Advanced Topics in Optimization

Direct and Indirect Search Methods

D Nagesh Kumar, IISc

Introduction

Real World Problems

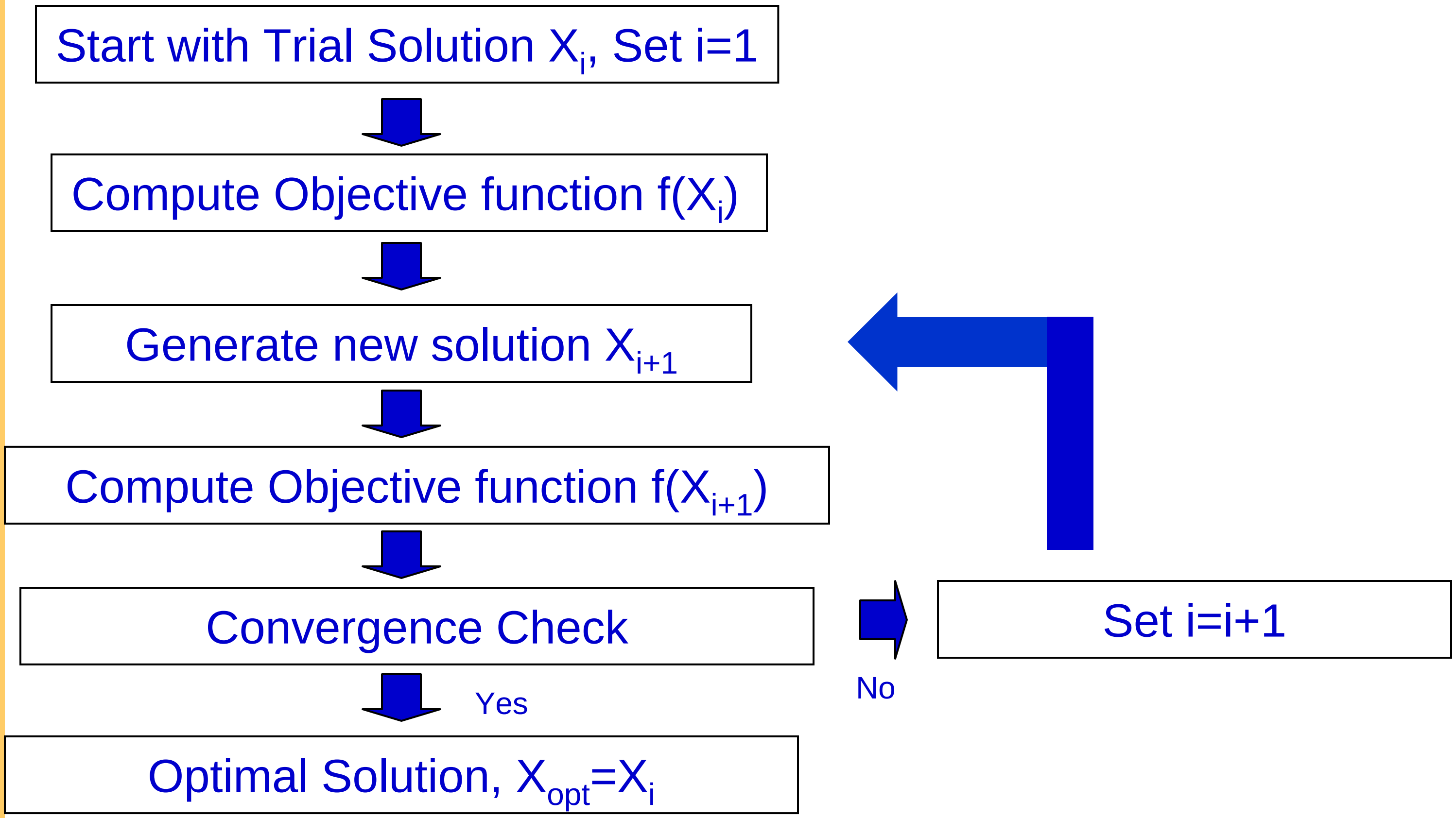
Nonlinear Optimization

Complicated Objective Function

~~Analytical Solution~~

<u>Possible Solution: Search Algorithm</u>

Flowchart for Search Algorithm



Classification of Search Algorithm

Search Algorithm

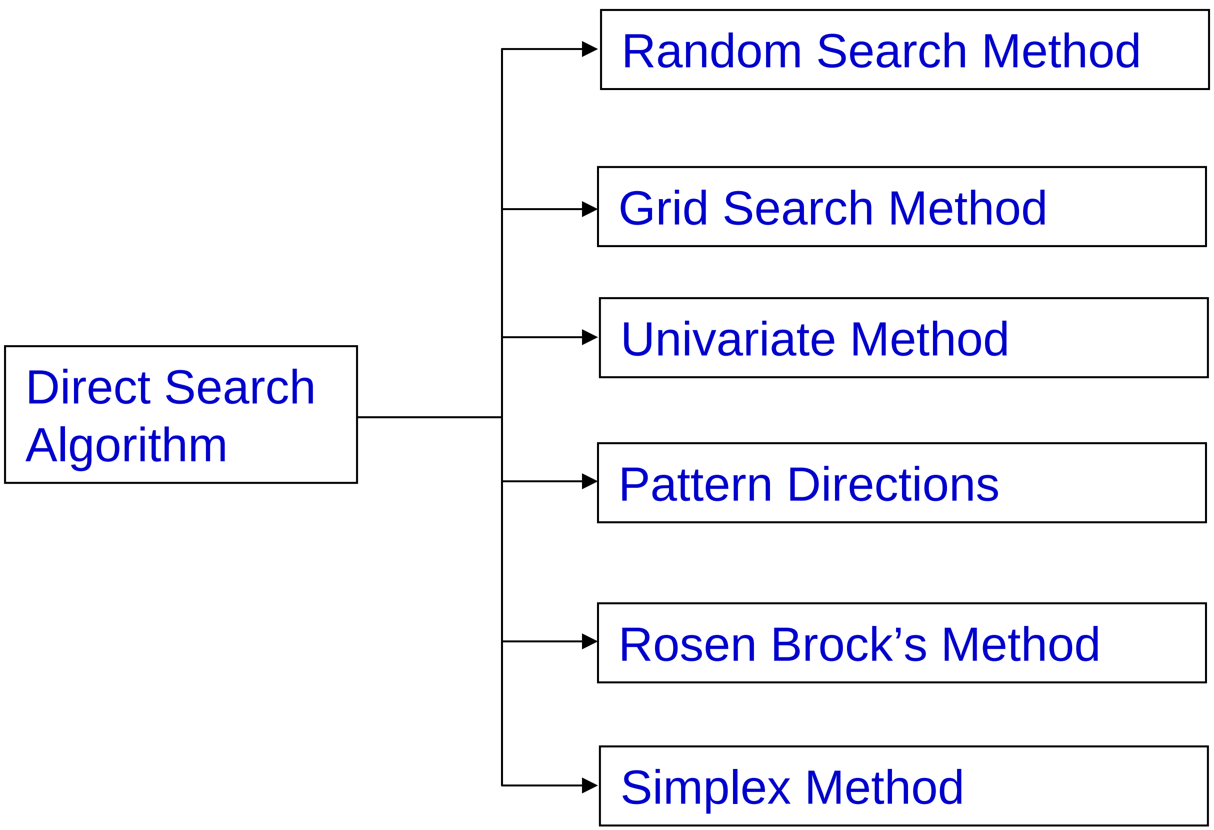
Direct Search Algorithm

- Considers only objective function
- Partial derivatives of objective function: not considered
- Called nongradient or zeroth order method

Indirect Search Algorithm

- Partial derivatives of objective function are considered
- Also called descent method

Direct Search Algorithm



Direct Search Algorithm (Contd..)

<u>Random Search Method:</u>

- Generates trial solution for the decision variables.
- Classification: Random jump, random walk, and random walk with direction exploitation.
- Random Jump: generates huge number of data points assuming uniform distribution of decision variables and selects the best one
- Random Walk: generates trial solution with sequential improvements using scalar step length and unit random vector.
- Random Walk with Direction Exploitation: Improved version of random walk search, successful direction of generating trial solution is found out and steps are taken along this direction.

Direct Search Algorithm (Contd..)

Grid Search Method

- Methodology involves setting up grids in the decision space.
- Objective function values are evaluated at the grid points.
- The point corresponding to the best objective function value considered as optimum solution.
- Major Drawback: number of grid points increases exponentially with the number of decision variables.

Direct Search Algorithm (Contd..)

Univariate Method:

- Generates trial solution for one decision variable keeping all others fixed.
- Best solution for each of the decision variables keeping others constant are obtained.
- The whole process is repeated iteratively till convergence.

Direct Search Algorithm (Contd..)

Pattern Directions:

- In Univariate method, search direction is same as coordinate axes directions.
- Makes the convergence process slow.
- In pattern directions, search is performed not along the coordinate directions, but along the direction towards the best solution.
- Popular Methods: Hooke and Jeeves' Method, and Powells method.
- Hooke and Jeeves' Method: uses exploratory move and pattern move.
- Powells method: uses conjugate gradient

Direct Search Algorithm (Contd..)

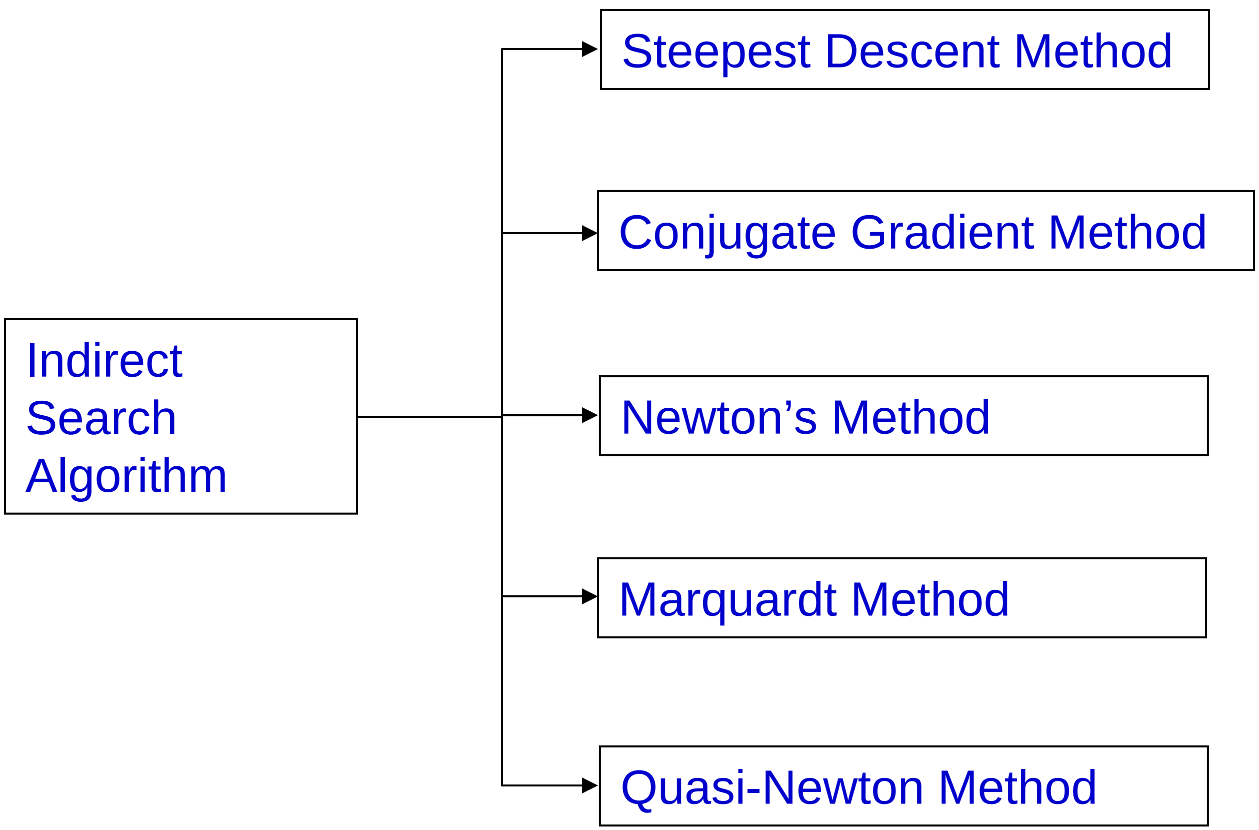
Rosen Brock's Method of Rotating Coordinates:

- Modified version of Hooke and Jeeves' Method.
- Coordinate system is rotated in such a way that the first axis always orients to the locally estimated direction of the best solution and all the axes are made mutually orthogonal and normal to the first one.

Simplex Method

- Conventional direct search algorithm.
- The best solution lies on the vertices of a geometric figure in N-dimensional space made of a set of N+1 points.
- The method compares the objective function values at the N+1 vertices and moves towards the optimum point iteratively.
- Movement of the simplex algorithm: reflection, contraction and expansion.

Indirect Search Algorithm



Indirect Search Algorithm (Contd..)

Steepest Descent Method:

- Search starts from an initial trial point X_1
- Iteratively moves along the steepest descent direction until optimum point is found.
- Drawback: Solution may get stuck to a local optima.

Conjugate Gradient Method

- Convergence technique uses the concept of conjugate gradient
- Uses the properties of quadratic convergence

Indirect Search Algorithm (Contd..)

Newton's Method:

- Based on Taylor series expansion.

$$f(X) = f(X_i) + \nabla f_i^T (X - X_i) + \frac{1}{2}(X - X_i)^T [J_i](X - X_i)$$

$[J_i]=[J]|x_i$ is the Hessian Matrix

- Setting the partial derivatives of the objective function to zero, minimum *f(x)* can be obtained

$$\frac{\partial f(X)}{\partial x_j} = 0, \qquad j = 1,2,\ldots,N$$

- From both the Eq. $\nabla f = \nabla f_i + [J_i](X - X_i) = 0$

- The improved solution is given by $X_{i+1} = X_i - [J_i]^{-1} \nabla f_i$

- Process continues till convergence

Indirect Search Algorithm (Contd..)

<u>Marquardt Method:</u>

- Combination of both steepest descent algorithm and Newton's method
- By modifying the diagonal elements of Hessian matrix iteratively optimum solution is obtained

<u>Quasi-Newton Method</u>

- Based on Newton's method
- They approximate the Hessian matrix, or its inverse, in order to reduce the amount of computation per iteration.
- The Hessian matrix is updated using the secant equation, a generalization of the secant method for multidimensional problems

Constrained Optimization

- Search algorithms cannot be directly applied to constrained optimization.
- Constrained optimization can be converted into unconstrained optimization using penalty function when there is a constraint violation.

$$f(X_i) = f(X_i) + \boxed{\lambda \times M \times \delta^2} \rightarrow \boxed{\text{Penalty Function}}$$

λ=1(for minimization problem) and -1 (for maximization problem), M=dummy variable with a very high value

Summary

- Nonlinear optimization with complicated objective functions or constraints: Can be solved by search algorithm.
- Care should be taken so that the search should not get stuck at local optima.
- Search algorithms are not directly applicable to constrained optimization
- Penalty functions can be used to convert constrained optimization into unconstrained optimization

Thank You