Module 5: Response Spectrum Method of Analysis

Exercise Problems :

5.8. For the stick model of a building shear frame shown in Figure 3.24, find the mean peak values of the base shear, base moment and top displacement. Take normalized response spectrum for pseudo rotational acceleration to be one fourth of that for translational one. 5% response spectrum for El Centro earthquake is the basic input ground motion. Take response spectrum for rotational component of ground motion as 1/6th of the translational ground motion.

5.9. For the shear frame, shown in Figure 3.29, find the mean peak values of base shear, top displacement, inter story drift between first and second floor, and column moments at the base considering contributions from 3 modes only and all six modes. Use SRSS, ABSSUM and CQC rules of combinations and compare the results. Use 5% response spectrum of El Centro earthquake as basic input ground motion. Also, compare the results with those of the time history analysis.

5.10. For the 3D frame, shown in Figure 3.30, find the mean peak values of base shear, top floor displacement and moment at the base of column A. Take normalized design response spectrum given in IBC (2000) and assume that the angle of incidence of earthquake is 30^0 with the x-axis. Compare the results with those obtained for zero angle of incidence. Use CQC rule of combination.

5.11. For the simplified model of a cable stayed bridge, shown in Figure 3.28, obtain the mean peak values of the vertical displacement of the centre of the deck, base moments of the piers and the axial forces in the central cables. Assume the time lag between the supports as 5s, El Centro earthquake spectrum $(\xi = 5\%)$ as seismic input and correlation function given by Equation 2.94 to be valid. Compare the results with those of the time history analysis.

5.12. For the pipeline, shown in Figure 3.15, find the mean peak values of displacements of the supports for the El Centro earthquake response spectrum. Assume time lag between supports as 5s and use Equation 2.94 as correlation function. Compare the results with those of the time history analysis.

5.13. For the frame with a secondary system, shown in Figure 3.26, find the mean peak value of the displacement of the secondary system by cascaded analysis. Take El Centro earthquake response spectrum as input excitation with a time lag of 5s between two supports, and 5% and 2% dampings for primary and secondary systems, respectively. Compare the results with those of the time history analysis.

5.14. For the same secondary system, as above, find the mean peak value of the displacement of the secondary system using an approximate modal response spectrum analysis. Use the El Centro earthquake response spectrum. Compare the results with those of the time history analysis.

5.15. For the frame, shown in Figure 3.27, find the base shear, top storey displacement and storey drift between the second and first floor by seismic coefficient method of analysis using the recommendations of IBC (2000), NBCC (1995), IS1893 (2002) and NZ 4203 (1992) and Euro 8. Take R=1, hard soil and PGA=0.2g.

5.16. For the frame, shown in Figure 3.29, find the mean peak values of the base shear, top storey displacement and moments at the bottom of the second storey using the response spectrum method of analysis. Compare the results between those obtained by using the above five codes. Take R=1, hard soil and PGA=0.2g. Further, compare the results with those obtained by the seismic coefficient method.

Take the relevant figures from the slides or from the reference book

Module 5: Response Spectrum Method of Analysis

Exercise Solution :

ERRATA FOR THE TEXT BOOK

pp 216, Second para, last line $s \neq 2$, but 3; $n \neq 3$, but 2

Equation 5.14a: $\bar{\phi}_2 \beta_{21}$ should be $\bar{\phi}_2 \beta_{12}$

pp 227, Figure 5.7 is wrong plots of Equations 5.46 and 5.47. The values of c_h and $\frac{A}{g}$

should be obtained directly from the given Equations (Figure 5.7 should not

be used)

pp 228, In Equation 5.51b, replace U by v; U in Equation 5.51a is defined as a calibration

factor.

pp 229, In Figure 5.9, top curve is for $Z_a > Z_v$; middle one is for $Z_a = Z_v$; last one is

for

Refer to the exercise problem 5.8 and Figure 3.24.

The mass, stiffness and damping matrices of the stick model are (exercise problem)

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} m \qquad \boldsymbol{K} = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} k$$

$$\boldsymbol{\phi}_1^T = \begin{bmatrix} -1 & -0.5 \end{bmatrix}, \ \omega_1 = 5\,\text{rad}\,\text{s}^{-1}, \ \boldsymbol{\phi}_2^T = \begin{bmatrix} -1 & 1 \end{bmatrix} \qquad , \ \omega_2 = 10\,\text{rad}\,\text{s}^{-1}$$

It is assumed that translational and rotational ground motions are perfectly correlated and are acting together. The influence coefficient vector on the RHS of the equation of motion is

$$\boldsymbol{I}^T = \begin{bmatrix} 1+\frac{6}{10} & 1+\frac{3}{10} \end{bmatrix} = \begin{bmatrix} 1.6 & 1.3 \end{bmatrix}$$

Mode participation factors: $\lambda_1 = -1.933$; $\lambda_2 = 0.33$; $\frac{S_{a_1}}{g} = 0.242$; $\frac{S_{a_2}}{g} = 0.176$

The lateral load vectors (using Equation 5.9)

$$\boldsymbol{F}_1^T = \begin{bmatrix} 4.6175 & 4.6175 \end{bmatrix} m \qquad \boldsymbol{F}_2^T = \begin{bmatrix} -2.4107 & 4.8215 \end{bmatrix} m$$

SRSS rule is used to compute the mean peak values as:

Top disp. (m)	0.1862
Base shear	10.4178m
Base moment	15.627m

Refer to the exercise problem 5.9 and Figure 3.29.

The mass, stiffness and damping matrices of the frame are (Exercise problem 3.20)

$$\boldsymbol{M} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} m \qquad \boldsymbol{K} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} k$$

$$\boldsymbol{C} = \begin{bmatrix} 1.442 & -0.664 & 0 & 0 & 0 & 0 \\ -0.664 & 1.442 & -0.664 & 0 & 0 & 0 \\ 0 & -0.664 & 1.442 & -0.664 & 0 & 0 \\ 0 & 0 & -0.664 & 1.442 & -0.664 & 0 \\ 0 & 0 & 0 & -0.664 & 1.442 & -0.664 \\ 0 & 0 & 0 & 0 & -0.664 & 0.778 \end{bmatrix} m$$

The mode shapes and frequencies are

$$\boldsymbol{\phi}_1^T = \begin{bmatrix} -0.24 & -0.46 & -0.66 & -0.82 & -0.94 & -1 \end{bmatrix} \qquad \omega_1 = 1.525\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_2^T = \begin{bmatrix} 0.66 & 1 & 0.82 & 0.24 & -0.46 & -0.94 \end{bmatrix} \qquad \omega_2 = 4.485\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_3^T = \begin{bmatrix} -0.94 & -0.66 & 0.468 & 1 & 0.24 & -0.82 \end{bmatrix} \qquad \omega_3 = 7.185\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_4^T = \begin{bmatrix} 1 & -0.24 & -0.94 & 0.46 & 0.82 & -0.66 \end{bmatrix} \qquad \omega_4 = 9.47\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_5^T = \begin{bmatrix} 0.82 & -0.94 & 0.24 & 0.66 & -1 & 0.46 \end{bmatrix} \qquad \omega_5 = 11.2\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_6^T = \begin{bmatrix} -0.46 & 0.82 & -1 & 0.94 & -0.66 & 0.24 \end{bmatrix} \qquad \omega_6 = 12.28\,\text{rad s}^{-1}$$

Mode participation factor are:

$$\lambda_1 = -1.251 \qquad \lambda_2 = 0.4017 \qquad \lambda_3 = -0.2212$$

$$\lambda_4 = 0.1353 \qquad \lambda_5 = 0.0804 \qquad \lambda_6 = -0.0376$$

$$\frac{S_{a_1}}{g} = 0.103\,; \frac{S_{a_2}}{g} = 0.181\,; \frac{S_{a_3}}{g} = 0.57\,; \frac{S_{a_4}}{g} = 0.662\,; \frac{S_{a_5}}{g} = 0.9025\,; \frac{S_{a_6}}{g} = 0.9054$$

The lateral load vectors are

$$F_1^T = \begin{bmatrix} 0.195 & 0.379 & 0.5415 & 0.672 & 0.763 & 0.8106 \end{bmatrix} m$$

$$F_2^T = \begin{bmatrix} 0.478 & 0.716 & 0.593 & 0.172 & -0.335 & -0.67 \end{bmatrix} m$$

$$F_3^T = \begin{bmatrix} 1.175 & 0.833 & -0.584 & -1.248 & -0.3 & 1.035 \end{bmatrix} m$$

$$F_4^T = \begin{bmatrix} 0.812 & -0.2103 & -0.8217 & 0.4084 & 0.723 & -0.582 \end{bmatrix} m$$

$$F_5^T = \begin{bmatrix} 0.564 & -0.641 & 0.164 & 0.454 & -0.681 & 0.318 \end{bmatrix} m$$

$$F_6^T = \begin{bmatrix} 0.154 & -0.273 & 0.33 & -0.31 & 0.22 & -0.0795 \end{bmatrix} m$$

Table 5.5: Peak responses in each mode of vibration

Responses	**1st Mode**	**2nd Mode**	**3rd Mode**	**4th Mode**	**5th Mode**	**6th Mode**
Base shear (in terms of *m*)	3.36	0.95	0.91	0.39	0.180	0.041
Top displacement (m)	0.3487	-0.0335	0.02	-0.0065	0.00254	-0.000527
Inter storey drift (m)	0.0697	-0.00608	-0.027	-0.0068	0.0064	0.004
Column base moment (in terms of *m*)	5.044	1.427	1.366	0.584	0.270	0.0614

Table 5.6: Mean peak responses

Response	**3 Mode results**			**All mode results**			**Absolute peak time history**
	SRSS	**ABSSUM**	**CQC**	**SRSS**	**ABSSUM**	**CQC**	
Base shear (in terms of *m*)	3.6115	5.225	3.6306	3.637	5.835	3.682	3.394
Top displacement (m)	0.3509	0.4023	0.3506	0.3509	0.4118	0.3506	0.3245
Drift (m)	0.075	0.1032	0.07519	0.0758	0.1205	0.0759	0.08242
Column base moment (in terms of *m*)	5.417	7.837	5.446	5.455	8.753	5.523	5.092

Since the mode shapes are well separated, the results of SRSS and CQC are nearly the same. Further, results by considering all modes compare well with the time history analysis.

Refer to the exercise problem 5.10 and Figure 3.30.

The stiffness and mass matrices of the frame are (exercise problem 3.21):

$$\boldsymbol{K} = \begin{bmatrix} 15 & & & & & \\ 0 & 15 & & & & \\ 1.5 & 7.5 & 67.5 & & & \\ -7.5 & 0 & -0.75 & 7.5 & & \\ 0 & -7.5 & -3.75 & 0 & 7.5 & \\ -0.75 & -3.75 & -33.75 & 0.75 & 3.75 & 33.75 \end{bmatrix} k$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1.5 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1.5 \end{bmatrix} m$$

$\omega_1 = 14.5\,\text{rad s}^{-1}$, $\omega_2 = 15.14\,\text{rad s}^{-1}$

$\omega_3 = 26.58\,\text{rad s}^{-1}$, $\omega_4 = 38\,\text{rad s}^{-1}$

$\omega_5 = 39.63\,\text{rad s}^{-1}$ $\omega_6 = 69.6\,\text{rad s}^{-1}$

$$\boldsymbol{\phi}_1^T = \begin{bmatrix} 0.12 & 0.618 & -0.1 & 0.2 & 1 & -0.16 \end{bmatrix}$$

$$\boldsymbol{\phi}_2^T = \begin{bmatrix} -0.618 & 0.123 & 0 & -1 & -0.2 & 0 \end{bmatrix}$$

$$\boldsymbol{\phi}_3^T = \begin{bmatrix} -0.03 & -0.148 & -0.618 & -0.048 & -0.24 & -1 \end{bmatrix}$$

$$\boldsymbol{\phi}_4^T = \begin{bmatrix} 0.2 & 1 & -0.166 & -0.124 & -0.618 & 0.1030 \end{bmatrix}$$

$$\boldsymbol{\phi}_5^T = \begin{bmatrix} 1 & -0.2 & 0 & -0.618 & 0.124 & 0 \end{bmatrix}$$

$$\boldsymbol{\phi}_6^T = \begin{bmatrix} -0.05 & -0.24 & -1 & 0.029 & 0.148 & 0.618 \end{bmatrix}$$

$$\lambda_i = \frac{\phi_i^T MI}{\phi_i^T M \phi_i}$$

in which $I^T = \begin{bmatrix} c & s & o & c & s & o \end{bmatrix}$; $c = \cos 30^0$; $s = \sin 30^0$

$\lambda_1 = -1.036$, $\lambda_2 = -0.369$, $\lambda_3 = -0.172$ $\lambda_4 = -0.2446$, $\lambda_5 = 0.0953$, $\lambda_6 = 0.0407$

$\frac{S_{a_1}}{g} = 0.77$, $\frac{S_{a_2}}{g} = 0.74$, $\frac{S_{a_3}}{g} = 0.62$, $\frac{S_{a_4}}{g} = 0.69$, $\frac{S_{a_5}}{g} = 0.66$, $\frac{S_{a_6}}{g} = 0.54$

The equivalent load vectors are

$$P_1^T(N) = [-986.63 \quad -4933.15 \quad 1231.39 \quad -1596.403 \quad -1982.016 \quad 1992.43]$$

$$P_2^T(N) = [1817.37 \quad -373.47 \quad 2.0 \quad 2940.579 \quad -588.115 \quad 3.177]$$

$$P_3^T(N) = [-32.019 \quad -160.096 \quad -1000.539 \quad -51.808 \quad -259.0412 \quad -1618.906]$$

$$P_4^T(N) = [-336.73 \quad -1683.68 \quad 420.274 \quad 208.115 \quad 1040.577 \quad -259.744]$$

$$P_5^T(N) = [618.596 \quad -123.719 \quad -2.222 \quad -382.313 \quad 76.462 \quad 1.438]$$

$$P_6^T(N) = [-10.618 \quad -53.091 \quad -331.798 \quad 6.5624 \quad 32.812 \quad 205.062]$$

Taking the contributions of the four modes and using CQC rule:

Table5.7: Response for different angles of incident of earthquake

Responses	$\theta = 30^0$	$\theta = 0^0$
V_x	$11041.3N$	36140N
V_y	$38175N$	7588N
$(\delta_x)_{top}$	0.0069 m	0.0226 m
$(\delta_y)_{top}$	0.01907 m	0.0120 m
θ_{top}	0.00392 rad	0.000819 rad
M_{xA}	2760.28 Nm	9033.75Nm
M_{yA}	9543.8 Nm	1897.21Nm

The results of the analysis show that the direction of earthquake in asymmetric building should be carefully considered. The directions coinciding with principal axes do not necessarily provide the worst effect.

Refer to the exercise problems 5.11, 3.19 and Figure 3.28.

The acceleration response spectrum compatible PSDF of the Elcentro ground motion is shown in Figure 5.3. This PSDF and the correlation function given by Equation 2.93 are used to calculate the correlation matrices l_{uu}, $l_{u\bar{z}}$ and $l_{\bar{z}\bar{z}}$. Following the steps 1-8 given in section 5.4.2, the quantities required for calculating the expected value of the responses are given below:

$$\boldsymbol{\phi} = \begin{bmatrix} 0.2426 & -1 & 1 \\ -0.2426 & -1 & -1 \\ 1 & 0 & -0.162 \end{bmatrix}$$

$$\boldsymbol{\phi}_{pier}^{T} = \begin{bmatrix} 2387.2 & -9840 & 9840 \end{bmatrix} m$$

$$\boldsymbol{\phi}_{axial}^{T} = \begin{bmatrix} 2.048 & 13.01 & -13.01 \end{bmatrix} m$$

$$r = \begin{bmatrix} 0.719 & 0.1684 & -0.0046 & 0.1947 \\ 0.1947 & -0.0046 & 0.1684 & 0.719 \\ 0.0839 & 0.0277 & -0.0277 & -0.0839 \end{bmatrix}$$

$\omega_1 = 3.05\,\mathrm{rad\,s^{-1}}$; $\omega_2 = 5.38\,\mathrm{rad\,s^{-1}}$; $\omega_3 = 5.45\,\mathrm{rad\,s^{-1}}$

$$a^T\left(\text{vertical displacement}\right) = \begin{bmatrix} 0.0839 & 0.0277 & -0.0277 & -0.0839 \end{bmatrix}$$

$$a^T\left(\text{base moment at the left pier}\right) = m \times 10^2 \begin{bmatrix} 70.76 & 16.57 & -0.451 & 19.16 \end{bmatrix}$$

$$a^T\left(\text{axial force in the left central cable}\right) = m \times \begin{bmatrix} -8.24 & 1.89 & -0.08 & -2.74 \end{bmatrix}$$

ϕ_β^T [vertical displacement of the centre of the deck] =

$$\begin{bmatrix} 0.1218 & 0.0401 & -0.0401 & -0.1218 & 0 & 0 & 0 & -0.038 & -0.0124 & 0.0124 & 0.038 \end{bmatrix}$$

ϕ_β^T [Base moment at the left pier] =

$$\begin{bmatrix} 290.7 & 394.6 & -394.6 & -290.7 & 4496 & 806.9 & 806.9 & 4496 & 2282.9 & 747.8 & -747.8 & -2282.9 \end{bmatrix}$$

ϕ_β^T [axial force in the left central cable] =

$$\begin{bmatrix} 0.249 & 0.082 & -0.082 & -0.249 & -5.94 & -1.07 & -1.07 & -5.94 & -3.02 & -0.997 & 0.997 & 3.02 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 0.1218 & -0.457 & 0.232 \\ 0.0401 & -0.082 & 0.0767 \\ -0.0401 & -0.082 & -0.0767 \\ -0.1218 & -0.457 & -0.232 \end{bmatrix} \text{ (} \beta_{ki}\ k = 1\cdots\cdots 4\,;\, i = 1\cdots\cdots 3 \text{ arranged in matrix form)}$$

$\boldsymbol{\phi}_{\beta D}^T$ [vertical displacement of the centre of the deck] =

$$\begin{bmatrix} 0.0108 & 0.0036 & -0.0036 & -0.0108 & 0 & 0 & 0 & 0 & -0.0037 & -0.0012 & 0.0012 & 0.0037 \end{bmatrix}$$

$$\boldsymbol{\phi}_{\beta D}^{T}\left[\text{Base moment at the left pier}\right]=$$
$$\left[25.87 \quad 35.12 \quad -35.12 \quad -25.87 \quad 463.1 \quad 83.11 \quad 83.11 \quad 463.1 \quad 223.72 \quad 73.28 \quad -73.28 \quad -223.72\right]$$

$$\boldsymbol{\phi}_{\beta D}^{T}\left[\text{axial force in the cable}\right]=$$
$$\left[0.022 \quad 0.007 \quad -0.007 \quad -0.022 \quad -0.613 \quad -0.11 \quad -0.11 \quad -0.613 \quad -0.316 \quad -0.098 \quad 0.098 \quad 0.316\right]$$

$$D_{11}=D_{21}=D_{31}=D_{41}=D\left(\omega_1=3.05\,\text{rad}\,\text{s}^{-1}\right)=\frac{0.084g}{\left(3.05\right)^2}=0.089\text{m}$$
$$D_{12}=D_{22}=D_{32}=D_{42}=D\left(\omega_2=5.6\,\text{rad}\,\text{s}^{-1}\right)=\frac{0.33g}{\left(5.6\right)^2}=0.103\text{m}$$
$$D_{13}=D_{23}=D_{33}=D_{43}=D\left(\omega_3=5.65\,\text{rad}\,\text{s}^{-1}\right)=\frac{0.32g}{\left(5.65\right)^2}=0.098\text{m}$$

$$coh\left(i,j\right)=\begin{bmatrix}1 & \rho_1 & \rho_2 & \rho_3\\ \rho_1 & 1 & \rho_1 & \rho_2\\ \rho_2 & \rho_1 & 1 & \rho_1\\ \rho_3 & \rho_2 & \rho_1 & 1\end{bmatrix}; \qquad \rho_1=\exp\left(\frac{-5\omega}{2\pi}\right) \quad \rho_2=\exp\left(\frac{-10\omega}{2\pi}\right) \quad \rho_3=\exp\left(\frac{-15\omega}{2\pi}\right)$$

u_p is the same for the all supports and is equal to 3.31 cm, since the value of the peak displacement of the El Centro record, for which the PSDF is given in Appendix 4A, is known. Therefore u_{pi} is not required to be calculated. Otherwise, it is to be calculated following the step 6 given in section 5.4.2.

$$b^T\left(\text{for vertical displacement}\right)=10^{-3}\times\left[2.77 \quad 0.91 \quad -0.91 \quad -2.77\right]$$

b^T for base moment and axial force are similarly obtained

h_i and PSDF of the generalized displacement $\bar{Z}_{ki}$ or $\bar{Z}_{lj}$ are obtained using the fundamental modal Equation 5.18 by removing the summation sign. The area under the curve provides $\sigma_{\bar{z}_{ki}}$ etc. Using Equations 5.28-5.30, the elements of the correlation matrices ℓ_{uu}, $\ell_{u\bar{z}}$ and $\ell_{\bar{z}\bar{z}}$ are determined.

The calculation of these matrices is left to the readers (see example problem 5.4 and 5.5)

Finally, use of Equation 5.26 provides mean peak values of the response quantities of interest:

Mean peak (total) vertical displacement of the deck = 0.023*m*

Mean peak bending moment at the left tower base = 592.7*m*

Mean peak bending moment at the right tower base = 519.5*m*

Mean peak tension in the left cable = -0.762*m*

Mean peak tension in the right cable = -0.424*m*

Refer to the exercise problems 5.12, example problem 3.10 and Figure 3.15.

Like the previous example, the quantities required for the calculation of responses are:

$$\boldsymbol{K} = \begin{bmatrix} 56 & -16 & 8 \\ -16 & 80 & -16 \\ 8 & -16 & 56 \end{bmatrix} m \qquad \boldsymbol{\phi} = \begin{bmatrix} 0.5 & -1 & 1 \\ 1 & 0 & -0.5 \\ 0.5 & 1 & 1 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0.813 & -0.035 & 0.017 \\ -0.035 & 0.952 & -0.035 \\ 0.017 & -0.035 & 0.813 \end{bmatrix} m \qquad \boldsymbol{r} = \begin{bmatrix} \frac{11}{12} & \frac{1}{6} & -\frac{1}{12} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ -\frac{1}{12} & \frac{1}{6} & \frac{11}{12} \end{bmatrix}$$

$$\boldsymbol{M} = \begin{bmatrix} 0.5 & & \\ & 1 & \\ & & 0.5 \end{bmatrix} m$$

$$\omega_1 = 8.1\,\mathrm{rad\,s^{-1}};\ \omega_2 = 9.8\,\mathrm{rad\,s^{-1}};\ \omega_3 = 12\,\mathrm{rad\,s^{-1}}$$

$$a^T = r$$

$$\beta = \begin{bmatrix} 0.3 & -0.5 & 0.267 \\ 0.6 & 0 & -0.133 \\ 0.3 & 0.5 & 0.267 \end{bmatrix} \ (\beta_{ki}, \qquad k = 1\cdots\cdots 3; i = 1\cdots\cdots 3 \text{ in matrix form})$$

$$\boldsymbol{\phi}_\beta^T\left(\text{left support}\right) = \begin{bmatrix} 0.15 & 0.3 & 0.15 & 0.5 & 0 & -0.5 & 0.267 & -0.133 & 0.267 \end{bmatrix}$$

$$\boldsymbol{\phi}_\beta^T\left(\text{central support}\right) = \begin{bmatrix} 0.3 & 0.6 & 0.3 & 0 & 0 & 0 & -0.134 & 0.067 & -0.134 \end{bmatrix}$$

$$\boldsymbol{\phi}_{\beta D}^{T}\left(\text{central support}\right)=\begin{bmatrix}0.0186 & 0.0372 & 0.0186 & 0 & 0 & 0 & -0.0073 & 0.0037 & -0.0073\end{bmatrix}$$

$$\boldsymbol{\phi}_{\beta D}^{T}\left(\text{left support}\right)=\begin{bmatrix}0.0093 & 0.0186 & 0.0093 & 0.0303 & 0 & -0.0303 & 0.0146 & -0.0073 & 0.0146\end{bmatrix}$$

$$D_{11}=D_{21}=D_{31}=D\left(\omega_1=8.1\right)=0.0621\text{m}$$

$$D_{12}=D_{22}=D_{32}=D\left(\omega_2=9.8\right)=0.0606\text{m}$$

$$D_{13}=D_{23}=D_{33}=D\left(\omega_3=9.8\right)12.2=0.0548\text{m}$$

$$coh\left(i,j\right)=\begin{bmatrix}1 & \rho_1 & 1\\ \rho_1 & 1 & \rho_1\\ \rho_2 & \rho_1 & 1\end{bmatrix};\qquad \rho_1=\exp\left(\frac{-5\omega}{2\pi}\right);\qquad \rho_2=\exp\left(\frac{-10\omega}{2\pi}\right)$$

The computations of correlations matrices ℓ_{uu}, $\ell_{u\bar{z}}$ and $\ell_{\bar{z}\bar{z}}$ are left to the readers

$$\boldsymbol{b}^T=\begin{bmatrix}0.03 & 0.0055 & -0.00275\\ 0.0055 & 0.022 & 0.0055\\ -0.00275 & 0.0055 & 0.03\end{bmatrix}$$

in which $u_{p1}=u_{p2}=u_{p3}=u_{p4}=3.31\text{cm}$

Mean peak value of the displacements for the d.o.f are:

0.043 (m)

0.0412 (m)

0.0461 (m)

Refer to the exercise problem 5.13 and Figure 3.26.

The mass, stiffness and damping matrices of the frame without the secondary system are:

$$\boldsymbol{M}=\begin{bmatrix}1 & 0\\ 0 & 1.25\end{bmatrix}m \qquad \boldsymbol{K}=\begin{bmatrix}4 & -2\\ -2 & 2\end{bmatrix}k$$

C is assumed to be classically damped with $\xi=5\%$. The mode shapes and frequencies are

$$\phi_1^T=\begin{bmatrix}-0.596 & -1\end{bmatrix};\qquad \omega_1=6.225\,\text{rad s}^{-1}$$

$$\phi_2^T=\begin{bmatrix}-1 & 0.477\end{bmatrix};\qquad \omega_2=17.2407\,\text{rad s}^{-1}$$

α and β are calculated as

$\alpha=0.45739$; $\beta=0.0042614$

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K} = \begin{bmatrix} 1.48 & -0.5114 \\ -0.5114 & 1.083 \end{bmatrix} m$$

$$r = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

For a time lag of 5 sec between the two supports, the time histories of excitations ($\ddot{x}_{g_1}$ and $\ddot{x}_{g_2}$) of duration 35s are constructed for the two supports as explained in Example problem 3.8. The time history of absolute acceleration of the top floor of the frame is determined by direct integration of the equation of motion. The time history of the absolute acceleration is shown in Figure 5.16.

The pseudo acceleration response spectrum for $\xi = 2\%$ for the time history of acceleration is obtained using the method described in Chapter2. The acceleration spectrum is shown in Figure 5.17.

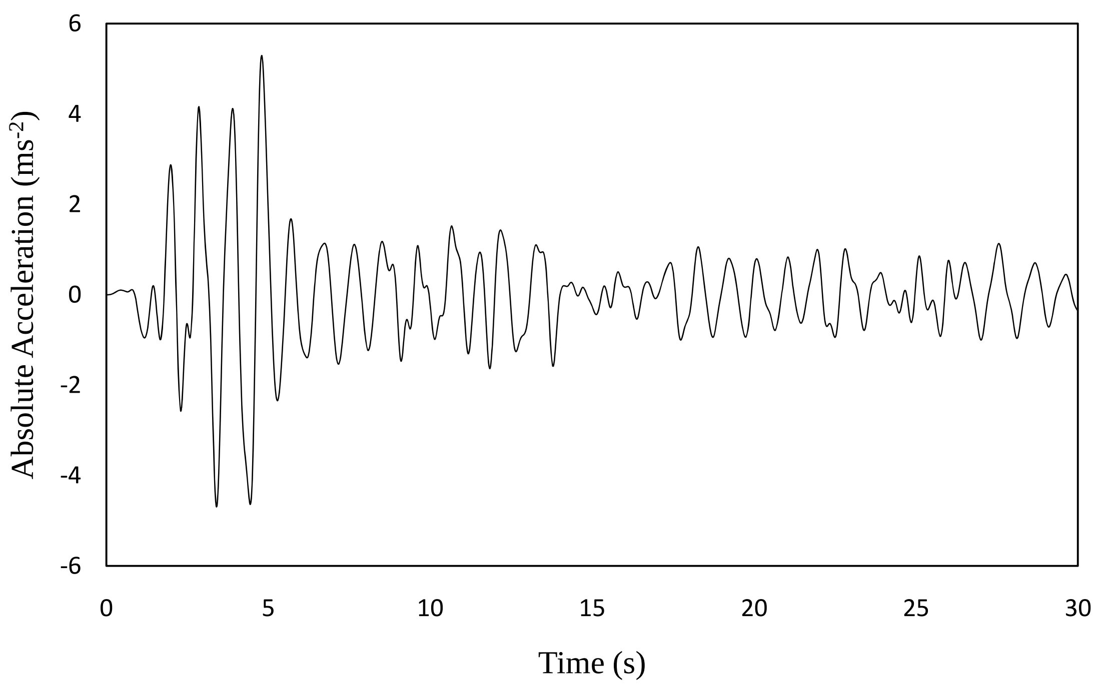

Figure 5.16 Time history of absolute acceleration of top floor

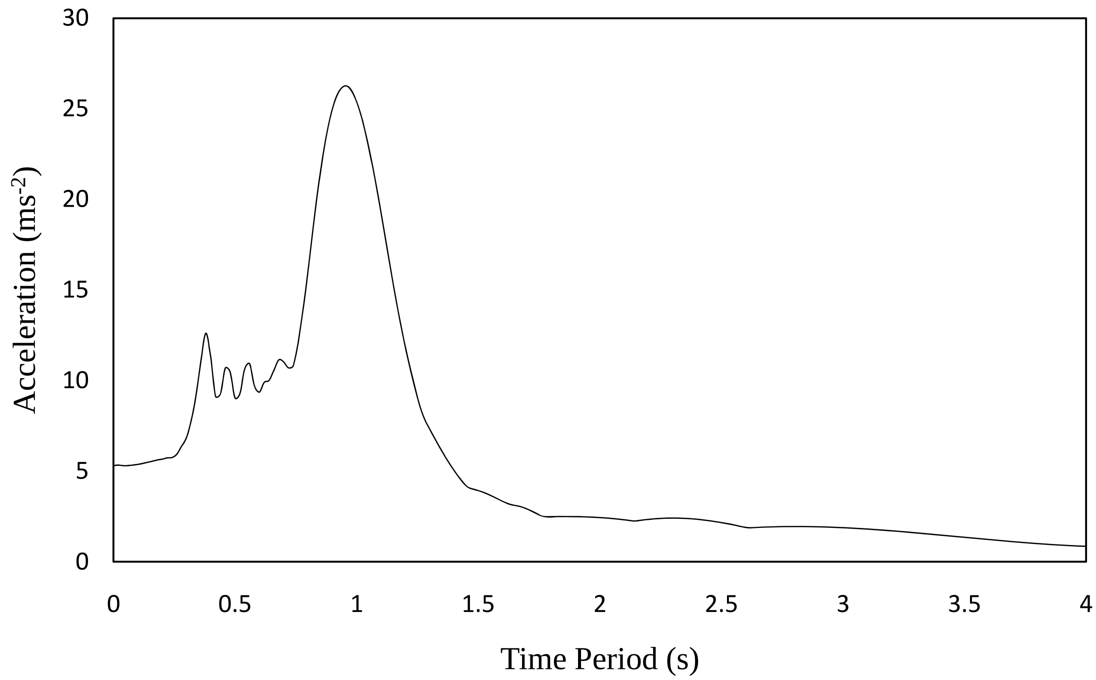

Figure 5.17 Top floor acceleration response spectrum (ψ = 2%)

The time period of the secondary system is obtained as

$$\omega_s = \sqrt{\frac{K}{m}} = \sqrt{60} = 7.746\,\text{rad}\,\text{s}^{-1} \qquad\qquad T_s = \frac{2\pi}{\omega_s} = 0.811s$$

For $\xi = 2\%$, spectral acceleration (S_a) for $T_s = 0.811s$ is 0.4407g

The mean peak value of the displacement of the secondary system is therefore,

$$x_3\big|_{peak} = \frac{S_a}{\omega_s^2} = \frac{0.4407 \times 9.81}{(7.746)^2} = 0.0720\text{m}$$

$x_3\big|_{peak} = 0.068\text{m}$ (from time history analysis)

Refer to the exercise problem 5.14.

The stiffness and mass matrices for the three degree of freedom problem are:

$$\boldsymbol{K} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 2.25 & -0.25 \\ 0 & -0.25 & 0.25 \end{bmatrix} k \qquad\qquad \boldsymbol{M} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0.25 \end{bmatrix} m$$

Undamped mode shapes and frequencies of the system are

$$\boldsymbol{\phi}_1^T = \begin{bmatrix} -0.2638 & -0.4558 & -1 \end{bmatrix}; \qquad \omega_1 = 5.71\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_2^T = \begin{bmatrix} 0.2904 & 0.38035 & -1 \end{bmatrix}; \qquad \omega_2 = 9.1\,\text{rad s}^{-1}$$

$$\boldsymbol{\phi}_3^T = \begin{bmatrix} -1 & 0.6627 & -0.1532 \end{bmatrix}; \qquad \omega_3 = 17.87\,\text{rad s}^{-1}$$

The damping matrix of the three degree of freedom system is constructed as

$$\boldsymbol{C} = \left[\begin{array}{c|c} \boldsymbol{C}_P & 0 \\ \hline 0 & \boldsymbol{C}_s \end{array}\right] = \begin{bmatrix} 1.48 & -0.511 & 0 \\ -0.511 & 1.083 & 0 \\ 0 & 0 & 0.0775 \end{bmatrix} m$$

in which $\boldsymbol{C}_P$ is the 2x2 damping matrix obtained in the previous problem and $\boldsymbol{C}_s = 2\xi\omega_s m_s = 0.0775$. The coupling terms of the damping matrix between the primary and secondary systems are assumed as zero (like problems of soil structure interaction discussed in Chapter 7).

$$\bar{\boldsymbol{C}} = \boldsymbol{\phi}^T \boldsymbol{C} \boldsymbol{\phi} = \begin{bmatrix} 0.282 & & \\ -0.104 & 0.246 & \\ -0.068 & -0.048 & 2.635 \end{bmatrix}$$

After ignoring the off diagonal terms of the $\bar{\boldsymbol{C}}$ matrix, approximate modal damping for the primary - secondary system is

$\xi_1 = 0.047$; $\xi_2 = 0.028$; $\xi_3 = 0.051$

Damping modifier (of 1.29) for $\frac{S_a}{g}$ value is used to find $\frac{S_a}{g}$ value for $\xi = 0.028$ as given in IS Code 2000. For $\xi_1 = 0.047$ and $\xi_2 = 0.051$, the values for 5% damping are used.

Assuming no time lag, the response spectrum analysis is performed which results in the following quantities

$\lambda_1 = -0.613$; $\lambda_2 = -0.27$; $\lambda_3 = -0.487$

$\frac{S_{a_1}}{g} = 0.353$; $\frac{S_{a_2}}{g} = 0.59$; $\frac{S_{a_3}}{g} = 0.773$

$$F_1^T = \begin{bmatrix} 0.584 & 1.0 & 0.553 \end{bmatrix} m\,; \qquad F_2^T = \begin{bmatrix} 0.5868 & 0.768 & -0.5 \end{bmatrix} m\,;$$

$$F_3^T = \begin{bmatrix} 3.67 & -2.43 & 0.14 \end{bmatrix} m$$

Using SRSS rule of combination, mean peak value of the top mass (secondary system) = 0.0721m

Refer to the exercise problem 5.15 and Figure 3.27.

In order to maintain uniformity, all multiplying factors are taken as unity. Further, the maximum value of the seismic co-efficient/spectral acceleration normalized with respect to g is taken as unity for different codes. This provides a uniform PGA 0f 0.4g for all codes except the Canadian code whose PGA turns out to be 0.65g.

The frame under consideration has a time period of 2.475s which corresponds to a flexible system (like tall buildings). In this range of time period, the seismic coefficient values differ significantly from code to code. Thus, this exercise problem illustrates the difference of base shear and hence, the lateral load computed by different codes for flexible systems.

The results are obtained for medium soil (not for the hard soil, as specified in the problem), PGA = 0.2g and $\xi = 5\%$

IBC (2000)

$$C_h = \frac{0.162}{2}\,; \quad V_b = \frac{8mg \times 0.162}{2} = 6.357m$$

Assuming a story height of 3m and using Equation 5.49 with k = 2

$$F^T = \begin{bmatrix} 3.39 & 1.91 & 0.847 & 0.212 \end{bmatrix} m$$

NBCC (1995) with (U = 1)

$$C_h = \frac{0.4}{1.2} \times \frac{0.2g}{0.65g} = 0.102\,; \qquad V_b = 8.205m$$

Using Equations 5.54 and 5.55

$$F^T = \begin{bmatrix} (2.715 + 1.418) & 2.03 & 1.357 & 0.678 \end{bmatrix} m$$
$$= \begin{bmatrix} 4.133 & 2.03 & 1.357 & 0.678 \end{bmatrix} m$$

Euro 8 (1995)

Using Equations 5.56 and 5.57 (with $q=1$) or Figure 5.11

$$C_s = C_h = \frac{0.389 \times 0.4}{2} = 0.0778; \qquad V_b = 6.105m$$

Using Equation 5.60

$$F^T = \begin{bmatrix} 2.442 & 1.832 & 1.221 & 0.611 \end{bmatrix} m$$

New Zealand (NZ4203:1992)

Using Figure 5.12 (category 2; $\mu = 1$; z factor such that PGA = 0.2g)

$$C_b = C_h = \frac{0.2}{2} = 0.1; \quad V_b = 7.848m$$

Using Equation 5.60 and multiplying by the factor 0.92

$$F^T = \begin{bmatrix} 2.89 & 2.167 & 1.44 & 0.722 \end{bmatrix} m$$

IS (1893:2002)

Using Equation 5.62 (z factor is taken such that PGA = 0.2g)

$$C_e = C_h = \frac{0.55 \times 0.4}{2} = 0.11; \quad V_b = 8.633m$$

Using Equation 5.65,

$$F^T = \begin{bmatrix} 4.604 & 2.59 & 1.151 & 0.288 \end{bmatrix} m$$

Table 5.8: Values of response quantities of interest

Codes	Base Shear in terms of m	Top story displacement (m)	Drift (m)
IBC	6.359	0.24196	0.0512
NBCC	8.198	0.2942	0.0627
NZ	7.219	0.2347	0.0541
Euro	6.106	0.1984	0.0458
IS	8.633	0.3285	0.0695

From the table it is seen that there are some variations of responses as determined by different codes. IS code provides the maximum values of responses, while Euro code provides the minimum values. The difference is primarily due to the difference in the values of C_h at higher time periods.

Refer to the exercise problem 5.16 and Figure 3.39.

The results are obtained for medium soil, PGA = 0.2g and $\xi = 5\%$

The frequencies and mode shapes are taken from Exercise problem 3.20

$\omega_1 = 1.525\,\text{rad s}^{-1}\ (T_1 = 4.125s)$; $\omega_2 = 4.48\,\text{rad s}^{-1}\ (T_2 = 1.4s)$

$\omega_3 = 7.18\,\text{rad s}^{-1}\ (T_3 = 0.87s)$; $\omega_4 = 9.47\,\text{rad s}^{-1}\ (T_4 = 0.66s)$

$\omega_5 = 11.2\,\text{rad s}^{-1}\ (T_5 = 0.565s)$; $\omega_6 = 12.28\,\text{rad s}^{-1}\ (T_6 = 0.51s)$

$$\boldsymbol{\phi}_1^T = [-0.24 \quad -0.47 \quad -0.67 \quad -0.83 \quad -0.94 \quad -1]$$

$$\boldsymbol{\phi}_2^T = [0.66 \quad 1 \quad 0.83 \quad 0.24 \quad -0.47 \quad -0.94]$$

$$\boldsymbol{\phi}_3^T = [-0.94 \quad -0.67 \quad 0.47 \quad 1 \quad 0.24 \quad -0.83]$$

$$\boldsymbol{\phi}_4^T = [1 \quad -0.24 \quad -0.94 \quad 0.47 \quad 0.83 \quad -0.67]$$

$$\boldsymbol{\phi}_5^T = [0.83 \quad -0.94 \quad 0.24 \quad 0.67 \quad -1 \quad 0.47]$$

$$\boldsymbol{\phi}_6^T = [-0.47 \quad 0.33 \quad -1 \quad 0.94 \quad -0.67 \quad 0.24]$$

Table 5.9: Spectral accelerations obtained from different codes

Period	**IBC**	**NBCC**	**Euro**	**NZ**	**IS**
T_1 **(4.12sec)**	0.048g	0.0317g	0.053g	0.075g	0.066g
T_2 **(1.4sec)**	0.143g	0.093g	0.214g	0.175g	0.194g
T_3 **(0.87sec)**	0.229g	0.15g	0.344g	0.26g	0.312g
T_4 **(0.66sec)**	0.303g	0.198g	0.454g	0.325g	0.412g
T_5 **(0.56sec)**	0.357g	0.233g	0.5g	0.35g	0.485g
T_6 **(0.51sec)**	0.392g	0.256g	0.5g	0.37g	0.5g

Table 5.10: Comparison of response quantities as obtained by different codes using different combination rules

	Base shear in	**Top displacement**	**Bending moment**

		terms of *m*	(m)	in terms of *m*
IBC	**SRSS**	2.6016	0.256	3.149
	CQC	2.619	0.256	3.1511
	ABSSUM	3.841	0.293	4.452
NBCC	**SRSS**	1.715	0.169	2.078
	CQC	1.727	0.169	2.079
	ABSSUM	2.525	0.193	2.93
Euro	**SRSS**	2.99	0.284	3.56
	CQC	3.03	0.284	3.566
	ABSSUM	4.77	0.339	5.429
NZ	**SRSS**	3.973	0.399	4.852
	CQC	3.99	0.3992	4.852
	ABSSUM	5.45	0.443	6.328
IS	**SRSS**	3.572	0.352	4.327
	CQC	3.596	0.352	4.329
	ABSSUM	5.257	0.402	6.092

Table 5.11: Contributions of different modes to responses for IS code.

	1st	2nd	3rd	4th	5th	6th
Top displacement (m)	0.3503	-0.036	0.0108	-0.004	0.0014	-0.000295
Base shear in terms of *m*	3.378	1.018	0.494	0.244	0.1008	0.02292

The same problem is solved using the Seismic coefficient method.

Assume the storey height as 3m

Table 5.12: Calculated force and base shears

Variables	IBC	NBCC	Euro	NZ	IS
C_h in terms of g	0.0485	0.0897	0.055	0.075	0.068
Base shear in terms of *m*	2.851	5.28	3.257	4.414	4

F_6 **in terms of *m***	1.11	2.452	0.9315	1.2624	1.56
F_5 **in terms of *m***	0.784	0.9425	0.775	1.0505	1.1
F_4 **in terms of *m***	0.501	0.7524	0.6188	0.8386	0.704
F_3 **in terms of *m***	0.282	0.5623	0.4625	0.6268	0.396
F_2 **in terms of *m***	0.125	0.3762	0.3094	0.4193	0.176
F_1 **in terms of *m***	0.031	0.18612	0.153	0.2074	0.044

Table 5.13: Response quantities of interest

Response	**IBC**	**NBCC**	**Euro**	**NZ**	**IS**
Top displacement (m)	0.3427	0.6265	0.3524	0.47769	0.4815
Bending moment in terms of *m*	4.0155	7.0638	4.1819	5.6674	5.64

It is seen from the tables that there is again a significant variation in the responses determined by different codes. NZ and IS codes provide comparable results (higher values), while NBCC code provides the minimum values. Further, the results show that the seismic coefficient method estimates higher values of the responses compared to the response spectrum method.