing less than 250 pounds is
- 0.4772
 - 0.9772
 - 0.0528
 - 0.5000

Exhibit 6-5

The travel time for a college student traveling between her home and her college is uniformly distributed between 40 and 90 minutes.

- _____ 5. Refer to Exhibit 6-5. The probability that her trip will take exactly 50 minutes is
- zero
 - 0.02
 - 0.06
 - 0.20

- _____ 6. A simple random sample of 100 observations was taken from a large population. The sample mean and the standard deviation were determined to be 80 and 12 respectively. The standard error of the mean is
- 1.20
 - 0.12
 - 8.00
 - 0.80
- _____ 7. The probability distribution of all possible values of the sample proportion \bar{p} is the
- probability density function of \bar{p}
 - sampling distribution of \bar{x}
 - same as \bar{p} , since it considers all possible values of the sample proportion
 - sampling distribution of \bar{p}
- _____ 8. The sample statistic s is the point estimator of
- μ
 - σ
 - \bar{x}
 - \bar{p}
- _____ 9. A population has a mean of 80 and a standard deviation of 7. A sample of 49 observations will be taken. The probability that the sample mean will be larger than 82 is
- 0.5228
 - 0.9772
 - 0.4772
 - 0.0228
- _____ 10. As a rule of thumb, the sampling distribution of the sample proportions can be approximated by a normal probability distribution whenever
- $np \geq 5$
 - $n(1 - p) \geq 5$ and $n \geq 30$
 - $n \geq 30$ and $(1 - p) = 0.5$
 - None of these alternatives is correct.

Numerical Problems

11. The Body Paint, an automobile body paint shop, has determined that the painting time of automobiles is uniformly distributed and that the required time ranges between 45 minutes to 1 1/2 hours.
 - a. Give a mathematical expression for the probability density function.
 - b. What is the probability that the painting time will be less than or equal to one hour?
 - c. What is the probability that the painting time will be more than 50 minutes?
 - d. Determine the expected painting time and its standard deviation.

12. A professor at a local community college noted that the grades of his students were normally distributed with a mean of 74 and a standard deviation of 10. The professor has informed us that 6.3 percent of his students received A's while only 2.5 percent of his students failed the course and received F's.
 - a. What is the minimum score needed to make an A?
 - b. What is the maximum score among those who received an F?
 - c. If there were 5 students who did not pass the course, how many students took the course?

13. "DRUGS R US" is a large manufacturer of various kinds of liquid vitamins. The quality control department has noted that the bottles of vitamins marked 6 ounces vary in content with a standard deviation of 0.3 ounces. Assume the contents of the bottles are normally distributed.
 - a. What percentage of all bottles produced contains more than 6.51 ounces of vitamins?
 - b. What percentage of all bottles produced contains less than 5.415 ounces?
 - c. What percentage of bottles produced contains between 5.46 to 6.495 ounces?
 - d. Ninety-five percent of the bottles will contain at least how many ounces?
 - e. What percentage of the bottles contains between 6.3 and 6.6 ounces?

14. The price of a bond is uniformly distributed between \$80 and \$85.
 - a. What is the probability that the bond price will be at least \$83?
 - b. What is the probability that the bond price will be between \$81 to \$90?
 - c. Determine the expected price of the bond.
 - d. Compute the standard deviation for the bond price.

15. The monthly income of residents of Daisy City is normally distributed with a mean of \$3000 and a standard deviation of \$500.
 - a. The mayor of Daisy City makes \$2,250 a month. What percentage of Daisy City's residents has incomes that are more than the mayor's?
 - b. Individuals with incomes of less than \$1,985 per month are exempt from city taxes. What percentage of residents is exempt from city taxes?
 - c. What are the minimum and the maximum incomes of the middle 95% of the residents?
 - d. Two hundred residents have incomes of at least \$4,440 per month. What is the population of Daisy City?

16. Approximate the following binomial probabilities by the use of normal approximation. Twenty percent of students who finish high school do not go to college. What is the probability that in a sample of 80 high school students
- exactly 10 will not go to college?
 - 70 or more will go to college?
 - fourteen or fewer will not go to college?
17. The average life expectancy of dishwashers produced by a company is 6 years with a standard deviation of 8 months. Assume that the lives of dishwashers are normally distributed.
- What is the probability that a randomly selected dishwasher will have a life expectancy of at least 7 years?
 - Dishwashers that fail operating in less than 4 1/2 years will be replaced free of charge. What percent of dishwashers are expected to be replaced free of charge?
 - What are the minimum and the maximum life expectancy of the middle 95% of the dishwashers' lives? Give your answer in months.
 - If 155 of this year's dishwasher production fail operating in less than 4 years and 4 months, how many dishwashers were produced this year?
18. Below you are given the values obtained from a random sample of 4 observations taken from an infinite population.
- 32 34 35 39
- Find a point estimate for μ . Is this an unbiased estimate of μ ? Explain.
 - Find a point estimate for σ^2 . Is this an unbiased estimate of σ^2 ? Explain.
 - Find a point estimate for σ .
 - What can be said about the sampling distribution of \bar{x} ? Be sure to discuss the expected value, the standard deviation, and the shape of the sampling distribution of \bar{x} .
19. MNM Corporation gives each of its employees an aptitude test. The scores on the test are normally distributed with a mean of 75 and a standard deviation of 15. A simple random sample of 25 is taken from a population of 500.
- What are the expected value, the standard deviation, and the shape of the sampling distribution of \bar{x} ?
 - What is the probability that the average aptitude test in the sample will be between 70.14 and 82.14?
 - What is the probability that the average aptitude test in the sample will be greater than 82.68?
 - What is the probability that the average aptitude test in the sample will be less than 78.69?
 - Find a value, C, such that $P(\bar{x} \geq C) = .015$.

Answer Section

MULTIPLE CHOICE

1. ANS: B
2. ANS: D
3. ANS: A
4. ANS: B
5. ANS: A
6. ANS: A
7. ANS: D
8. ANS: B
9. ANS: D
10. ANS: B

PROBLEM

11. ANS:
- a. $f(x) = \begin{cases} \frac{1}{45} & \text{for } 45 \leq x \leq 90 \\ 0 & \text{elsewhere} \end{cases}$
- b. 0.333
- c. 0.889
- d. 67.5, 12.99

12. ANS:
- a. 89.3
- b. 54.4
- c. 200

13. ANS:
- a. 4.46%
- b. 2.56%
- c. 91.46%
- d. 5.5065 ounces
- e. 13.59%

14. ANS:

- a. 0.4
- b. 0.8
- c. \$82.50
- d. \$1.44

15. ANS:

- a. 93.32%
- b. 2.12%
- c. Min = 2020 Max = 3980
- d. 100,000

16. ANS:

- a. 0.0274
- b. 0.0618
- c. 0.3372

17. ANS:

- a. 0.0668
- b. 1.22%
- c. 56.32 and 87.68 (Months)
- d. 25,000

18. ANS:

- a. 35; Yes; $E(x) = \mu$
- b. 8.667; Yes; $E(s^2) = \sigma^2$
- c. 2.944
- d. $E(\bar{x}) = \mu$, the standard deviation = σ^2/n , and the sampling distribution of \bar{x} is normally distributed if the population is normally distributed.

19. ANS:

- a. 75; 3; normal
- b. 0.9387
- c. 0.0052
- d. 0.8907
- e. 81.51

Confidence Interval Estimation

Multiple Choice

Identify the choice that best completes the statement or answers the question.

- _____ 1. A random sample of 1000 people was taken. Four hundred fifty of the people in the sample favored Candidate A. The 95% confidence interval for the true proportion of people who favors Candidate A is
- 0.419 to 0.481
 - 0.40 to 0.50
 - 0.45 to 0.55
 - 1.645 to 1.96

Exhibit 8-1

In order to estimate the average time spent on the computer terminals per student at a local university, data were collected for a sample of 81 business students over a one-week period. Assume the population standard deviation is 1.8 hours.

- _____ 2. Refer to Exhibit 8-1. With a 0.95 probability, the margin of error is approximately
- 0.39
 - 1.96
 - 0.20
 - 1.64

Exhibit 8-2

A random sample of 121 automobiles traveling on an interstate showed an average speed of 65 mph. From past information, it is known that the standard deviation of the population is 22 mph.

- _____ 3. Refer to Exhibit 8-2. If the sample size was 100 (other factors remain unchanged), the interval for μ would
- not change
 - become narrower
 - become wider
 - become zero

Exhibit 8-6

A sample of 75 information system managers had an average hourly income of \$40.75 with a standard deviation of \$7.00.

- _____ 4. Refer to Exhibit 8-6. If we want to determine a 95% confidence interval for the average hourly income, the value of "t" statistics is
- 1.96
 - 1.64
 - 1.28
 - 1.993

- _____ 5. Refer to Exhibit 8-6. The 95% confidence interval for the average hourly wage of all information system managers is
- 40.75 to 42.36
 - 39.14 to 40.75
 - 39.14 to 42.36
 - 30 to 50
- _____ 6. The probability of committing a Type I error when the null hypothesis is true is
- the confidence level
 - β
 - greater than 1
 - the Level of Significance
- _____ 7. The probability of making a Type I error is denoted by
- α
 - β
 - $1 - \alpha$
 - $1 - \beta$
- _____ 8. For a one-tailed test (upper tail), a sample size of 26 at 90% confidence, $t =$
- 1.316
 - 1.316
 - 1.740
 - 1.740
- _____ 9. For a one-tailed test (lower tail) with 22 degrees of freedom at 95% confidence, the value of $t =$
- 1.383
 - 1.383
 - 1.717
 - 1.721

Exhibit 9-8

The average gasoline price of one of the major oil companies in Europe has been \$1.25 per liter. Recently, the company has undertaken several efficiency measures in order to reduce prices. Management is interested in determining whether their efficiency measures have actually **reduced** prices. A random sample of 49 of their gas stations is selected and the average price is determined to be \$1.20 per liter. Furthermore, assume that the standard deviation of the population (σ) is \$0.14.

- _____ 10. Refer to Exhibit 9-8. The p -value for this problem is
- 0.4938
 - 0.0062
 - 0.0124
 - 0.05

Problem

11. A random sample of 49 lunch customers was taken at a restaurant. The average amount of time the customers in the sample stayed in the restaurant was 45 minutes with a standard deviation of 14 minutes.
- Compute the standard error of the mean.
 - With a .99 probability, what statement can be made about the size of the margin of error?
 - Construct a 99% confidence interval for the true average amount of time customers spent in

the restaurant.

- d. With a .99 probability, how large of a sample would have to be taken to provide a margin of error of 2.5 minutes or less?

12. The monthly incomes from a random sample of workers in a factory are shown below.

Monthly Income

(In \$1,000)

4.0

5.0

7.0

4.0

6.0

6.0

7.0

9.0

- a. Compute the standard error of the mean (in dollars).
 - b. Compute the margin of error (in dollars) at 95% confidence.
 - c. Compute a 95% confidence interval for the mean of the population. Assume the population has a normal distribution. Give your answer in dollars.
13. The proprietor of a boutique in New York wanted to determine the average age of his customers. A random sample of 53 customers revealed an average age of 28 years with a standard deviation of 4 years. Determine a 98% confidence interval estimate for the average age of all his customers.
 14. A coal company wants to determine a 95% confidence interval estimate for the average daily tonnage of coal that they mine. Assuming that the company reports that the standard deviation of daily output is 200 tons, how many days should they sample so that the margin of error will be 39.2 tons or less?
 15. The average score of a sample of 87 senior business majors at UTC who took the Graduate Management Admission Test was 510 with a standard deviation of 36. Provide a 98% confidence interval for the mean of the population.
 16. In order to determine the average weight of carry-on luggage by passengers in airplanes, a sample of 25 pieces of carry-on luggage was collected and weighed. The average weight was 18 pounds. Assume that we know the *standard deviation of the population* to be 7.5 pounds.
 - a. Determine a 97% confidence interval estimate for the mean weight of the carry-on luggage.
 - b. Determine a 95% confidence interval estimate for the mean weight of the carry-on luggage.
 17. In order to determine the average price of hotel rooms in Atlanta, a sample of 64 hotels was selected. It was determined that the average price of the rooms in the sample was \$108.50 with a standard deviation of \$16.
 - a. Formulate the hypotheses to determine whether or not the average room price is significantly different from \$112.
 - b. Compute the test statistic.
 - c. At 95% confidence using the *p*-value approach, test the hypotheses. Let $\alpha = 0.1$.

18. A carpet company advertises that it will deliver your carpet within 15 days of purchase. A sample of 49 past customers is taken. The average delivery time in the sample was 16.2 days. The standard deviation of the population (σ) is known to be 5.6 days.
 - a. State the null and alternative hypotheses.
 - b. Using the critical value approach, test to determine if their advertisement is legitimate. Let $\alpha = .05$.
 - c. Using the p -value approach, test the hypotheses at the 5% level of significance.
19. A student believes that no more than 20% (i.e., $\leq 20\%$) of the students who finish a statistics course get an A. A random sample of 100 students was taken. Twenty-four percent of the students in the sample received A's.
 - a. State the null and alternative hypotheses.
 - b. Using the critical value approach, test the hypotheses at the 1% level of significance.
 - c. Using the p -value approach, test the hypotheses at the 1% level of significance.
20. Two thousand numbers are selected randomly; 960 were even numbers.
 - a. State the hypotheses to determine whether the proportion of odd numbers is significantly different from 50%.
 - b. Compute the test statistic.
 - c. At 90% confidence using the p -value approach, test the hypotheses.

Answer Section

MULTIPLE CHOICE

1. ANS: A
2. ANS: A
3. ANS: C
4. ANS: A
5. ANS: C
6. ANS: D
7. ANS: A
8. ANS: A
9. ANS: C
10. ANS: B

PROBLEMS

11. ANS:
 - a. 2
 - b. There is a .99 probability that the sample mean will provide a margin of error of 7.193 or less.
 - c. 39.636 to 50.364
 - d. 226
12. ANS:
 - a. \$597.61
 - b. \$1,413.10
 - c. \$4,586.90 to \$7,413.10
13. ANS:
26.68 to 29.32
14. ANS:
100
15. ANS:
500.85 to 519.15
16. ANS:
 - a. 14.745 to 21.255
 - b. 15.06 to 20.94

17. ANS:

- a. $H_0: \mu = 112$
 $H_a: \mu \neq 112$
- b. $t = -1.75$
- c. p -value is between 0.025 and 0.05; therefore, do not reject H_0

18. ANS:

- a. $H_0: \mu \leq 15$
 $H_a: \mu > 15$
- b. test statistic $Z = 1.5 < 1.645$; therefore do not reject H_0
- c. Do not reject H_0 ; p -value is $(.5 - .4332) = 0.0668$

19. ANS:

- a. $H_0: P \leq 0.2$
 $H_a: P > 0.2$
- b. Do not reject H_0 ; test statistic $Z = 1 < 2.33$
- c. Do not reject H_0 ; p -value = $0.1587 > 0.01$

20. ANS:

- a. $H_0: P = 0.5$
 $H_a: P \neq 0.5$
- b. $Z = 1.79$
- c. p -value = $.0734 < 0.10$; reject H_0

Estimation and Test of Hypothesis basics

Multiple Choice

Identify the choice that best completes the statement or answers the question.

- _____ 1. From a population with a variance of 900, a sample of 225 items is selected. At 95% confidence, the margin of error is
- 15
 - 2
 - 3.92
 - 4
- _____ 2. It is known that the variance of a population equals 1,936. A random sample of 121 has been taken from the population. There is a .95 probability that the sample mean will provide a margin of error of
- 7.84
 - 31.36
 - 344.96
 - 1,936
- _____ 3. When constructing a confidence interval for the population mean and the standard deviation of the sample is used, the degrees of freedom for the t distribution equals
- n-1
 - n
 - 29
 - 30

Exhibit 8-3

The manager of a grocery store has taken a random sample of 100 customers. The average length of time it took these 100 customers to check out was 3.0 minutes. It is known that the standard deviation of the population of checkout times is one minute.

- _____ 4. Refer to Exhibit 8-3. The standard error of the mean equals
- 0.001
 - 0.010
 - 0.100
 - 1.000

Exhibit 8-4

In order to estimate the average electric usage per month, a sample of 81 houses was selected, and the electric usage was determined. Assume a population standard deviation of 450-kilowatt hours.

- _____ 5. Refer to Exhibit 8-4. At 95% confidence, the size of the margin of error is
- 1.96
 - 50
 - 98
 - 42
- _____ 6. If a hypothesis test leads to the rejection of the null hypothesis,
- a Type II error must have been committed
 - a Type II error may have been committed
 - a Type I error must have been committed
 - a Type I error may have been committed
- _____ 7. Your investment executive claims that the average yearly rate of return on the stocks she recommends is more than 10.0%. You plan on taking a sample to test her claim. The correct set of hypotheses is
- $H_0: \mu < 10.0\%$ $H_a: \mu \geq 10.0\%$
 - $H_0: \mu \leq 10.0\%$ $H_a: \mu > 10.0\%$
 - $H_0: \mu > 10.0\%$ $H_a: \mu \leq 10.0\%$
 - $H_0: \mu \geq 10.0\%$ $H_a: \mu < 10.0\%$

Exhibit 9-2

$$n = 64 \qquad \bar{x} = 50 \qquad s = 16 \qquad H_0: \mu \geq 54$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad H_a: \mu < 54$$

- _____ 8. Refer to Exhibit 9-2. If the test is done at 95% confidence, the null hypothesis should
- not be rejected
 - be rejected
 - Not enough information is given to answer this question.
 - None of these alternatives is correct.

Exhibit 9-4

The manager of a grocery store has taken a random sample of 100 customers. The average length of time it took the customers in the sample to check out was 3.1 minutes with a standard deviation of 0.5 minutes. We want to test to determine whether or not the mean waiting time of all customers is significantly more than 3 minutes.

- _____ 9. Refer to Exhibit 9-4. The test statistic is
- 1.96
 - 1.64
 - 2.00
 - 0.056

Exhibit 9-9

The sales of a grocery store had an average of \$8,000 per day. The store introduced several advertising campaigns in order to *increase* sales. To determine whether or not the advertising campaigns have been effective in increasing sales, a sample of 64 days of sales was selected. It was found that the average was \$8,250 per day. From past information, it is known that the standard deviation of the *population* is \$1,200.

- _____ 10. Refer to Exhibit 9-9. The p -value is
- a. 1.67
 - b. 0.4525
 - c. 0.0475
 - d. 0.5475

Problems

- 11. Choo Choo Paper Company produces papers of various thickness. A random sample of 256 cuts had a mean thickness of 30.3 mils with a standard deviation of 4 mils. Develop a 95% confidence interval for the mean thickness of the population.
- 12. Two hundred students are enrolled in an Economics class. After the first examination, a random sample of 6 papers was selected. The grades were 65, 75, 89, 71, 70 and 80.
 - a. Determine the standard error of the mean.
 - b. What assumption must be made before we can determine an interval for the mean grade of all the students in the class? Explain why.
 - c. Assume the assumption of Part b is met. Provide a 95% confidence interval for the mean grade of all the students in the class.
- 13. Computer Services, Inc. wants to determine a confidence interval for the average CPU time of their teleprocessing transactions. A sample of 64 transactions yielded a mean of 6 seconds with a standard deviation of 0.8 seconds. Determine a 98% confidence interval for the average CPU time.
- 14. The proprietor of a boutique in New York wanted to determine the average age of his customers. A random sample of 53 customers revealed an average age of 28 years with a standard deviation of 4 years. Determine a 98% confidence interval estimate for the average age of all his customers.
- 15. A sample of 36 patients in a doctor's office showed that they had to wait an average of 45 minutes with a standard deviation of 10 minutes before they could see the doctor. Provide a 90% confidence interval estimate for the average waiting time of all the patients who visit this doctor.
- 16. Six hundred consumers were asked whether they would like to purchase a domestic or a foreign automobile. Their responses are given below.

Preference	Frequency
Domestic	240
Foreign	360

Develop a 95% confidence interval for the proportion of all consumers who prefer to purchase domestic automobiles.

17. In order to determine the average price of hotel rooms in Atlanta, a sample of 64 hotels was selected. It was determined that the average price of the rooms in the sample was \$108.50 with a standard deviation of \$16.
 - a. Formulate the hypotheses to determine whether or not the average room price is significantly different from \$112.
 - b. Compute the test statistic.
 - c. At 95% confidence using the p -value approach, test the hypotheses. Let $\alpha = 0.1$.
18. Identify the null and alternative hypotheses for the following problems.
 - a. The manager of a restaurant believes that it takes a customer more than 25 minutes to eat lunch.
 - b. Economists have stated that the marginal propensity to consume is at least 90¢ out of every dollar.
 - c. It has been stated that 75 out of every 100 people who go to the movies on Saturday night buy popcorn.
19. A carpet company advertises that it will deliver your carpet within 15 days of purchase. A sample of 49 past customers is taken. The average delivery time in the sample was 16.2 days. The standard deviation of the population (σ) is known to be 5.6 days.
 - a. State the null and alternative hypotheses.
 - b. Using the critical value approach, test to determine if their advertisement is legitimate. Let $\alpha = .05$.
 - c. Using the p -value approach, test the hypotheses at the 5% level of significance.
20. A law enforcement agent believes that at least 88% of the drivers stopped on Saturday nights for speeding are under the influence of alcohol. A sample of 66 drivers who were stopped for speeding on a Saturday night was taken. Eighty percent of the drivers in the sample were under the influence of alcohol.
 - a. State the null and alternative hypotheses.
 - b. Compute the test statistic.
 - c. Using the p -value approach, test the hypotheses at the .05 level of significance.

Answer Section

MULTIPLE CHOICE

1. ANS: C
2. ANS: A
3. ANS: A
4. ANS: C
5. ANS: C
6. ANS: D
7. ANS: B
8. ANS: B
9. ANS: C
10. ANS: C

PROBLEMS

11. ANS:
29.81 to 30.79
12. ANS:
 - a. 3.474
 - b. Since σ is estimated from s , we must assume the distribution of all the grades is normal.
 - c. 66.07 to 83.93
13. ANS:
5.761 to 6.239
14. ANS:
26.68 to 29.32
15. ANS:
42.18 to 47.82
16. ANS:
0.3608 to 0.4392
17. ANS:
 - a. $H_0: \mu = 112$
 $H_a: \mu \neq 112$
 - b. $t = -1.75$

c. p -value is between 0.025 and 0.05; therefore, do not reject H_0

18. ANS:

- a. $H_0: \mu \leq 25$
 $H_a: \mu > 25$
- b. $H_0: p \geq 0.9$
 $H_a: p < 0.9$
- c. $H_0: p = 0.75$
 $H_a: p \neq 0.75$

19. ANS:

- a. $H_0: \mu \leq 15$
 $H_a: \mu > 15$
- b. test statistic $Z = 1.5 < 1.645$; therefore do not reject H_0
- c. Do not reject H_0 ; p -value is $(.5 - .4332) = 0.0668$

20. ANS:

- a. $H_0: P \geq 0.88$
 $H_a: P < 0.88$
- b. $Z = -2$
- c. p -value = $0.0228 < 0.05$; reject H_0