Example 1

Three contractors A , B, and C are bidding for a project. A has half the chance that B has. B has two thirds as likely as C for the award of contract. What is the probability of each contractor, if only he gets the contract?

Answer:

There are three contractors, one will be successful

P[A + B + C] = 1

As they are mutually exclusive, P[A] + P[B] + P[C] = 1

But, P[A] = $\frac{1}{2}$ P[B] and P[B] = $\frac{2}{3}$ P[C]

$\therefore \frac{1}{2}$ P[B]+ P[B]+ $\frac{3}{2}$ P[B] = 1

➔ P[B] = $\frac{1}{3}$, P[A] = $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ and P[C] = $\frac{3}{2}\times\frac{1}{3}=\frac{1}{2}$

Example 2

A load of 100 kg can be anywhere on the beam, consequently the reaction R_A and R_B can be anywhere between 0 to 100 depending on the position of load.

1. Identify the possibilities of $(10 \le R_A \le 20\ Kg)$ and $R_A > 50$
2. Identify the event of interest

Answer:

Possibilities are 1. $(10 \le R_A \le 20\ Kg)$

2. $R_A > 50$

Event of interest: 1. $P(10 \le R_A \le 20\ Kg) = \frac{10}{100} = 0.1$

2. P (R_A > 50) = $\frac{50}{100} = 0.5$

Example 3

A contractor is planning to purchase of equipment , including bulldozer needed for a project in a remote area. He knows that bulldozer can last at least six months without any breakdown. If he has 3 bulldozers, what is the probability that there will be at least one bulldozer after six months.

Answer:

If denote the condition of each bulldozer after six months as G ➔ Good and B➔ Bad, the possible combinations are:

1	**2**	**3**
G	G	G
G	G	B
G	B	G
G	**B**	**B**
B	G	G
B	**B**	**G**
B	**G**	**B**
B	B	B

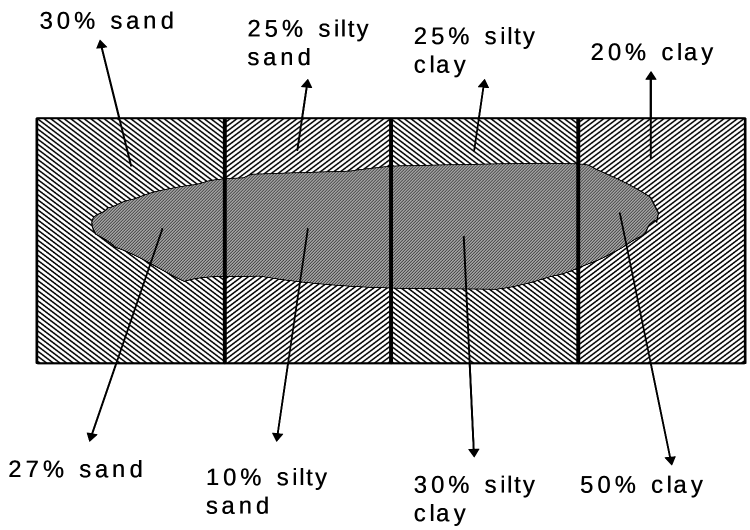
There are at least three combinations out of 8, where there is possibility of having at least one bull dozer after six months.

∴ The probability of having at least one bulldozer in good condition after six months is $\frac{3}{8}$.

Example 4

In a bore log of interest,the soil profile the results are as follows; sand = 30%' silty sand = 25%, silty clay = 25% and clay = 20%. This is the arrived result of bore logs at many places. However there is some doubt that not all soil sampling is reliable. The adequacy is represented by sand = 27%, silty sand = 10%, silty clay = 30% and clay = 50%. What is the reliability of information from sampling at one of the random points?

Answer



Using the total probability theorem,

$$P[A_i B] = P[A_i]P\left[\frac{B}{A_i}\right]$$

$$P[A_S] = 0.3$$
$$P[A_{SS}] = 0.25$$
$$P[A_{SC}] = 0.25$$
$$P[A_C] = 0.20$$

If B denotes the sample that is considered reliable, P[B] is given by

$$P\left[\frac{B}{A_S}\right] = 0.27$$

$$P\left[\frac{B}{A_{SS}}\right] = 0.10$$

$$P\left[\frac{B}{A_{SC}}\right] = 0.75$$

$$P\left[\frac{B}{A_C}\right] = 0.50$$

$$P[B] = 0.3 * 0.27 + 0.25 * 0.10 + 0.25 * 0.30 + 0.2 * 0.5 = 0.28$$

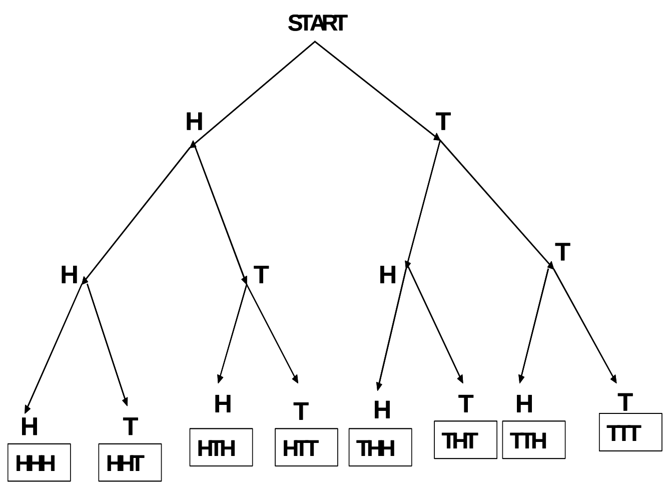
Example 5

A fair coin [P[head] = P[Tail] = ½] is tossed three times. What is the probability of getting three heads.

Answer

The three events are independent, hence

1. Probability of getting three heads ➔ P[Three heads] = ½ X ½ X ½ = 1/8
2. Probability of getting two heads and one tail ➔ P[Two Heads + Tail] = 3/ 8



Example 6

A sample from a pit containing sand is to be examined, if the pit can furnish adequate fine aggregate for making road. Specifications suggest that the pit should be rejected if at least one lot is not satisfactory of the four lots made from the pit. What is the probability that sand from the pit will be accepted, if 100 bags are available and contain fine aggregate of poor quality? Find the probability of acceptance.

Answer

Let E_i be the probability of finding that the sand can be accepted. For four lots, each time sampling is done it is not replaced. After four trials,

P[acceptance] = $\frac{95}{100} \times \frac{94}{99} \times \frac{93}{98} \times \frac{92}{97} = 0.812$

If there are 50 bags

P[acceptance] = $\frac{45}{50} \times \frac{44}{49} \times \frac{43}{48} \times \frac{42}{47} = 0.650$

If there are only 25 bags available,

P[acceptance] = $\frac{20}{25} \times \frac{19}{24} \times \frac{18}{23} \times \frac{17}{22} = 0.380$

Hence it can be noted that sample size has influence on the acceptance criteria.

Example 7

Consider a 100 Km highway; assume that the road condition and traffic conditions are uniform throughout, So that the accidents are equally likely anywhere on the highway.

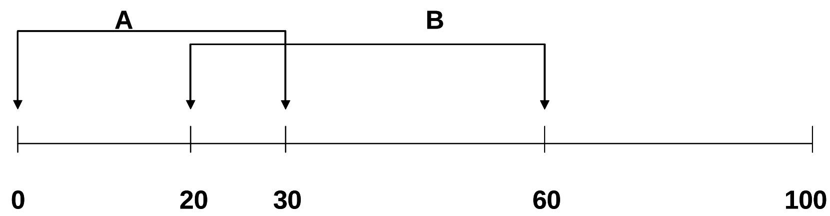
(a) Find the probabilities of events A and B if A = an event denoting accidents between 0 to 30 km and B = an event denoting accidents between 20 to 60 km.

Answer

Since the accidents are equally likely along the highway , it may be assumed that the probability of an accident in a given interval of highway is proportional to the distance of travel . If the accident occurs along 100 km highway then;

P[A] = $\frac{30}{100}$ and P[B] = $\frac{40}{100}$

(b) If an accident occurs in the interval (20, 60) what is the probability that an accident occurs between 20 to 30 Km from A



$$P(A / B) = \frac{10}{40} = \frac{P(A \cap B)}{P(B)} = \frac{10/100}{40/100} = 0.25$$

Example 8

Consider a chain system of two links. If the applied force is 1000 kg and if the strength anywhere is less than 100 kg, it will fail. However probability of this happening to any link is 0.05. What is the probability of failure of chain?

Answer

If E_1 and E_2 denote the probability of failure of links 1 and 2 , then failure of the chain is

$P[E_1 \cup E_2] = P[E_1] + P[E_2] – P[E_1E_2]$

$= 0.5 + 0.5 – P(E_1 / E_2) P[E_1]$

$= 0.5 + 0.5 - 0.05 P(E_2 / E_1)$

$= 1- 0.05 P(E_2 / E_1)$

This depends on the mutual dependence of $E_1$and E_2. For example if the links randomly selected from two suppliers then E_1 and E_2 may be assumed to be statistically independent. In such a case,

$P[E_2 / E_1] = P[E_2] = 0.05$ hence

$P[E_1 / E_2] = P[E_1] = 0.05$

$P[E_1 \cup E_2] = 0.1 – 0.05 * 0.05 = 0.0975$

If the links are form the same manufacturer $P[E_1/ E_2] = 1.0$

$P(E_1 \cup E_2) = 0.1-0.05*0.1 = 0.05$ → Probability of one of the links

Hence the failure probability of the chain system ranges from 0.05 to 0.095 depending on the conditional probability $P[E_2 / E_1]$

Example 9

The following problem demonstrates the use of Bayes theorem in updating the information. Consider that Mohammad Gazini while planning his attack on India from West Coast, assumed that he had 1% chance of winning (probability of win) if he attacked. This represents the level of confidence in the first trial. Probability of win in the subsequent trials can be obtained from Bayes theorem. Probability of win is improved in the second and subsequent attacks and finally at the end of 17 trials, the probability of win becomes 0.99 as given in the following .

Ans:

$$P(A/T) = \frac{P\left(\frac{T}{A}\right)*P(A)}{P\left(\frac{T}{A}\right)*P(A)+P\left(\frac{T}{\overline{A}}\right)P(\overline{A})}$$

1	$\frac{(1*0.01)}{(1*0.01)+(0.5*0.99)}$	0.02
2	$\frac{(1*0.02)}{0.02+(0.5+0.98)}$	0.04
3	$\frac{0.04}{0.04+0.5+0.96}$	0.08

4	$\frac{0.08}{0.08+0.5+0.92}$	0.15
5	$\frac{0.15}{0.15+0.5+0.85}$	0.26
6	$\frac{0.26}{0.26+0.5*0.74}$	0.41
7	$\frac{0.41}{0.41+0.5*0.59}$	0.58
8	$\frac{0.58}{0.58+0.5*0.42}$	0.73
9	$\frac{0.73}{0.73+0.5*0.27}$	0.84
10	$\frac{0.84}{0.84+0.5*0.16}$	0.91
11	$\frac{0.91}{0.91+0.5*0.09}$	0.91
12	$\frac{0.95}{0.95+0.5*0.05}$	0.974
13	$\frac{0.974}{0.974+0.5*0.0256}$	0.974
14	$\frac{0.987}{0.987+0.5*0.013}$	0.993
15	$\frac{0.993}{0.993+0.5*0.0065}$	0.9967
16	$\frac{0.9967}{0.9967+0.5*0.0098}$	0.9983
17	$\frac{0.9983}{0.9983+0.5*0.005}$	0.9991