



Integer Programming

Mixed Integer Linear Programming



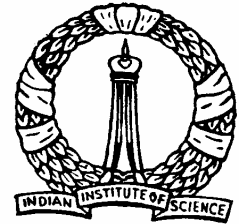
Objectives

- To discuss about the Mixed Integer Programming (MIP)
- To discuss about the Mixed Integer Linear Programming (MILP)
- To discuss the generation of Gomory constraints
- To describe the procedure for solving MILP



Introduction

- ***Mixed Integer Programming:***
 - Some of the decision variables are real valued and some are integer valued
- ***Mixed Integer Linear Programming:***
 - MIP with linear objective function and constraints
- The procedure for solving an MIP is similar to that of All Integer LP with some exceptions in the generation of Gomory constraints.



Generation of Gomory Constraints

- Consider the final tableau of an LP problem consisting of n basic variables (original variables) and m non basic variables (slack variables)
- Let x_i be the basic variable which has integer restrictions



Generation of Gomory Constraints ...contd.

- From the i^{th} equation,

- $$x_i = b_i - \sum_{j=1}^m c_{ij} y_j$$

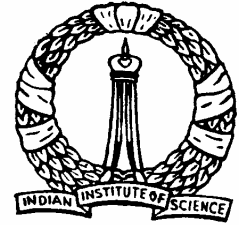
- Expressing b_i as an integer part plus a fractional part,

$$b_i = \bar{b}_i + \beta_i$$

- Expressing c_{ij} as $c_{ij} = \bar{c}_{ij}^+ + \bar{c}_{ij}^-$ where

$$\bar{c}_{ij}^+ = \begin{cases} c_{ij} & \text{if } c_{ij} \geq 0 \\ 0 & \text{if } c_{ij} < 0 \end{cases}$$

$$\bar{c}_{ij}^- = \begin{cases} 0 & \text{if } c_{ij} \geq 0 \\ c_{ij} & \text{if } c_{ij} < 0 \end{cases}$$

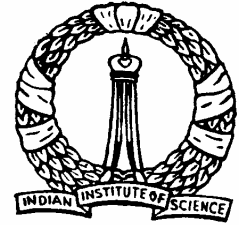


Generation of Gomory Constraints ...contd.

➤ Thus,

$$\sum_{j=1}^m (\bar{c}_{ij}^+ + \bar{c}_{ij}^-) y_j = \beta_i + (\bar{b}_i - x_i)$$

- Since x_i and \bar{b}_i are restricted to take integer values and also $0 < \beta_i < 1$ the value of $\beta_i + (\bar{b}_i - x_i)$ can be ≥ 0 or < 0
- Thus we have to consider two cases.



Generation of Gomory Constraints ...contd.

Case I: $\beta_i + (\bar{b}_i - x_i) \geq 0$

- For x_i to be an integer,

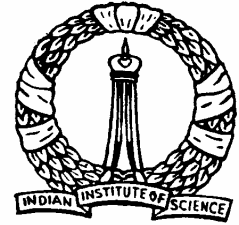
$$\beta_i + (\bar{b}_i - x_i) = \beta_i \text{ or } \beta_i + 1 \text{ or } \beta_i + 2, \dots$$

- Therefore,

$$\sum_{j=1}^m (\bar{c}_{ij}^+ + \bar{c}_{ij}^-) y_j \geq \beta_i$$

- Finally it takes the form,

$$\sum_{j=1}^m \bar{c}_{ij}^+ y_j \geq \beta_i$$



Generation of Gomory Constraints ...contd.

Case II: $\beta_i + (\bar{b}_i - x_i) < 0$

- For x_i to be an integer,

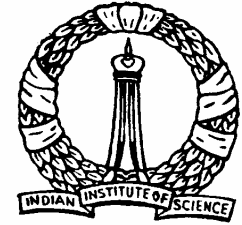
$$\beta_i + (\bar{b}_i - x_i) = -1 + \beta_i \text{ or } -2 + \beta_i \text{ or } -3 + \beta_i, \dots$$

- Therefore,

$$\sum_{j=1}^m (\bar{c}_{ij}^+ + \bar{c}_{ij}^-) y_j \leq \beta_i - 1$$

- Finally it takes the form,

$$\sum_{j=1}^m \bar{c}_{ij}^- y_j \leq \beta_i - 1$$



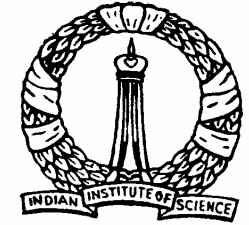
Generation of Gomory Constraints ...contd.

- Dividing this inequality by $(\beta_i - 1)$ and multiplying with β_i , we have

$$\frac{\beta_i}{\beta_i - 1} \sum_{j=1}^m \bar{c}_{ij}^- y_j \geq \beta_i$$

- Now considering both cases I and II, the final form of the Gomory constraint after adding one slack variable s_i is,

$$s_i - \sum_{j=1}^m \bar{c}_{ij}^+ y_j - \frac{\beta_i}{\beta_i - 1} \sum_{j=1}^m \bar{c}_{ij}^- y_j = -\beta_i$$



Procedure for solving Mixed-Integer LP

- Solve the problem as an ordinary LP problem neglecting the integrality constraints.
- Generate Gomory constraint for the fractional valued variable that has integer restrictions.
- Insert a new row with the coefficients of this constraint, to the final tableau of the ordinary LP problem.
- Solve this by applying the dual simplex method
- The process is continued for all variables that have integrality constraints



Thank You