# Dynamic Programming

## Computational Procedure in Dynamic Programming



D Nagesh Kumar, IISc

# Objectives

- To explain the computational procedure of solving the multistage decision process using recursive equations for backward approach

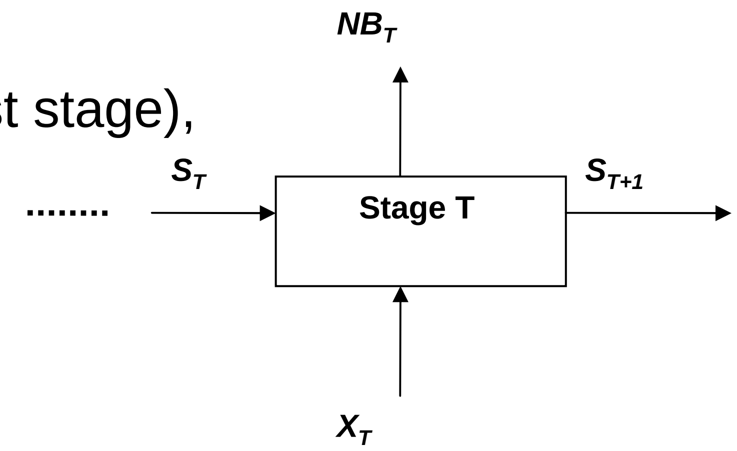
# Computational Procedure

- Consider a serial multistage problem and the recursive equations developed for backward recursion
- The objective function is

$$f = \sum_{t=1}^{T} NB_t = \sum_{t=1}^{T} h_t(X_t, S_t)$$

- Considering first sub-problem (last stage), the objective function is

$$f_T^*(S_T) = \underset{X_T}{opt}\left[h_T(X_T, S_T)\right]$$



## *Computational Procedure …contd.*

- The input variable is $S_T$ and the decision variable is $X_T$
- Optimal value of the objective function $f_T^*$ depend on the input $S_T$
- But at this stage, the value of $S_T$ is not known
- Value of $S_T$ depends upon the values taken by the upstream components
- Therefore, $S_T$ is solved for all possible range of values
- The results are entered in a graph or table which contains the calculated optimal values of $X_T^*$, $S_{T+1}$ and also $f_T^*$.

# Computational Procedure …contd.

- The results are entered in a graph or table which contains the calculated optimal values of $X_T^*$, $S_{T+1}$ and also $f_T^*$

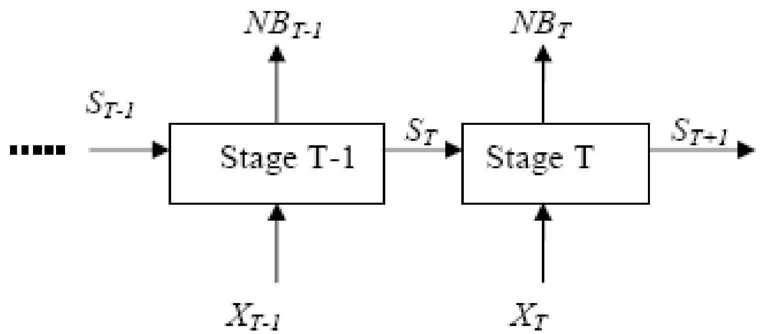
Typical table showing the results from the sub-optimization of stage 1

**TABLE - 1**

| Sl no | $S_T$ | $X_T^*$ | $f_T^*$ | $S_{T+1}$ |
|---|---|---|---|---|
| 1 | - | - | - | - |
| - | - | - | - | - |
| - | - | - | - | - |

# Computational Procedure …contd.

- Consider the second sub-problem by grouping the last two components




- The objective function is

$$f_{T-1}^*(S_{T-1}) = \underset{X_{T-1}, X_T}{opt} \left[h_{T-1}(X_{T-1}, S_{T-1}) + h_T(X_T, S_T)\right]$$

## *Computational Procedure …contd.*

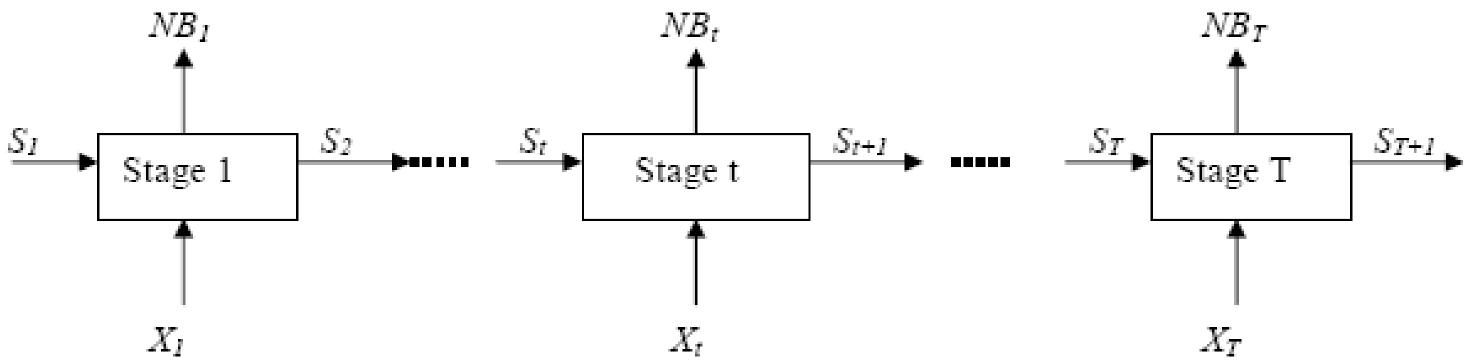
- From the earlier lecture,

$$f^*_{T-1}(S_{T-1}) = \underset{X_{T-1}}{opt}\left[h_{T-1}\left(X_{T-1}, S_{T-1}\right) + f^*_T(S_T)\right]$$

- The information of first sub-problem is obtained from the previous table
- A range of values are considered for $S_{T-1}$
- The optimal values of $X^*_{T-1}$ and $f^*_{T-1}$ are found for these range of values

# *Computational Procedure …contd.*

- In general, consider the sub-optimization of *i+1th* sub-problem (*T-ith* stage)




- The objective function can be written as

$$f^*_{T-i}(S_{T-i}) = \underset{X_{T-i},\ldots,X_{T-1},X_T}{opt} \left[h_{T-i}\left(X_{T-i}, S_{T-i}\right) + \ldots + h_{T-1}\left(X_{T-1}, S_{T-1}\right) + h_T\left(X_T, S_T\right)\right]$$

$$= \underset{X_{T-i}}{opt}\left[h_{T-i}\left(X_{T-i}, S_{T-i}\right) + f^*_{T-(i-1)}\right] \qquad \ldots(7)$$

## Computational Procedure …contd.

- At this stage, the sub-optimization has been carried out for all last $i$ components
- The information regarding the optimal values of $i^{th}$ sub-problem will be available in the form of a table
- Substituting this information in the objective function and considering a range of values, the optimal values of $f^*_{T-i}$ and $X^*_{T-i}$ can be calculated

# *Computational Procedure …contd.*

- The table showing the sub-optimization of *i+1*$^{th}$ sub-problem is shown

**TABLE - 2**

| Sl no | $S_{T-i}$ | $X^*_{T-i}$ | $S_{T-(i-1)}$ | $f^*_{T-(i-1)}(S_{T-(i-1)})$ | $f^*_{T-i}$ |
|---|---|---|---|---|---|
| 1 | - | - | - | - | |
| - | - | - | - | - | |
| - | - | - | - | - | |

- This procedure is repeated until stage 1 is reached
- For initial value problems, only one value $S_1$ need to be analyzed for stage 1

# *Computational Procedure …contd.*

- After completing the sub-optimization of all the stages, retrace the steps through the tables generated to find the optimal values of *X*
- The $T^{th}$ sub-problem gives the values of $X_1^*$ and $f_1^*$ for a given value of $S_1$ (since the value of $S_1$ is known for an initial value problem)
- Calculate the value of $S_2^*$ using the transformation equation $S_2 = g(X_1, S_1)$, which is the input to the 2nd stage ( $T\text{-}1^{th}$ sub-problem)
- From the tabulated results for the 2nd stage, the values of $X_2^*$ and $f_2^*$ are found out
- Again use the transformation equation to find out $S_3^*$ and the process is repeated until the 1st sub-problem or $T^{th}$ stage is reached
- The final optimum solution vector is given by $X_1^*, X_2^*, \ldots, X_T^*$

# Thank You