Module 6: Inelastic Seismic Response of Structures

Exercise Problems:

(Use any standard software like SAP2000, ABAQUS and MATLAB for solving the problems; you may use your own developed program based on the methods presented in the Chapter)

6.8 A SDOF system has a nonlinear spring having the bilinear force-displacement characteristics as shown in Figure 6.29 Find the time histories of relative displacement and acceleration of the mass for El Centro earthquake. Also, find the ductility ratio. Take $\xi = 5\%$ and $m = 10\text{kg}$.

6.9 A five storey frame as shown in Figure 6.30 is subjected to El Centro earthquake. The columns are weaker than the beams and have the nonlinear properties as shown in the figure. Find (i) the envelope of the peak relative displacements of the floors; (ii) the time history of the base shear; and (iii) the stiffness matrix of the system at time $t = 6.2\,\text{s}$. Assume $\xi = 5\%$.

6.10 A three storey 3D frame with rigid diaphragms, shown in Figure 6.31, is subjected to El Centro earthquake acting along x-direction. The elasto-plastic force-deformation characteristics of the columns are also shown in the same figure. Find (i) the time histories of rotation and x-displacement of the top floor; (ii) the time histories of base shears of column A; (iii) the ductility demand of the column A at each floor level; and (iv) the stiffness matrix of the system at time $t = 7\,\text{s}$. Assume $\xi = 5\%$. For performing the analysis, the effect of bi-directional interaction on yielding of columns may be ignored. However, for computing the stiffness matrix at *t*=7s, this interaction effect should be included and the responses obtained from the no interaction analysis at *t*=7s may be used.

6.11 For the frame shown in Figure 6.30, find the top floor response spectrum (for 5% damping), and compare it with that if the columns are assumed to be un-yielding. Also, find the ductility demand of each floor. [Hint: floor response spectrum is the plot of pseudo acceleration of a SDOF attached to the floor with its time period for a given value of ξ].

6.12 A five storey strong column-weak beam frame shown in Figure 6.31 is subjected to El Centro earthquake. The mass at each floor level is $m = 2500\text{kg}$. The potential locations of the plastic hinges in the beams are shown in the same figure along with the moment-rotation back bone curves. Obtain the time histories of (i) the top floor relative displacement; (ii) the base shear of the column A; (iii) the moment-rotation plot for section B; and (iv) ductility demand of each beam.

6.13 Using the displacement control push over analysis, obtain the plots of base shear vs. top displacement of the frames shown in Figures 6.31 (with changed beams and columns cross sections) and 6.30. Changed cross sections are: all columns - 40×40 cm and all beams- 30×30 cm. Floor masses are not changed. Take the values of M_y, θ_y and θ_{max} from Table 6.4. For the other problem (i.e., Figure 6.30), take the shear displacement capacity as 3 times the yield value. Also, show the formation of hinges at different displacement stages. (NOTE: for push over analysis, flat portion of the elasto plastic curve is replaced by a line with a very mild slope.

Table 6.4: Properties of the frame

Member	Cross section (mm)	M_y (kNm)	θ_y (rad)	θ_{max} (rad)
Beam	300×300	168.9	9.025E-3	0.0271
Column	400×400	153.88	8.397E-3	0.0252

6.14 Construct an inelastic design response spectrum in tripartite plot for $\mu = 2$, 3 and 4 for the idealized elastic response spectrum of El Centro earthquake (Figure 2.17; Chapter 2). Compare the inelastic design response spectrums with the actual inelastic response spectrums of El Centro earthquake for $\mu = 2$ and 4 (for $\xi = 5\%$).

Take the relevant figures from the slides or from the reference book

Module 6: Inelastic Seismic Response of Structures

Exercise Solution:

ERRATA FOR THE TEXT BOOK
pp 273, Exercise problem 6.12: Take $\xi = 5\%$
pp 274, In Figure 6.32, section B is at the left end of the third beam from the bottom

6.8. Refer to the exercise problem 6.8.

Initial stiffness of the spring = $\dfrac{0.15mg}{0.0147} = 100.1m$

Frequency based on the initial stiffness = $\sqrt{\dfrac{100.1m}{m}} \approx 10\,\text{rad s}^{-1}$

$c = 2\xi\omega_n m = 2 \times 0.05 \times 10 \times 10 = 10\ \text{Ns m}^{-1}$

The incremental equation of motion is

$10\Delta\ddot{x} + 10\Delta x + R(t) = -10\Delta\ddot{x}_g$

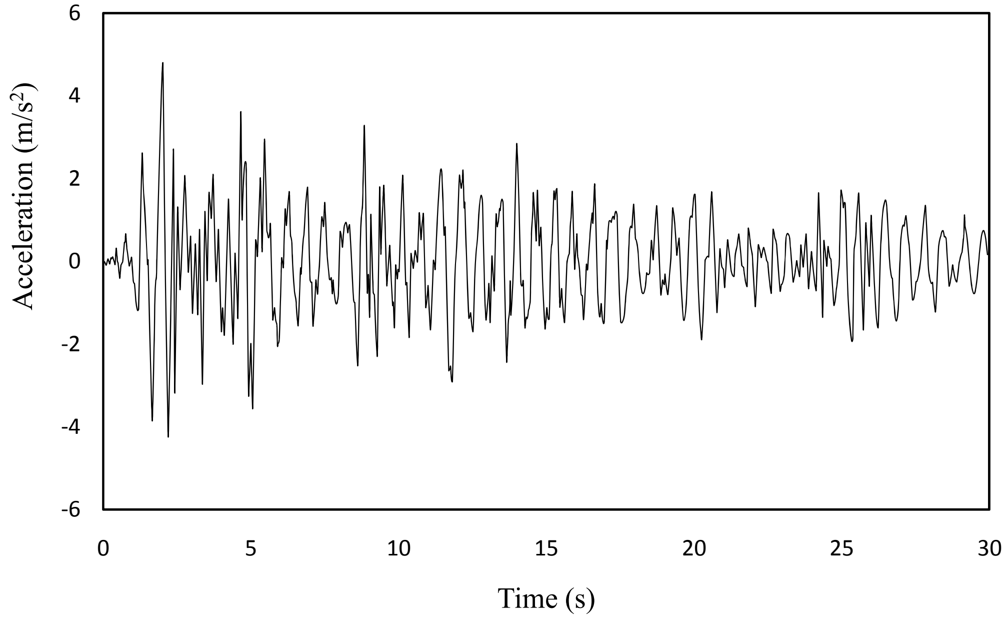
The incremental equation of motion is solved using the method outlined in section 6.2.2. Initial condition at $t = 0$ is taken as $x = \dot{x} = 0$.

The time histories of relative displacement and acceleration are shown in Figure 6.33

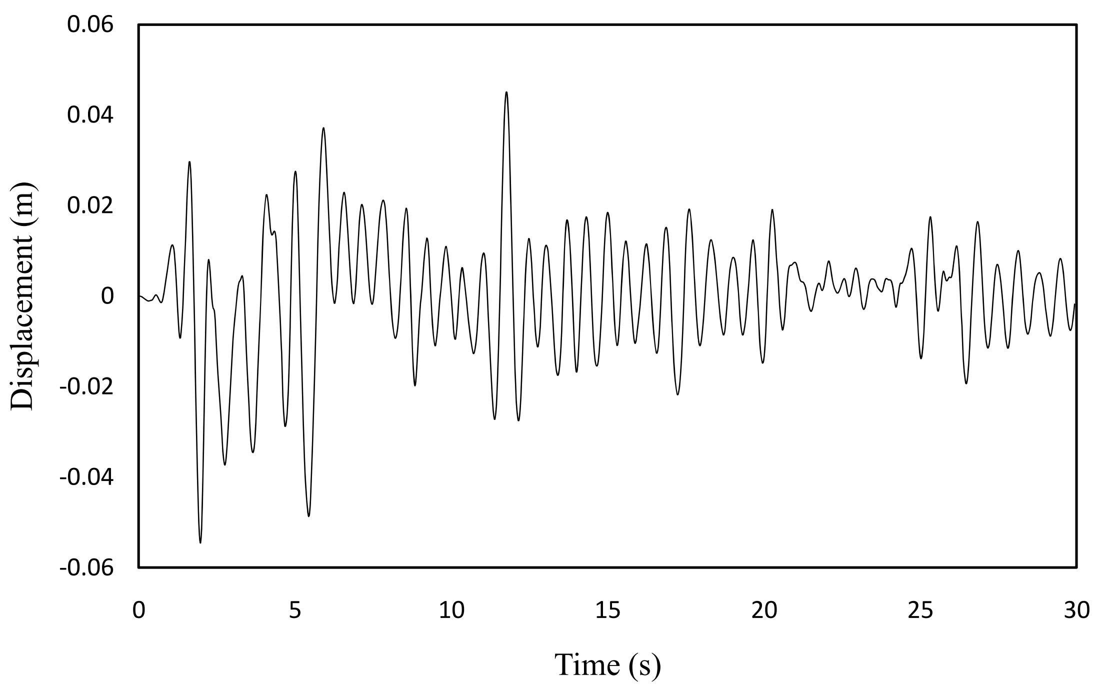
Absolute maximum displacement = 0.055m

Yield displacement = 0.0417m

Ductility ratio = 3.7



(a)



(b)

Figure 6.33 Time histories of responses (a) relative acceleration (b) relative displacement

6.9. Refer to the exercise problem 6.9.

The incremental equations of motion based on initial stiffness are

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{x}} + \boldsymbol{C}\Delta\dot{\boldsymbol{x}} + K\Delta\boldsymbol{x} = -\boldsymbol{M}\boldsymbol{I}\Delta\ddot{\boldsymbol{x}}_g$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} m \qquad \boldsymbol{K} = \begin{bmatrix} 4 & & & & & \\ -2 & 4 & & sym & & \\ 0 & -2 & 4 & & & \\ 0 & 0 & -2 & 4 & & \\ 0 & 0 & 0 & -2 & 4 & \\ 0 & 0 & 0 & 0 & -2 & 2 \end{bmatrix} k$$

$\omega_1 = 4.02\,\text{rads}^{-1}$; $\omega_2 = 11.75\,\text{rads}^{-1}$ (based on the initial stiffness)

$\alpha = 0.2998$; $\beta = 0.00634$

$$\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K} = \begin{bmatrix} 2.835 & & & & \\ -1.267 & 2.835 & & sym & \\ 0 & -1.267 & 2.835 & & \\ 0 & 0 & -1.267 & 2.835 & \\ 0 & 0 & 0 & -1.267 & 1.567 \end{bmatrix} m$$

For solving the problem using SAP2000, the following numerical values are used:

$E = 2.637 \times 10^8\ \text{Nm}^{-2}$; $m = 2500\,\text{kg}$; $k = 250000\,\text{Nm}^{-1}$; $\xi = 5\%$

$F_y = 3678.75\text{N}$; Column size = $40\,\text{cm} \times 40\,\text{cm}$

Beam is assumed flexurally rigid.

E is computed from $\dfrac{12EI}{l^3} = k$

The absolute peak values of the displacements are obtained from the time histories of displacements of each floor. From these peak values, the envelop of the peak displacements of the floors is shown in Table 6.4

Table 6.4: Peak relative displacements of the floor

Floor	Peak floor displacement (m)

	Max	Min
1st	0.042	-0.088
2nd	0.053	-0.113
3rd	0.061	-0.146
4th	0.065	-0.162
5th	0.066	-0.172

(ii) The time history of base shear is shown in Figure 6.34

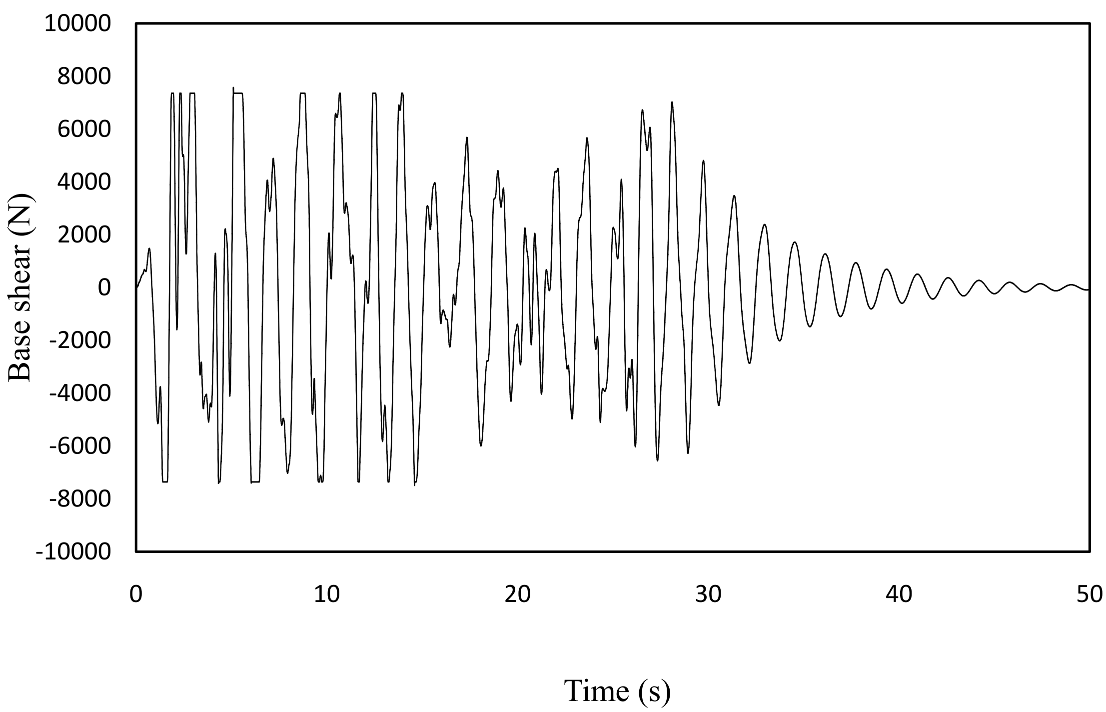


Figure 6.34 Time history of base shear

(iii) At time t = 6.25 s, the shear forces in the columns are computed as:

Table 6.5: Shear forces in columns

Columns in floor	Shear force (N)
1	3678.75
2	3678.75
3	3678.75
4	2764.44

5	1411.58

$$f_y = 3678.75\,\text{N}$$

Thus, first three columns from the bottom have undergone yielding and therefore, they do not contribute to the overall stiffness of the frame. The resulting stiffness matrix is given as

$$\boldsymbol{K} = \begin{bmatrix} 0 & & & & \\ 0 & 0 & & sym & \\ 0 & 0 & 2 & & \\ 0 & 0 & -2 & 4 & \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix} k$$

6.10. Refer to the exercise problem 6.10.

The incremental equations of motion for the 3D frame based on the initial stiffness are given as

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{x}} + \boldsymbol{C}\Delta\dot{\boldsymbol{x}} + K\Delta\boldsymbol{x} = -\boldsymbol{M}\boldsymbol{I}\Delta\ddot{x}_g$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 2.042 & & & & & & \\ & & & 1 & & & & & \\ & & & & 1 & & & & \\ & & & & & 2.042 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 2.042 \end{bmatrix} m$$

$$
\boldsymbol{K}=\begin{bmatrix}
6 & & & & & & & & \\
0 & 6 & & & & & & & \\
-1.75 & -1.75 & 20 & & & & sym & & \\
-6 & 0 & -1.75 & 12 & & & & & \\
0 & -6 & -1.75 & 0 & 12 & & & & \\
1.75 & 1.75 & -20 & -3.5 & -3.5 & 40 & & & \\
0 & 0 & 0 & -6 & 0 & -1.75 & 12 & & \\
0 & 0 & 0 & 0 & -6 & -1.75 & 0 & 12 & \\
0 & 0 & 0 & 1.75 & 1.75 & -20 & -3.5 & -3.5 & 40
\end{bmatrix} k
$$

$\omega_1 = 5.34\,\text{rad s}^{-1}$; $\omega_2 = 5.45\text{rad s}^{-1}$; $\omega_3 = 9.65\text{rad s}^{-1}$; $\omega_4 = 15\text{rad s}^{-1}$ (based on initial stiffness)

$\alpha = 0.2698$; $\beta = 0.0093$

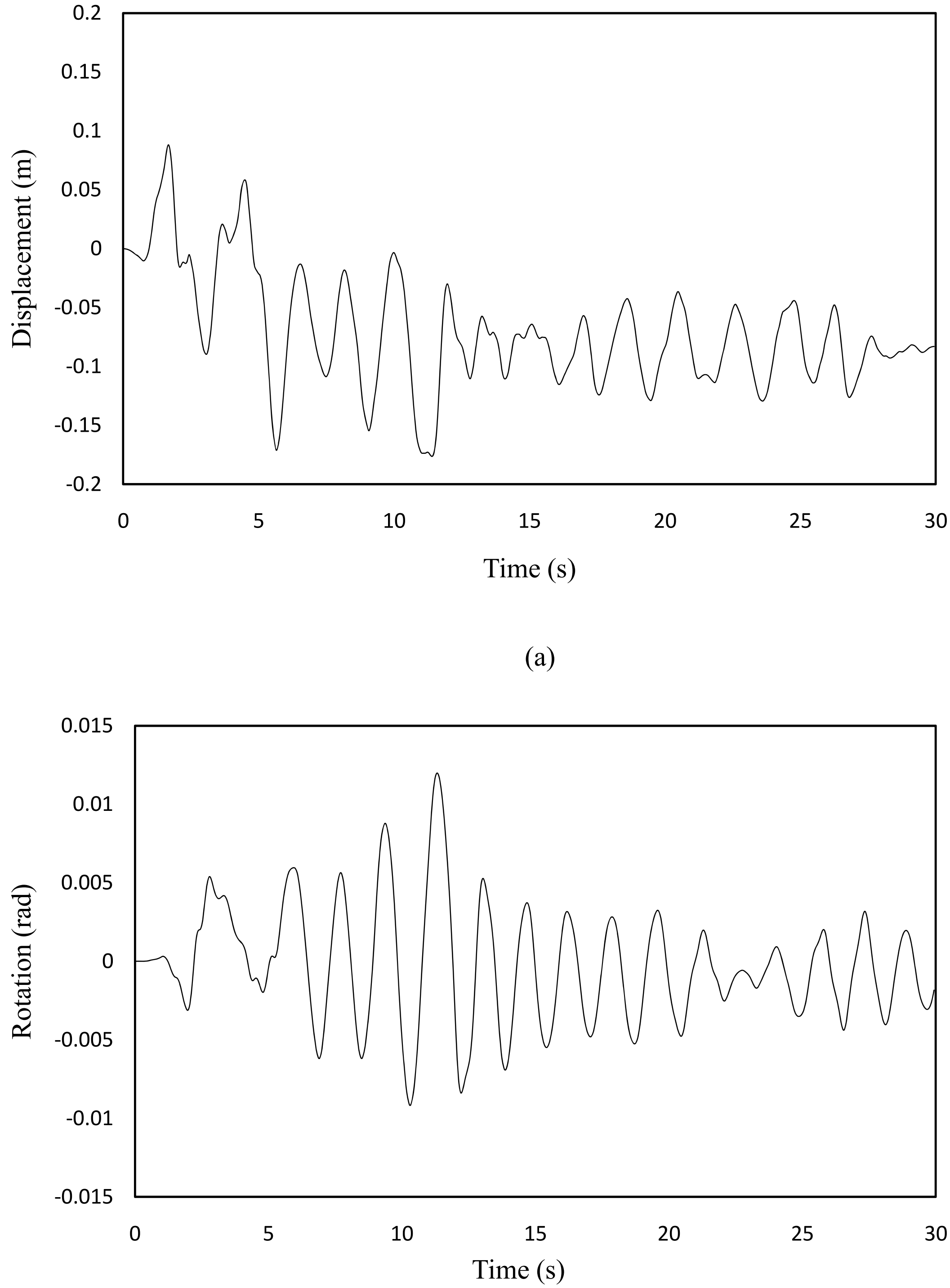
$$
\boldsymbol{C}=\begin{bmatrix}
4.15 & & & & & & & & \\
0 & 4.15 & & & & & & & \\
1.013 & 1.013 & 22.8 & & & & sym & & \\
-3.47 & 0 & -1.013 & 7.62 & & & & & \\
0 & -3.47 & -1.013 & 0 & 7.63 & & & & \\
-1.013 & -1.013 & -21.28 & 2.02 & 2.02 & 44.07 & & & \\
0 & 0 & 0 & -3.47 & -3.47 & -1.013 & 7.62 & & \\
0 & 0 & 0 & 0 & 0 & -1.013 & 0 & 7.62 & \\
0 & 0 & 0 & 0 & -1.013 & -21.28 & 2.02 & 2.02 & 44.07
\end{bmatrix} \times 10^3\,\text{Nsm}^{-1}
$$

$$
I = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T
$$

The problem is solved using, SAP 2000 (force-displacement plots or the moment rotation plots of the yield sections are to be specified).

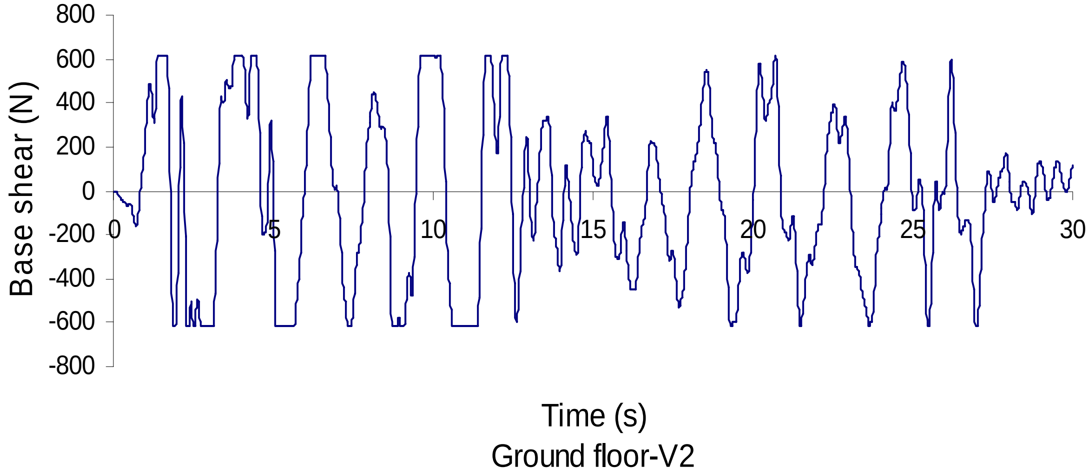
The time histories of the rotation and x displacement of the floor are shown in Figure 6.35. Absolute maximum values of displacement and rotation are 0.18m and 0.0124 rad.

The time histories of base shears are shown in Figure 6.36 for column A. absolute maximum value of base shear = 700N.

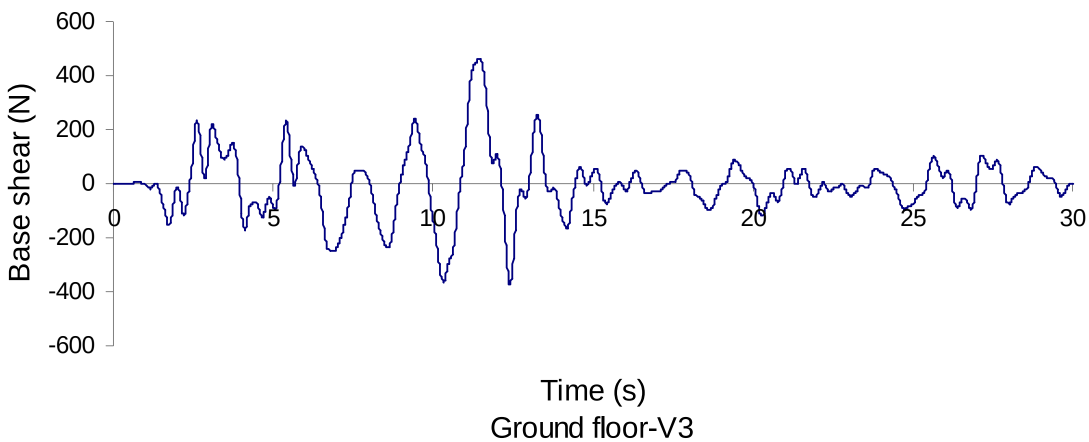


(b)

Figure 6.35 Time histories of response of top floor (a) x-displacement (b) rotation



(a)



(b)

Figure 6.36 Time histories of base shears of column A (a) x-direction (b) y-direction

For obtaining the ductility demand of column A, the absolute peak displacement at each floor level in x and y directions are determined. From these displacements, the ductility ratio are computed as given in Table 6.6

Table 6.6: Ductility demand of column A at different floors

Columns at Floor	Yield Dis. (m)	U_x (m)	Ductility demand (x)	U_y (m)	Ductility demand (y)
Ground	0.00981	0.11894	12.12	0.01101	1.122
1st	0.00981	0.15577	15.88	0.020289	2.130
2nd	0.00981	0.17631	17.97	0.02526	2.575

With the help of the displacements, velocities and accelerations computed without considering the bidirectional interactions (i.e. SAP2000 results), the shears in each column are determined for t = 7s and then the ϕ values (used for bidirectional interaction) are calculated for checking the yielding of columns. Based on this inform action, the stiffness matrix is constructed at t = 7s. Table 6.7 shows the ϕ values for columns at t = 7s. Since no column undergoes plasticization, the stiffness matrix of the structure remains the same as the elastic stiffness matrix.

Table 6.7: Values of ϕ for different columns

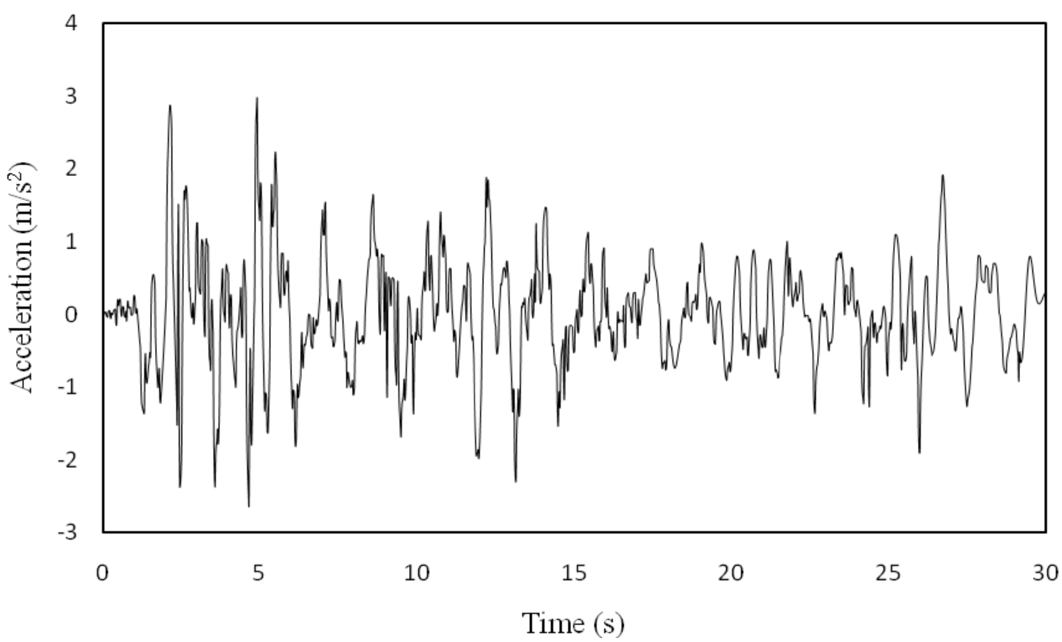
Columns	Floor	V_p	V_x	V_y	ϕ
A	**Ground**	613	13.36	-237.9	0.151
	1st	613	-99.54	-88.89	0.047
	2nd	613	86.59	-74.12	0.035
B	**Ground**	919.5	17.39	-315.1	0.118
	1st	919.5	-210.06	-82.4	0.06
	2nd	919.5	-157.8	-82.55	0.038
C	**Ground**	1226	171	273.74	0.069
	1st	1226	-452.1	80.83	0.069
	2nd	1226	-351.5	66.6	0.085
D	**Ground**	919.5	164	224.36	0.091
	1st	919.5	115.84	83.69	0.024
	2nd	919.5	-40.8	64.79	0.007

6.11. Refer to the exercise problem 6.11.

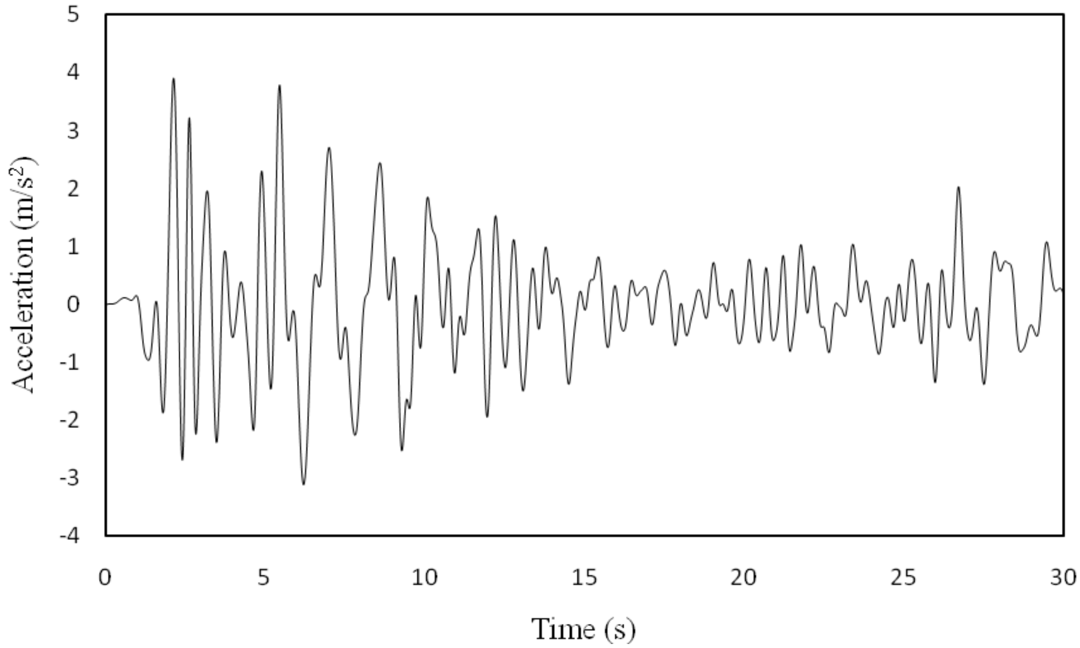
The acceleration time history of the top floor for elasto-plastic columns are shown in Figure 6.37. In the same figure, the time history of acceleration for the case of unyielding column i.e., the force displacement relationship of the columns assumed to be linear (with no yield point) is shown. The corresponding response spectrum for the two cases for 5% damping are shown in Figure 6.38. It is seen that response spectrum ordinates for the elasto plastic case are smaller than those of the elastic analysis. The maximum floor displacements and the ductility demand for each floor are given in Table 6.8.

Table 6.8: Max floor displacements and the ductility demand

Floor	Floor displacement (m)				Yield displacement	Ductility demand
	Elasto plastic analysis		Elastic analysis			
	Max	**Min**	**Max**	**Min**	U_y	
1st Floor	0.042	-0.088	0.043	-0.0404	0.01472	5.980
2nd Floor	0.053	-0.113	0.07902	-0.0736	0.01472	7.679
3rd Floor	0.061	-0.146	0.1103	-0.0986	0.01472	9.922
4th Floor	0.065	-0.162	0.1363	-0.1169	0.01472	11.009
5th Floor	0.066	-0.172	0.1511	-0.1292	0.01472	11.689

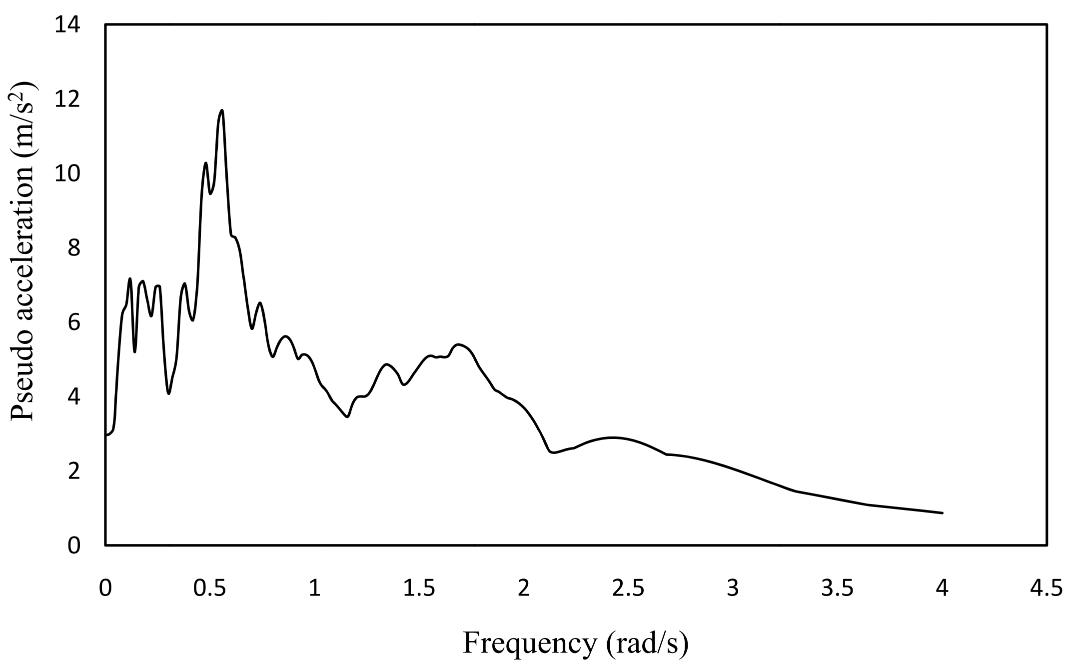


(a)

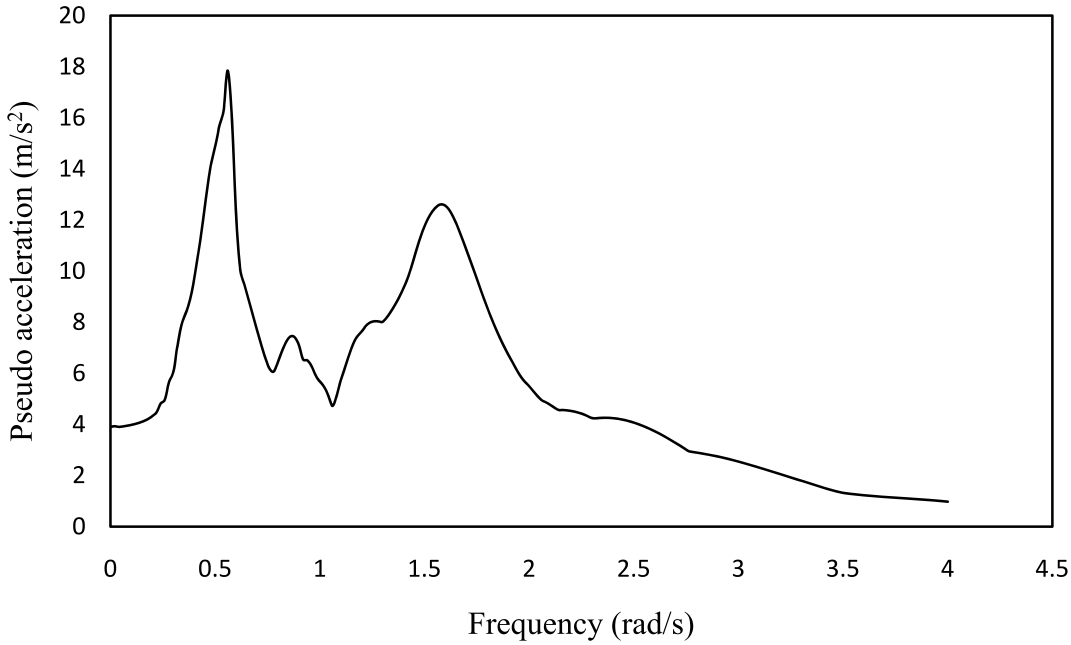


(b)

Figure 6.37 Time histories of absolute acceleration of top floor (a) inelastic analysis (b) elastic analysis



(a)



(b)

Figure 6.38 Floor response spectrum (a) inelastic analysis (b) elastic analysis

6.12. Refer to the exercise problem 6.12.

The mass, stiffness (condensed) and damping matrices for the frame for sway degrees of freedom are shown (for initial stiffness)

$$\boldsymbol{K} = \begin{bmatrix} 5.86 & & & & \\ -5.95 & 11.97 & & sym & \\ 0.113 & -6.14 & 12.06 & & \\ -0.024 & 1.53 & -6.16 & 12.06 & \\ 0.005 & -0.034 & 0.165 & -6.2 & 12.22 \end{bmatrix} \times 10^3 \text{ Nm}^{-1}$$

$\phi_1^T = \begin{bmatrix} 0.275 & -0.75 & 1 & -0.94 & 0.58 \end{bmatrix}$; $\quad \omega_1 = 0.433 \text{ rad s}^{-1}$

$\phi_2^T = \begin{bmatrix} -0.54 & 1 & -0.32 & -0.72 & 0.922 \end{bmatrix}$; $\quad \omega_2 = 1.27 \text{ rad s}^{-1}$

$\phi_3^T = \begin{bmatrix} 1 & 0.92 & 0.76 & 0.54 & 0.28 \end{bmatrix}$; $\quad \omega_3 = 2.012 \text{ rad s}^{-1}$

$\phi_4^T = \begin{bmatrix} -0.72 & 0.57 & 0.92 & -0.32 & -1 \end{bmatrix}$; $\quad \omega_4 = 2.60 \text{ rad s}^{-1}$

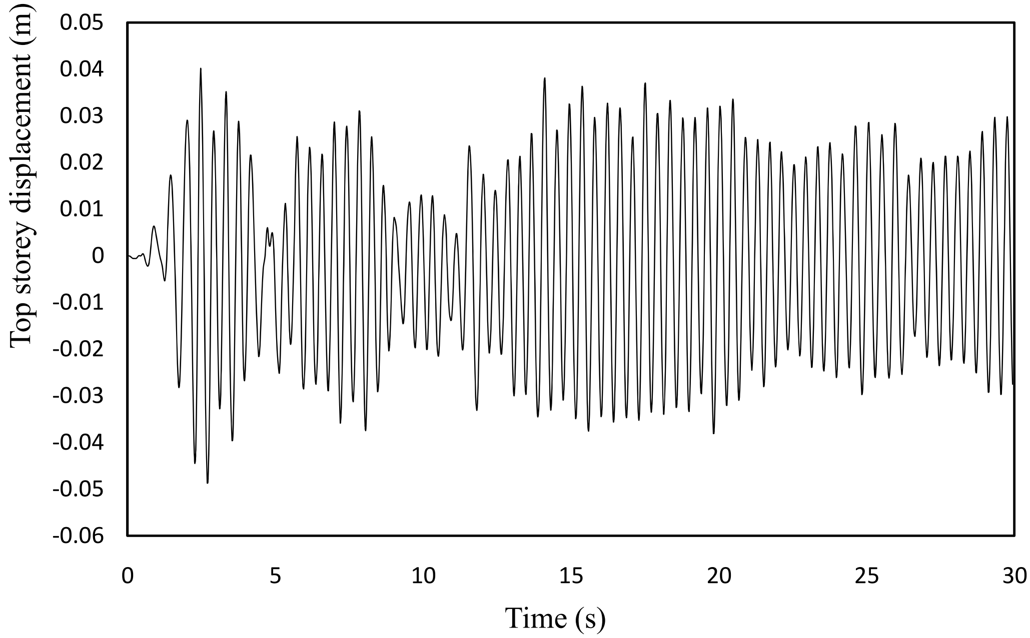
$\phi_5^T = \begin{bmatrix} 0.92 & 0.275 & -0.55 & -1 & -0.744 \end{bmatrix}$; $\quad \omega_5 = 3 \text{ rad s}^{-1}$

$\alpha = 0.0531 \qquad \beta = 0.0587$

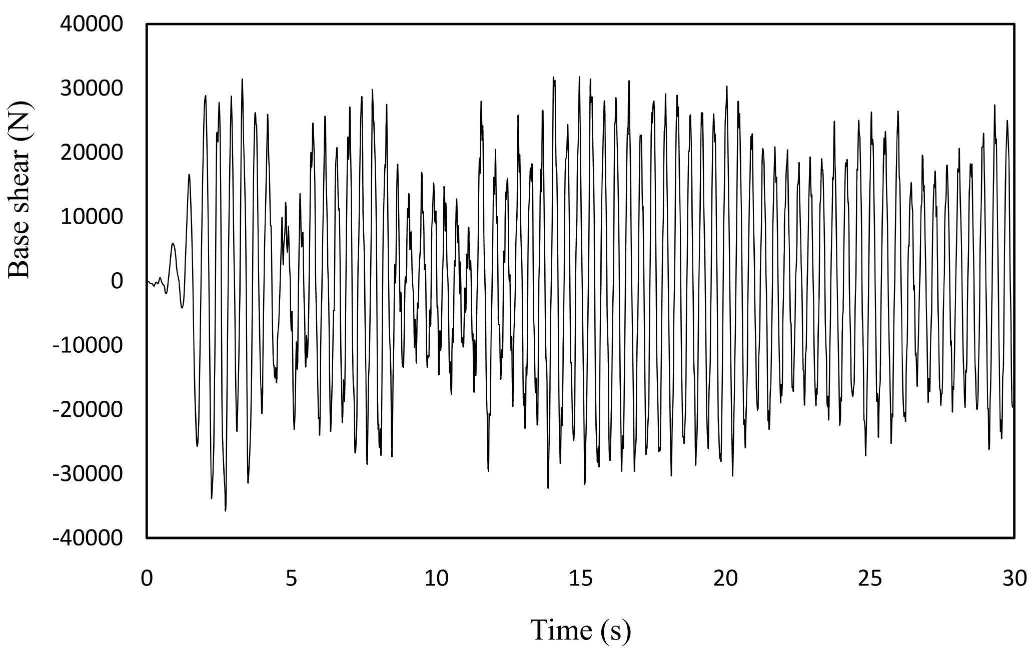
$$
\boldsymbol{C} = \begin{bmatrix} 4.77 & & & & \\ -3.5 & 8.95 & & sym & \\ 0.006 & -3.6 & 8.41 & & \\ -0.001 & 0.009 & -3.6 & 8.41 & \\ 0.0003 & -0.002 & 0.009 & -3.6 & 8.50 \end{bmatrix} \times 10^{2} \text{ Ns-m}^{-1}
$$

The problem is solved using SAP2000 (force displacement or moment rotation plots for the yield sections are specified).

The time histories of the top relative displacement and the base shear for the column A are shown in Figure 6.39



(a)



(b)

Figure 6.39 Time histories of (a) top floor displacement (b) base shear of column A

The moment rotation plot for the section B (which is the left end section of the third beam from the bottom, not shown in Figure 6.32) is shown in Figure 6.40. The elasto plastic hysteretic behavior of the yielding section is clearly observed from the figure.

The maximum rotations and the corresponding ductility demands of the beams are shown in Table 6.9.

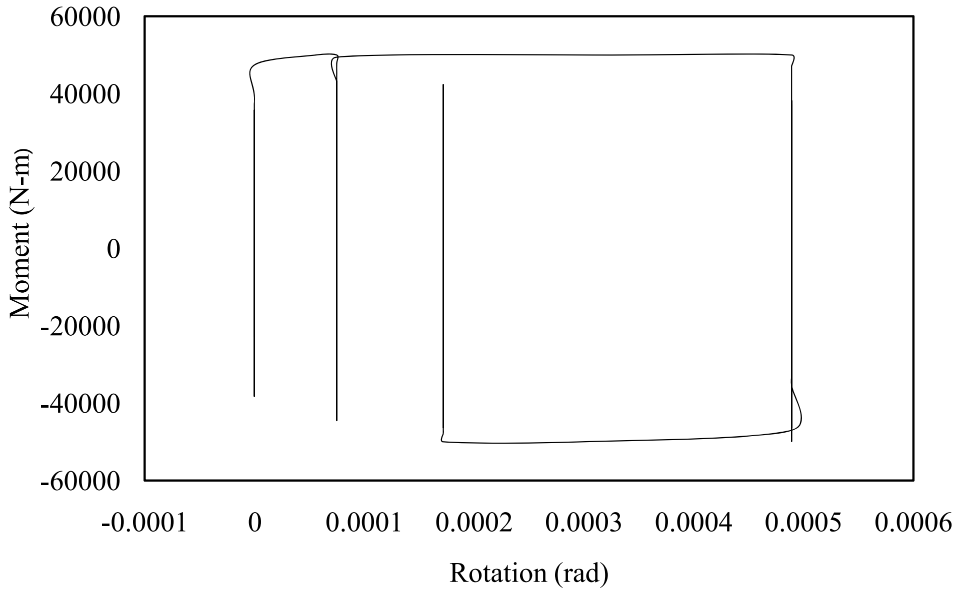


Figure 6.40 Plot of moment-rotation at section B

Table 6.9: Ductility demand of different beams

Beams at floor	**Yield rotation (rad)**	**Maximum rotation (rad)**	**Ductility demand**
1st	0.00109	0.003024	2.77
2nd	0.00109	0.00418	3.83
3rd	0.00109	0.003987	3.66
4th	0.00109	0.003007	2.76
5th	0.00109	0.002126	1.95

6.13. Refer to the exercise problem 6.13.

The push over analysis is carried out in SAP2000. The backbone curves for the moment-rotation of the cross sections of the beams and columns are provided. Whenever the plastic hinge is formed, the structure is unloaded and then reloaded. The results of the frame shown in Figure 6.32 are shown in Figures 6.41 and 6.42.

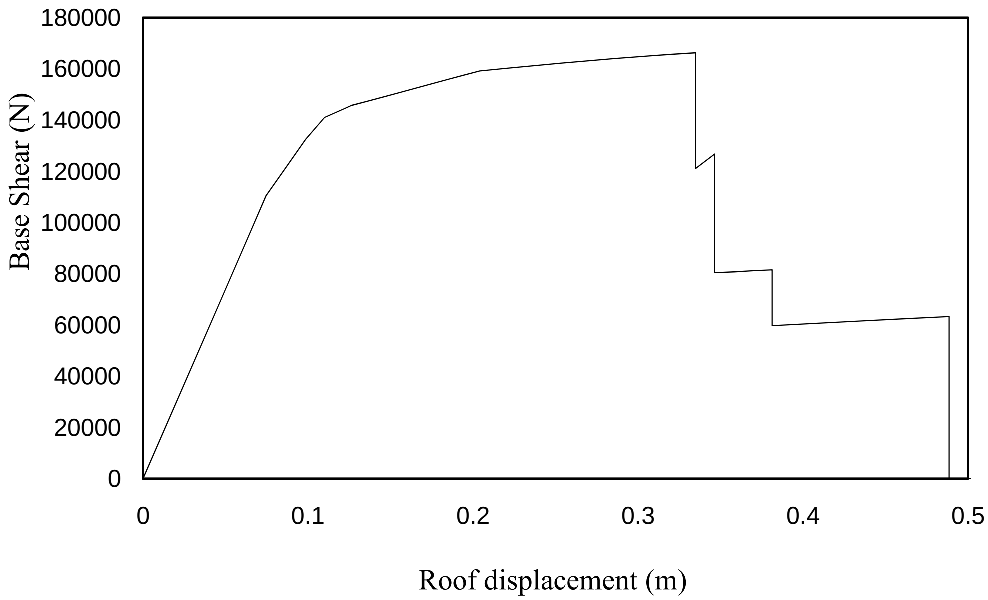


Figure 6.41 Variation of base shear with roof displacement

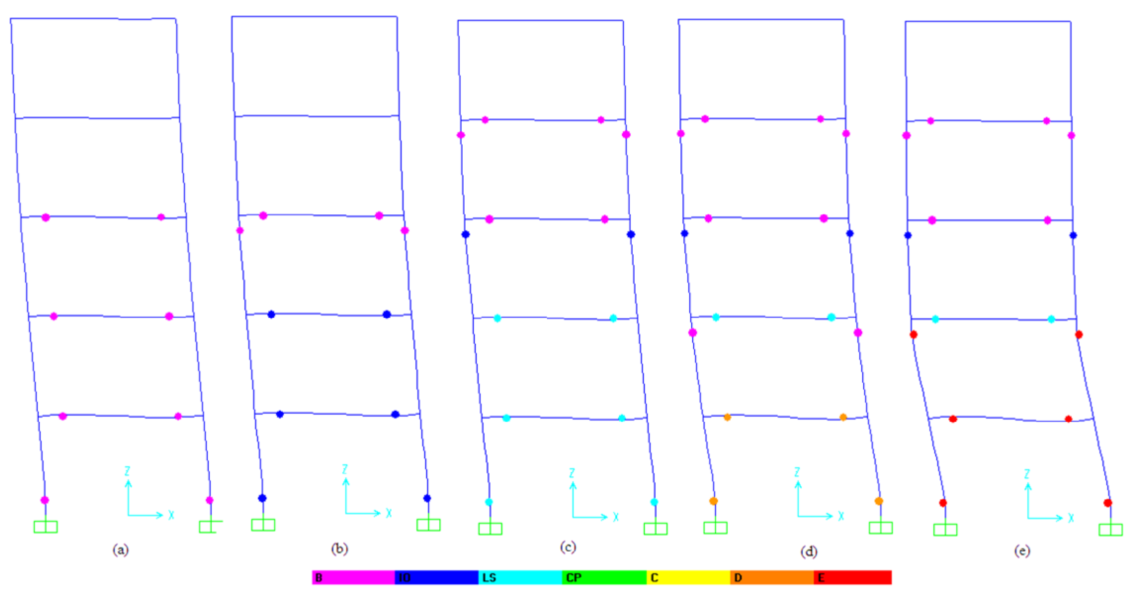


Figure 6.42 Plastic hinge formations at different displacement stages (a) 0.15m (b) 0.224m (c) 0.37m (d) 0.6m

The results of the frame shown in Figure 6.30 are shown in Figures 6.43 and 6.44. It is seen from the Figure 6.44 that the plastic hinges are formed only at the ground floor columns. Note that while displaying the hinge formation, SAP2000 denotes the classification of the hinges based on the computed values of rotations, as immediate occupancy, collapse state etc. and differentiate them by different colors.

6.14. Refer to the exercise problem 6.14.

The inelastic design response spectrums for $\mu =$ 2, 3 and 4 are drawn following the procedure given in Example 6.7 and are shown in Figure 6.45. The inelastic design response spectrums for $\mu = 2{,}4$ and the inelastic response spectrum for the same values of μ for Elcentro earthquake (Figure 6.25) are compared in Figure 6.46. The similarity between the two can be seen from the figures.

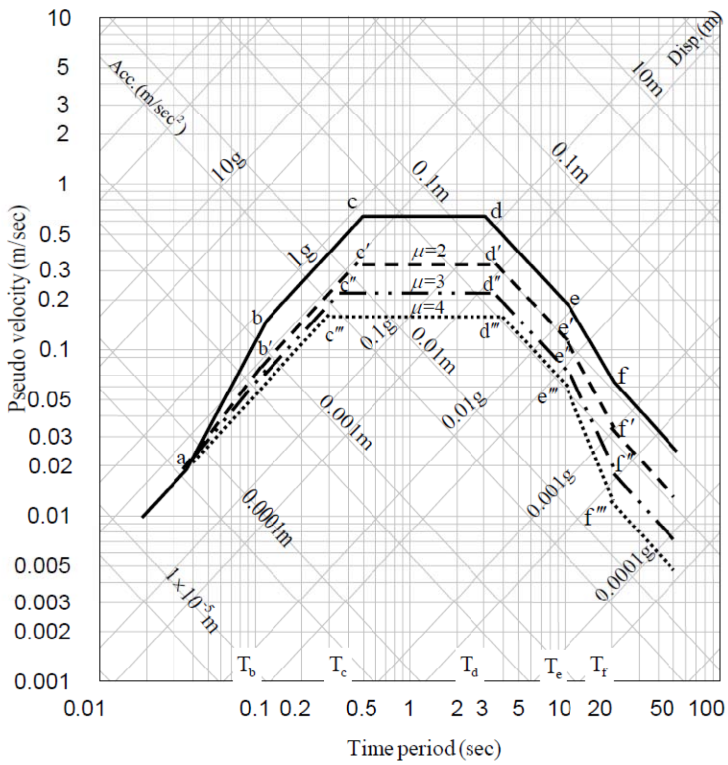
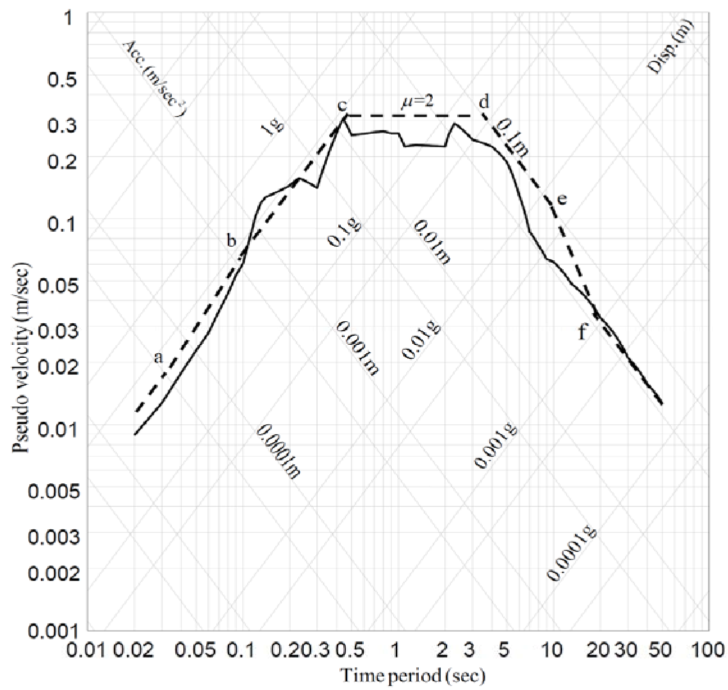
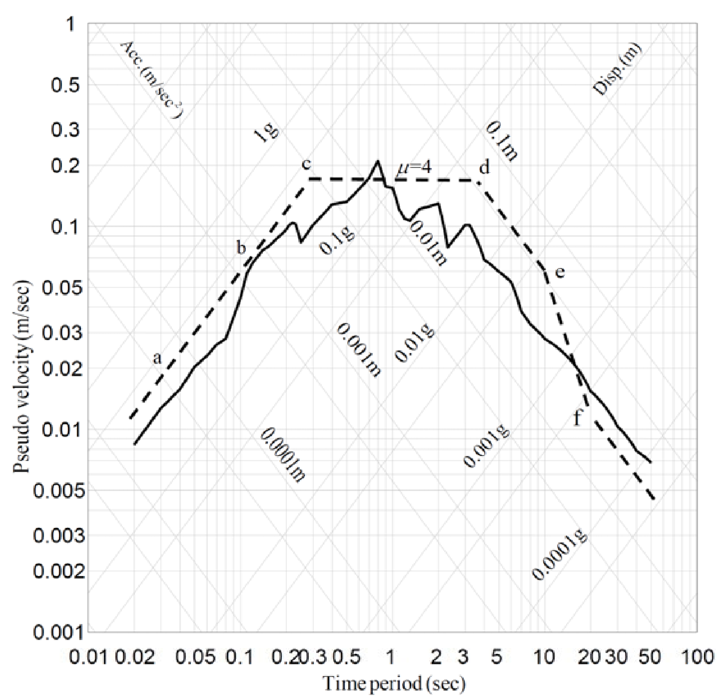


Figure 6.45 Inelastic design spectrum for μ = 2, 3, and 4



(a)



(b)

Figure 6.46 Comparison of inelastic design spectrums with those of El Centro earthquake (a) μ = 2 (b) μ = 4