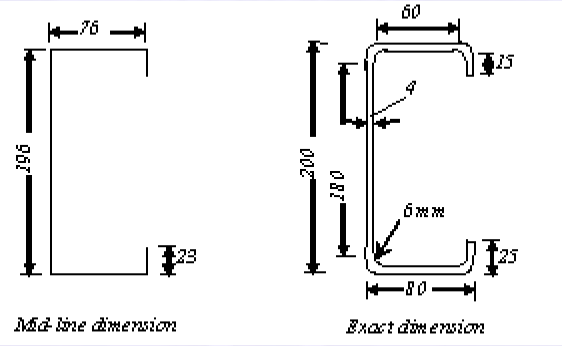
5.10 Examples

5.10.1 Analysis of effective section under compression

To illustrate the evaluation of reduced section properties of a section under axial compression.

Section: 200 x 80 x 25 x 4.0 mm

Using mid-line dimensions for simplicity. Internal radius of the corners is 1.5t.



Effective breadth of web (flat element)

$h = B_2 / B_1 = 60 / 180 = 0.33$

$$K_1 = 7 - \frac{1.8h}{0.15 + h} - 1.43h^3$$

$$= 7 - \frac{1.8 \text{ x } 0.33}{0.15 + 0.33} - 1.43 \text{ x } 0.33^3$$

= 5.71 or 4 (minimum) = 5.71

$p_{cr} = 185000 \ K_1 \ (t / b)^2$

$= 185000 \text{ x } 5.71 \text{ x } (4 / 180)^2 = 521.7 \text{ N / mm}^2$

$$\frac{f_{cr}}{p_{cr} \text{ x } \gamma_m} = \frac{240}{521.7 \text{ x } 1.15} = 0.4 > 0.123$$

$$\frac{b_{eff}}{b} = \left[1 + 14\left\{\sqrt{f_{cr}/(p_{cr}\ x\ \gamma_m)} - 0.35\right\}^4\right]^{-0.2}$$

$$= \left[1 + 14\left\{\sqrt{0.4} - 0.35\right\}^4\right]^{-0.2} \qquad = 0.983$$

or b_{eff} = 0.983 x 180 = 176.94 mm

Effective width of flanges (flat element)

$K_2 = K_1 h^2 (t_1 / t_2)^2$

$= K_1 h^2 (t_1 = t_2)$

$= 5.71 \times 0.33^2 = 0.633$ or 4 (minimum) = 4

$p_{cr} = 185000 \times 4 \times (4 / 60)^2 = 3289$ N /mm^2

$$\frac{f_c}{p_{cr}\ x\ \gamma_m} = \frac{240}{3289\,x\,1.15} = 0.063 > 0.123$$

$$\therefore \quad \frac{b_{eff}}{b} = 1$$ b_{eff} = 60 mm

Effective width of lips (flat element)

K = 0.425 (conservative for unstiffened elements)

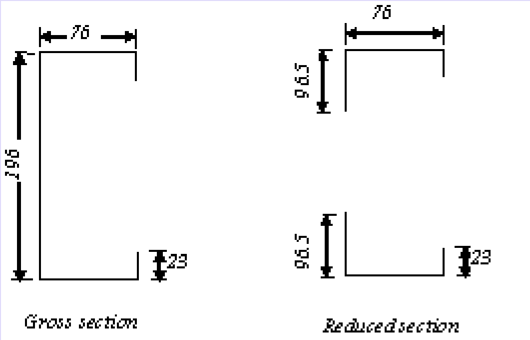
$p_{cr} = 185000 \times 0.425 \times (4 / 15)^2 = 5591$ N /mm^2

$$\frac{f_c}{p_{cr}\ x\ \gamma_m} = \frac{240}{5591\,x\,1.15} = 0.04 > 0.123$$

$$\therefore \quad \frac{b_{eff}}{b} = 1$$ b_{eff} = 15 mm

Effective section in mid-line dimension

As the corners are fully effective, they may be included into the effective width of the flat elements to establish the effective section.



The calculation for the area of gross section is tabulated below:

	A_i (mm^2)
Lips	2 x 23 x 4 = 184
Flanges	2 x 76 x 4 = 608
Web	196 x 4 = 784
Total	1576

The area of the gross section, A = 1576 mm^2

The calculation of the area of the reduced section is tabulated below:

	A_i (mm^2)
Lips	2 x 15 x 4 = 120
Corners	4 x 45.6 = 182.4
Flanges	2 x 60 x 4 = 480
Web	176.94 x 4 = 707.8
Total	1490.2

The area of the effective section, A_{eff} = 1490.2 mm^2

Therefore, the factor defining the effectiveness of the section under compression,

$$Q = \frac{A_{eff}}{A} = \frac{1490}{1576} = 0.95$$

The compressive strength of the member = Q A f_y / γ_m

= 0.95 x 1576 x 240 / 1.15

= 313 kN

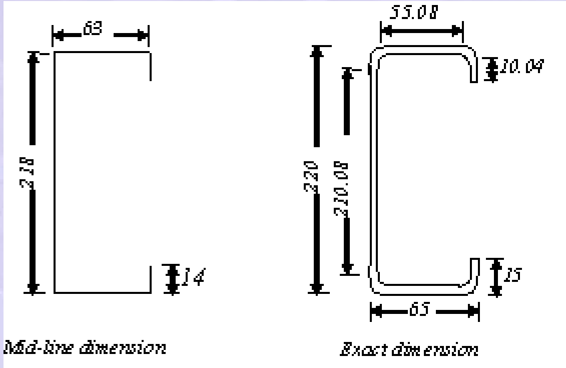
5.10.2 Analysis of effective section under bending

To illustrate the evaluation of the effective section modulus of a section in bending.

We use section: 220 x 65 x 2.0 mm Z28 Generic lipped Channel (from "Building Design using Cold Formed Steel Sections", Worked Examples to BS 5950: Part 5, SCI PUBLICATION P125)

Only the compression flange is subject to local buckling.

Using mid-line dimensions for simplicity. Internal radius of the corners is 1.5t.



Thickness of steel (ignoring galvanizing), t = 2 - 0.04 = 1.96 mm

Internal radius of the corners = 1.5 x 2 = 3 mm

Limiting stress for stiffened web in bending

$$p_0 = \left\{1.13 - 0.0019\frac{D}{t}\sqrt{\frac{f_y}{280}}\right\}p_y$$

and p_y = 280 / 1.15 = 243.5 N / mm^2

$$p_0 = \left\{1.13 - 0.0019 \text{ x } \frac{220}{1.96}\sqrt{\frac{280}{280}}\right\}\frac{280}{1.15}$$

= 223.2 N / mm^2

Which is equal to the maximum stress in the compression flange, i.e.,

f_c = 223.2 N / mm^2

Effective width of compression flange

h = B_2 / B_1 = 210.08 / 55.08 = 3.8

$$K_1 = 5.4 - \frac{1.4h}{0.6+h} - 0.02h^3$$

$$= 5.4 - \frac{1.4 \text{ x } 3.8}{0.6 + 3.8} - 0.02 \text{ x } 3.8^3$$

= 3.08 or 4 (minimum) = 4

$$p_{cr} = 185000 \text{ x } 4 \text{ x} \left(\frac{1.96}{55.08}\right)^2 = 937 \text{ N/mm}^2$$

$$\frac{f_c}{p_{cr}} = \frac{223.2}{937} = 0.24 > 0.123$$

$$\frac{b_{eff}}{b} = \left[1 + 14\left\{\sqrt{f_c/p_{cr}} - 0.35\right\}^4\right]^{-0.2}$$

$$= \left[1 + 14\left\{\sqrt{0.24} - 0.35\right\}^4\right]^{-0.2} = 0.998$$

b_{eff} = 0.99 x 55 = 54.5

Effective section in mid-line dimension:

The equivalent length of the corners is 2.0 x 2.0 = 4 mm

The effective width of the compression flange = 54.5 + 2 x 4 = 62.5

The calculation of the effective section modulus is tabulated as below:

Elements	A_i (mm^2)	y_i (mm)	A_i y_i (mm^3)	I_g + A_i y_i^2 (mm^4)
Top lip	27.44	102	2799	448 + 285498
Compression flange	122.5	109	13352.5	39.2 + 1455422.5
Web	427.3	0	0	39.2 + 1455422.5
Tension flange	123.5	-109	-13459.3	39.5 + 1467064
Bottom lip	27.4	-102	-2799	448 + 285498
Total	728.2		-106.8	5186628.4

The vertical shift of the neutral axis is

$$\bar{y} = \frac{-106.8}{728.2} = -0.15\,\text{mm}$$

The second moment of area of the effective section is

$I_{xr} = (5186628.4 + 728.2 \times 0.15^2) \times 10^{-4}$

$= 518.7\ cm^4$ at $p_0 = 223.2\ N/mm^2$

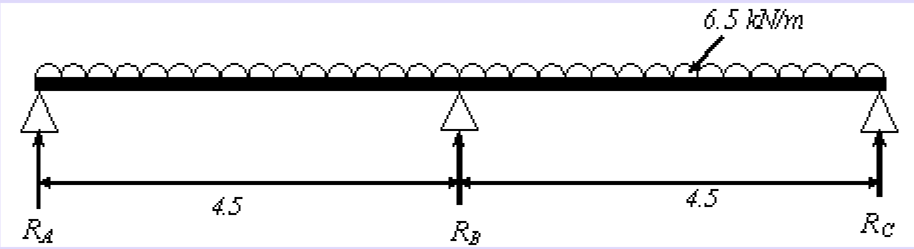
or = $518.7 \times \dfrac{223.2 \times 1.15}{280} = 475.5\,\text{cm}^4$ at $p_y = 280 / 1.15\ N/mm^2$

The effective section modulus is,

$$Z_{xr} = \frac{475.5}{(109 + 0.15)/10} = 43.56\,\text{cm}^3$$

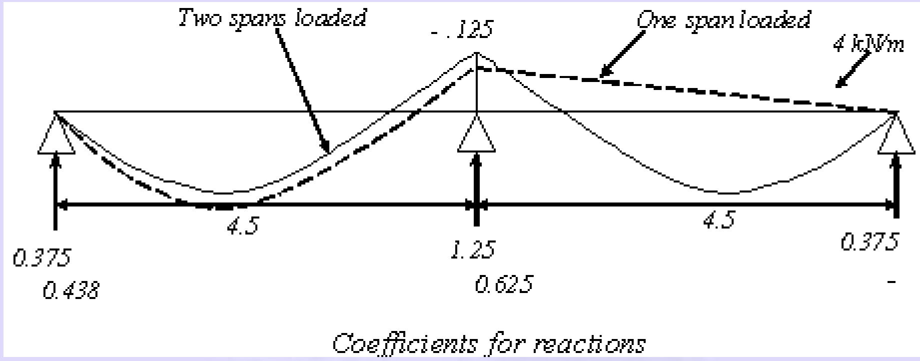
5.10.3 Two span design

Design a two span continuous beam of span 4.5 m subject to a UDL of 4kN/m as shown in Fig.1.



Factored load on each span = 6.5 x 4.5 = 29.3 kN

Bending Moment



Maximum hogging moment = 0.125 x 29.3 x 4.5 = 16.5 kNm

Maximum sagging moment = 0.096 x 29.3 x 4.5 = 12.7 kNm

Shear Force

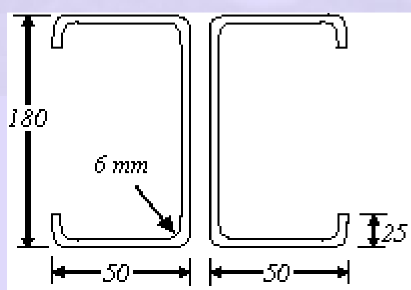
Two spans loaded: R_A = 0.375 x 29.3 = 11 kN

R_B = 1.25 x 29.3 = 36.6 kN

One span loaded: R_A = 0.438 x 29.3 = 12.8 kN

Maximum reaction at end support, $F_{w,max}$ = 12.8 kN

Maximum shear force, $F_{v,max}$ = 29.3 - 11 = 18.3 kN



Try 180 x 50 x 25 x 4 mm Double section (placed back to back)

Material Properties: E = 205 kN/mm

p_y = 240 / 1.15

= 208.7 N/mm^2

Section Properties: t = 4.0 mm

D = 180 mm

r_{yy} = 17.8 mm

I_{xx} = 2 x 518 x 10^4 mm^4

Z_{xx} = 115.1 x 10^3 mm^3

Only the compression flange is subject to local buckling

Limiting stress for stiffened web in bending

$$p_0 = \left\{1.13 - 0.0019\frac{D}{t}\sqrt{\frac{f_y}{280}}\right\}p_y$$

and p_y = 240 / 1.15 = 208.7 N / mm^2

$$p_0 = \left\{1.13 - 0.0019\,x\,\frac{180}{4}\sqrt{\frac{240}{280}}\right\}x\,208.7$$

= 219.3 N / mm^2

Which is equal to the maximum stress in the compression flange, i.e.,

f_c = 219.3 N / mm^2

Effective width of compression flange

h = B_2 / B_1 = 160 / 30 = 5.3

$$K_1 = 5.4 - \frac{1.4h}{0.6 + h} - 0.02h^3$$
$$= 5.4 - \frac{1.4 \text{ x } 3.8}{0.6 + 3.8} - 0.02 \text{ x } 5.3^3$$

= 1.1 or 4 (minimum) = 4

$$p_{cr} = 185000 \text{ x } 4 \text{ x} \left(\frac{4}{30}\right)^2 = 13155 \text{ N/mm}^2$$
$$\frac{f_c}{p_{cr}} = \frac{219.3}{13155} = 0.017 > 0.123$$
$$\frac{b_{eff}}{b} = 1$$

b_{eff} = 30 mm

i.e. the full section is effective in bending.

I_{xr} = 2 x 518 x 10^4 mm^4

Z_{xr} = 115.1 x 10^3 mm^3

Moment Resistance

The compression flange is fully restrained over the sagging moment region but it is unrestrained over the hogging moment region, that is, over the internal support.

However unrestrained length is very short and lateral torsional buckling is not critical.

The moment resistance of the restrained beam is:

$$M_{cx} = Z_{xr}\, p_y$$

$= 115.1 \times 103 \times (240 / 1.15)\ 10^{-6} = 24$ kNm > 16.5 kNm O.K

Shear Resistance

Shear yield strength,

$$p_v = 0.6\ p_y = 0.6 \times 240 / 1.15 = 125.2 \text{ N/mm}^2$$

Shear buckling strength, $q_{cr} = \left(\frac{1000t}{D}\right)^2 = \left(\frac{1000 \times 4}{180}\right)^2 = 493.8 \text{ N/mm}^2$

Maximum shear force, $F_{v,max} = 18.3$ kN

Shear area = 180 x 4 = 720 mm^2

Average shear stress $f_v = \frac{18.3 \times 10^3}{720} = 25.4 \text{ N/mm}^2 < q_{cr}$

O.K

Web crushing at end supports

Check the limits of the formulae.

$$\frac{D}{t} = \frac{180}{4} = 45 \leq 200 \quad \therefore \text{O.K}$$

$$\frac{r}{t} = \frac{6}{4} = 1.5 \leq 6 \quad \therefore \text{O.K}$$

At the end supports, the bearing length, N is 50 mm (taking conservatively as the flange width of a single section)

For c=0, N/ t = 50 / 4 = 12.5 and restrained section.

C is the distance from the end of the beam to the load or reaction.

Use

$$P_w = 2\,x\,t^2\,C_r\,\frac{f_y}{\gamma_m}\left\{8.8 + 1.11\sqrt{N/t}\right\}$$

$$C_r = 1 + \frac{D/t}{750}$$

$$= 1 + \frac{45}{750} = 1.06$$

$$P_w = 2\,x\,4^2\,x\,1.06\,x\,\frac{240}{1.15}\left\{8.8 + 1.11\sqrt{12.5}\right\}10^{-3}$$

Web Crushing at internal support

t the internal support, the bearing length, N, is 100mm (taken as the flange width of a double section)

For c > 1.5D, N / t = 100 / 4 = 25 and restrained section.

$$P_w = t^2\,C_5 C_6\,\frac{f_y}{\gamma_m}\left\{13.2 + 1.63\sqrt{N/t}\right\}$$

$$k = \frac{f_y}{228\,x\,\gamma_m} = \frac{240}{1.15\,x\,228} = 0.9$$

C_5 = (1.49 - 0.53 k) = 1.49 - 0.53 x 0.92 = 1.0 > 0.6

C_6 = (0.88 - 0.12 m)

m = t / 1.9 = 4 / 1.9 = 2.1

C_6 = 0.88 - 0.12 x 2.1 = 0.63

$$\therefore\ P_w = 2\,x\,4^2\,x\,1\,x\,0.63\,x\,\frac{240}{1.15}\left\{13.2 + 1.63\sqrt{25}\right\}10^{-3}$$

= 89.8 kN > R_B (= 36 kN)

Deflection Check

A coefficient of $\frac{3}{384}$ is used to take in account of unequal loading on a double span. Total unfactored imposed load is used for deflection calculation.

$$\delta_{max} = \frac{3}{384}\frac{W L^3}{E I_{av}}$$

$$I_{av} = \frac{I_{xx} + I_{xy}}{2} = \frac{1036+1036}{2} = 1036 \, x \, 10^4 \; mm^4$$

W = 29.3 / 1.5 = 19.5 kN

$$\delta_{max} = \frac{3}{384}\frac{19.5 \, x \, 10^3 \; x \, 4500^3}{205 \, x \, 10^3 \; x \, 1036 \, x \, 10^4} = 6.53 \, mm$$

Deflection limit = L / 360 for imposed load

= 4500 / 360 = 12.5 mm > 6.53 mm O.K

In the double span construction: Use double section 180 x 50 x 25 x 4.0 mm lipped channel placed back to back.

5.10.4 Column design

Design a column of length 2.7 m for an axial load of 550 kN.

Axial load P = 550 kN

Length of the column, L = 2.7 m

Effective length, le = 0.85L = 0.85 x 2.7 = 2.3 m

Try 200 x 80 x 25 x 4.0 mm Lipped Channel section

Material Properties: E = 205 kN/mm^2

f_y = 240 N/ mm^2

p_y = 240 / 1.15 = 208.7 N/mm^2

Section Properties: A = 2 x 1576 = 3152 mm^2

I_{xx} = 2 x 903 x 10^4 mm^4

I_{yy} = 2 [124 x 10^4 + 1576 x 24.8^2]

= 442 x 10^4 mm^4

$$r_{min} = \sqrt{\frac{442 \times 10^4}{2 \times 1576}} = 37.4 \text{ mm}$$

Load factor Q = 0.95 (from worked example 1)

The short strut resistance, P_{cs} = 0.95 ? 2 ? 1576 ? 240/ 1.15 = 625 kN

P = 550 kN < 625 kN O. K

Axial buckling resistance

Check for maximum allowable slenderness

$$\frac{l_e}{r_y} = \frac{2.3 \times 10^3}{37.4} = 61.5 < 180 \quad \text{O.K}$$

In a double section, torsional flexural buckling is not critical and thus α = 1

Modified slenderness ratio,

$$\bar{\lambda} = \frac{\alpha \frac{l_e}{r_y}}{\lambda_y}$$

$$\lambda_y = \pi\sqrt{\frac{E}{p_y}} = \pi\sqrt{\frac{2.05 \times 10^5}{208.7}} = 98.5$$

$$\therefore \bar{\lambda} = \frac{1 \times 61.5}{98.5} = 0.62$$

$$\frac{P_c}{P_{cs}} = 0.91$$

P_c = 0.91 x 625 = 569 kN > P O. K