

Module – 1 Lecture Notes – 1

Historical Development and Model Building

Introduction

In this lecture, historical development of optimization methods is glanced through. Apart from the major developments, some recently developed novel approaches, such as, goal programming for multi-objective optimization, simulated annealing, genetic algorithms, and neural network methods are briefly mentioned tracing their origin. Engineering applications of optimization with different modeling approaches are scanned through from which one would get a broad picture of the multitude applications of optimization techniques.

Historical Development

The existence of optimization methods can be traced to the days of Newton, Lagrange, and Cauchy. The development of differential calculus methods for optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations, which deals with the minimization of functions, were laid by Bernoulli, Euler, Lagrange, and Weistrass. The method of optimization for constrained problems, which involve the addition of unknown multipliers, became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained optimization problems. By the middle of the twentieth century, the high-speed digital computers made implementation of the complex optimization procedures possible and stimulated further research on newer methods. Spectacular advances followed, producing a massive literature on optimization techniques. This advancement also resulted in the emergence of several well defined new areas in optimization theory.

Some of the major developments in the area of numerical methods of unconstrained optimization are outlined here with a few milestones.

- Development of the simplex method by Dantzig in 1947 for linear programming problems
- The enunciation of the principle of optimality in 1957 by Bellman for dynamic programming problems,

- Work by Kuhn and Tucker in 1951 on the necessary and sufficient conditions for the optimal solution of programming problems laid the foundation for later research in non-linear programming.
- The contributions of Zoutendijk and Rosen to nonlinear programming during the early 1960s have been very significant.
- Work of Carroll and Fiacco and McCormick facilitated many difficult problems to be solved by using the well-known techniques of unconstrained optimization.
- Geometric programming was developed in the 1960s by Duffin, Zener, and Peterson.
- Gomory did pioneering work in integer programming, one of the most exciting and rapidly developing areas of optimization. The reason for this is that most real world applications fall under this category of problems.
- Dantzig and Charnes and Cooper developed stochastic programming techniques and solved problems by assuming design parameters to be independent and normally distributed.

The necessity to optimize more than one objective or goal while satisfying the physical limitations led to the development of multi-objective programming methods. Goal programming is a well-known technique for solving specific types of multi-objective optimization problems. The goal programming was originally proposed for linear problems by Charnes and Cooper in 1961. The foundation of game theory was laid by von Neumann in 1928 and since then the technique has been applied to solve several mathematical, economic and military problems. Only during the last few years has game theory been applied to solve engineering problems.

Simulated annealing, genetic algorithms, and neural network methods represent a new class of mathematical programming techniques that have come into prominence during the last decade. Simulated annealing is analogous to the physical process of annealing of metals and glass. The genetic algorithms are search techniques based on the mechanics of natural selection and natural genetics. Neural network methods are based on solving the problem using the computing power of a network of interconnected 'neuron' processors.

Engineering applications of optimization

To indicate the widespread scope of the subject, some typical applications in different engineering disciplines are given below.

- Design of civil engineering structures such as frames, foundations, bridges, towers, chimneys and dams for minimum cost.
- Design of minimum weight structures for earth quake, wind and other types of random loading.
- Optimal plastic design of frame structures (e.g., to determine the ultimate moment capacity for minimum weight of the frame).
- Design of water resources systems for obtaining maximum benefit.
- Design of optimum pipeline networks for process industry.
- Design of aircraft and aerospace structure for minimum weight
- Finding the optimal trajectories of space vehicles.
- Optimum design of linkages, cams, gears, machine tools, and other mechanical components.
- Selection of machining conditions in metal-cutting processes for minimizing the product cost.
- Design of material handling equipment such as conveyors, trucks and cranes for minimizing cost.
- Design of pumps, turbines and heat transfer equipment for maximum efficiency.
- Optimum design of electrical machinery such as motors, generators and transformers.
- Optimum design of electrical networks.
- Optimum design of control systems.
- Optimum design of chemical processing equipments and plants.
- Selection of a site for an industry.
- Planning of maintenance and replacement of equipment to reduce operating costs.
- Inventory control.
- Allocation of resources or services among several activities to maximize the benefit.
- Controlling the waiting and idle times in production lines to reduce the cost of production.
- Planning the best strategy to obtain maximum profit in the presence of a competitor.

- Designing the shortest route to be taken by a salesperson to visit various cities in a single tour.
- Optimal production planning, controlling and scheduling.
- Analysis of statistical data and building empirical models to obtain the most accurate representation of the statistical phenomenon.

However, the list is incomplete.

Art of Modeling: Model Building

Development of an optimization model can be divided into five major phases.

- Data collection
- Problem definition and formulation
- Model development
- Model validation and evaluation of performance
- Model application and interpretation

Data collection may be time consuming but is the fundamental basis of the model-building process. The availability and accuracy of data can have considerable effect on the accuracy of the model and on the ability to evaluate the model.

The **problem definition and formulation** includes the steps: identification of the decision variables; formulation of the model objective(s) and the formulation of the model constraints.

In performing these steps the following are to be considered.

- Identify the important elements that the problem consists of.
- Determine the number of independent variables, the number of equations required to describe the system, and the number of unknown parameters.
- Evaluate the structure and complexity of the model
- Select the degree of accuracy required of the model

Model development includes the mathematical description, parameter estimation, input development, and software development. The model development phase is an iterative process that may require returning to the model definition and formulation phase.

The **model validation and evaluation** phase is checking the performance of the model as a whole. **Model validation** consists of validation of the assumptions and parameters of the

model. The performance of the model is to be evaluated using standard performance measures such as Root mean squared error and R^2 value. A sensitivity analysis should be performed to test the model inputs and parameters. This phase also is an iterative process and may require returning to the model definition and formulation phase. One important aspect of this process is that in most cases data used in the formulation process should be different from that used in validation. Another point to keep in mind is that no single validation process is appropriate for all models.

Model application and implementation include the use of the model in the particular area of the solution and the translation of the results into operating instructions issued in understandable form to the individuals who will administer the recommended system.

Different modeling techniques are developed to meet the requirements of different types of optimization problems. Major categories of modeling approaches are: classical optimization techniques, linear programming, nonlinear programming, geometric programming, dynamic programming, integer programming, stochastic programming, evolutionary algorithms, etc. These modeling approaches will be discussed in subsequent modules of this course.