Integer Programming

Mixed Integer Linear Programming

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Objectives

- To discuss about the Mixed Integer Programming (MIP)
- To discuss about the Mixed Integer Linear Programming (MILP)
- To discuss the generation of Gomory constraints
- To describe the procedure for solving MILP

Introduction

- ***Mixed Integer Programming:***
  - Some of the decision variables are real valued and some are integer valued
- ***Mixed Integer Linear Programming:***
  - MIP with linear objective function and constraints
- The procedure for solving an MIP is similar to that of All Integer LP with some exceptions in the generation of Gomory constraints.

Generation of Gomory Constraints

- Consider the final tableau of an LP problem consisting of n basic variables (original variables) and m non basic variables (slack variables)
- Let x_i be the basic variable which has integer restrictions

Generation of Gomory Constraints …contd.

- From the i^th equation,
- $$x_i = b_i - \sum_{j=1}^{m} c_{ij} y_j$$
- Expressing b_j as an integer part plus a fractional part,

$$b_i = \overline{b_i} + \beta_i$$

- Expressing c_{ij} as $c_{ij} = \overline{c}_{ij}^{+} + \overline{c}_{ij}^{-}$ where

$$\overline{c}_{ij}^{+} = \begin{cases} c_{ij} \text{ if } c_{ij} \geq 0 \\ 0 \text{ if } c_{ij} < 0 \end{cases}$$

$$\overline{c}_{ij}^{-} = \begin{cases} 0 \text{ if } c_{ij} \geq 0 \\ c_{ij} \text{ if } c_{ij} < 0 \end{cases}$$

Generation of Gomory Constraints …contd.

- Thus,

$$\sum_{j=1}^{m}\left(\overline{c}_{ij}^{+}+\overline{c}_{ij}^{-}\right)y_j=\beta_i+\left(\overline{b}_i-x_i\right)$$

- Since x_i and $\overline{b}_i$ are restricted to take integer values and also $0<\beta_i<1$ the value of $\beta_i+\left(\overline{b}_i-x_i\right)$ can be ≥ 0 or <0
- Thus we have to consider two cases.

Generation of Gomory Constraints …contd.

Case I: $\beta_i + \left(\overline{b}_i - x_i\right) \geq 0$

- For x_i to be an integer,

$$\beta_i + \left(\overline{b}_i - x_i\right) = \beta_i \; or \; \beta_i + 1 \, or \; \beta_i + 2, \ldots.$$

- Therefore,

$$\sum_{j=1}^{m} \left(\overline{c}_{ij}^{+} + \overline{c}_{ij}^{-}\right) y_j \geq \beta_i$$

- Finally it takes the form,

$$\sum_{j=1}^{m} \overline{c}_{ij}^{+} y_j \geq \beta_i$$

Generation of Gomory Constraints …contd.

Case II: $\beta_i + \left(\overline{b}_i - x_i\right) < 0$

- For x_i to be an integer,

$$\beta_i + \left(\overline{b}_i - x_i\right) = -1 + \beta_i \; or -2 + \beta_i \; or -3 + \beta_i, ...$$

- Therefore,

$$\sum_{j=1}^{m} \left(\overline{c}_{ij}^{+} + \overline{c}_{ij}^{-}\right) y_j \le \beta_i - 1$$

- Finally it takes the form,

$$\sum_{j=1}^{m} \overline{c}_{ij}^{-} y_j \le \beta_i - 1$$

Generation of Gomory Constraints …contd.

- Dividing this inequality by $(\beta_i - 1)$ and multiplying with β_i, we have

$$\frac{\beta_i}{\beta_i - 1} \sum_{j=1}^{m} \overline{c}_{ij}^{-} y_j \geq \beta_i$$

- Now considering both cases I and II, the final form of the Gomory constraint after adding one slack variable s_i is,

$$s_i - \sum_{j=1}^{m} \overline{c}_{ij}^{+} y_j - \frac{\beta_i}{\beta_i - 1} \sum_{j=1}^{m} \overline{c}_{ij}^{-} y_j = -\beta_i$$

Procedure for solving Mixed-Integer LP

- Solve the problem as an ordinary LP problem neglecting the integrality constraints.
- Generate Gomory constraint for the fractional valued variable that has integer restrictions.
- Insert a new row with the coefficients of this constraint, to the final tableau of the ordinary LP problem.
- Solve this by applying the dual simplex method
- The process is continued for all variables that have integrality constraints

Thank You