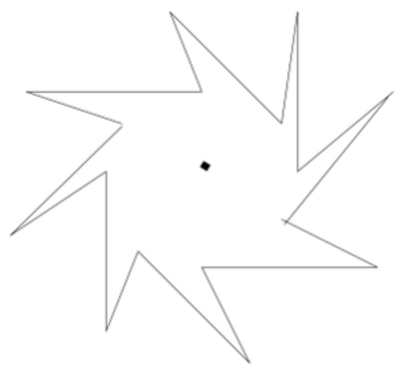
**CSL 852, Computational Geometry: Practice Problems**

# Introduction using basic visibility problems

1. Construct a polygon $P$ and a placement of guards such that the guards can see every point on the boundary of $P$ but there is at least one point in the interior that is not visible to the guards.

   Solution: Place the guards in the figure at all corner points. Each guard sees all points on 2 edges intersecting his point. But no guard sees the centre bold point of the figure.



2. Design a polyhedron (3 dimensional version of a polygon) such that guards placed at every vertex may not be able to cover the entire interior.

3. Let $\mathcal{M}(S)$ denote the set of maximal points of a planar point set $S$. Denote $L_0 = \mathcal{M}(S)$ and $S_0 = S$ and let $S_i = S_{i-1} - L_{i-1}$ and $L_i = \mathcal{M}(S_i)$ for $i \geq 1$.
   You can think about $L_i$'s as the *maximal layers* that are successively obtained by stripping away the previous layers. Design an $O(npolylog(n))$ algorithm for computing all the maximal layers.
   Solution: Do a line sweep in the decreasing order of $x$ (i.e. sort the points on their x coordinate value) let this sorted set be $p'1, p'_2 \ldots p'_n$. Initialize $L_0 = p'_n$ and as we sweep left, assume that we have inductively computed the layers correctly till $p'_{i+1}$. When we consider $p'_i$ then suppose the layers are $L_0, L_2 \ldots L_j$ and let $Y_0, Y_1, \ldots Y_j$ denote the highest $y$ coordinates of the points in the respective layers.
   Claim: $p'_i$ belongs to $L_k$ iff $Y_{k+1} > y'_i > Y_k$ if such a $k$ exists or start a new layer $j+1$ if $y'_i < Y_j$
   Using a dynamic dictionary, this can be found in $O(\log n)$ steps and therefore the entire algorithm takes $O(n \log n)$ time.