Module 2 – Seismic Inputs

Exercise Problems:

(Use standard MATLAB programs for solving the problems; you may use your own developed program based on the methods presented in the Chapter)

2.12. Take the Lomaprieta earthquake acceleration record from the website http://peer.berkely.edu/smcat and obtain the Fourier amplitude and phase spectra using FFT. Retrieve the time history of acceleration taking IFFT of the Fourier components. Find the values of the following:

(i) Maximum ordinates of fourier amplitudes and frequencies of their occurrences before and after smoothening.

(ii) Maximum phase angle and nyquest frequency.

(iii) Rms accelerations obtained from the time history and fourier components.

(iv) Absolute maximum ordinates of original time history and the retrieved one.

2.13. Assuming the above time history of acceleration to be a sample of an ergodic stationary process, obtain a smoothed PSDF of acceleration. Find the rms and expected peak values of acceleration from the PSDF and compare them with the rms and absolute peak values of the actual time history of acceleration.

2.14. Take the San Farnendo earthquake acceleration from the website http://peer.berkely.edu/smcat and obtain the normalized smoothed fourier spectrum, energy spectrum and pseudo acceleration spectrum $\xi = 0.05$. Compare between the time periods corresponding to the maximum values of the ordinates of the three spectra.

2.15. Draw the response spectrums of the San Fernando earthquake in tripartite plot and then, idealize them by a series of straight lines. Compare T_a, T_b, T_c, T_d, T_e, and T_f with those of Elcentro earthquake.

2.16. Construct design response spectrum for 50th and 84th percentile on a four way log graph paper for 5% damping for the hard soil for a site with R = 75km and M = 7.5. Use

Equations 2.44, 2.49 and 2.57 for finding PHA, PHV and PHD. Take $T_a = \frac{1}{33}$ s; $T_b = \frac{1}{8}$ s; $T_e = 10$s and $T_f = 33$s.

2.17. For a site, mean annual rate of exceedances of normalized ordinates of acceleration response spectrum and PGA are given below. Obtain a uniform hazard spectrum that has a 10% probability of exceedance in 50 years:

Table 2.5: Annual rate of exceedance a_r of acceleration response spectrum ordinates

$\bar{S}$ / **T**	**0.05**	**0.2**	**0.6**	**1**	**1.5**	**2**	**2.5**	**3**
0.1	0.2	0.2	0.2	0.1	0.05	0.004	0.003	0.0025
0.3	0.15	0.3	0.15	0.08	0.08	0.003	0.003	0.002
0.5	0.12	0.22	0.12	0.06	0.06	0.002	0.0015	0.0015
0.7	0.1	0.1	0.1	0.07	0.05	0.02	0.008	0.005
0.9	0.01	0.01	0.01	0.006	0.003	0.0018	0.0012	0.0012
1.0	0.01	0.01	0.08	0.005	0.002	0.0015	0.001	0.0008
1.2	0.008	0.008	0.008	0.006	0.005	0.004	0.0009	0.0007
1.4	0.003	0.003	0.002	0.002	0.0015	0.001	0.0007	0.0005
1.6	0.003	0.003	0.0015	0.002	0.0015	0.001	0.0006	0.0004
1.8	0.003	0.0024	0.0014	0.0012	0.0012	0.001	0.0015	0.0003
2.0	0.0025	0.002	0.0012	0.0012	0.0008	0.0006	0.0004	0.0002

Table 2.6: Annual rate of exceedance a_r of PGA

PGA	0.01g	0.05g	0.1g	0.15g	0.2g	0.3g	0.4g	0.45g
a_r	0.45	0.2	0.1	0.05	0.02	0.006	0.002	0.001

2.18. A site specific acceleration spectrum is to be constructed for a site which has a layer of soft soil over rock bed. Past earthquake records on the rock out crop at some distance away from the site shows that the pseudo velocity spectrum of the expected earthquake can be fairly represented by Equation 2.66(use coefficient for random component). The values of the ordinates of the pseudo acceleration spectrum are to be obtained from pseudo velocity spectrum. The soft soil deposit has a predominant period of 1.4s. The soil condition modifies the shape of the normalized spectrum at rock level in the following manner.

(i) At the predominant period (T = 1.4s) ordinate of the normalized modified spectrum (S_i) is 3 times that of the normalized rock bed spectrum (S_{oi}).

(ii) At other periods, the ordinates of the modified spectrum are given by $S_i = S_{oi} \times \lambda$ in which $\lambda \approx 1.5$.

(iii) At T = 0, $S_o\left(0\right) = S\left(0\right) = 1$

If the PGA amplification is 3.0 due to the soft soil deposit, construct the site specific acceleration response spectrum for $M_w = 7$. Use attenuation relationship (at rock bed) proposed by Esteva (Equation 2.43) for finding PGA with a value of R taken as 100km.

2.19. Assuming $\omega_f = 0.1\omega_g$, $\xi_g = \xi_f$, $\omega_g = 10\pi$ and $\xi_g = 0.4$ obtain an expression for the PSDF of ground acceleration in terms of *M*, *R* and filter characteristics using double filter PSDF (Equation 2.74) and attenuation relationship given by Campbell (Equation 2.44). From this expression, find the peak values and corresponding frequencies of the PSDF for *M* = 7, *R* = 50 km and 100 km. If Equations 2.54 and 2.51 were used in place of Equation 2.44, find the same expression in terms of only *M*. Also, find the peak value of the PSDF for *M* = 7.

2.20. A travelling train of seismic wave moves with a shear wave velocity of $V_s = 150\text{m/sec}$. The direction of wave propagation is at an angle of 20^0 with the line joining three bridge piers (A, B, C). AB = BC = 400m. If the ground acceleration produced by seismic wave is modelled as a stationary random process, then find PSDF matrices of ground accelerations in the major principal direction (assumed as the direction of wave propagation) at the base of the piers (A, B, C) for a frequency of 3 rad/s using the following data:

(i) PSDF of ground acceleration is represented by the expression given by Clough and Penzien (Equation 2.74) with a PGA = 0.4g; $\omega_g = 10\pi$; $\omega_f = 0.1\omega_g$ and $\xi_g = \xi_f$ = 0.4

(ii) Coherence functions to be used are

(a) that given by Hindy and Novak (Equation 2.93)

(b) that Given by Clough and Penzien (Equation 2.99)

(c) that given by Harichandran and Vanmarke (Equation 2.92)

(d) that given by Loh (Equation 2.94)

2.21. A time history of ground motion is artificially generated using the following fourier series co-efficients for a duration of 20s.

$\ddot{x}\ t\ = \sum_{n=1}^{16} A_n Cos\ \ \omega_n t + \varphi_n$; ω_n is in rad/s and φ_n is in rad.

$A_1 = 0.25g\ \ \omega_1 = 0.85;\ \phi_1 = 0.1$	$A_9 = 0.05g\ \ \phi_9 = 0.05$
$A_2 = 0.2g\ \ \phi_2 = 0.18$	$A_{10} = 0.2g\ \ \phi_{10} = -0.12$
$A_3 = 0.1g\ \ \phi_3 = 0.3$	$A_{11} = 0.15g\ \ \phi_{11} = -0.05$
$A_4 = 0.05g\ \ \phi_4 = 1.25$	$A_{12} = 0.1g\ \ \phi_{12} = -0.02$
$A_5 = 0.12g\ \ \phi_5 = 2$	$A_{13} = 0.12g\ \ \phi_{13} = 0.12$
$A_6 = 0.18g\ \ \phi_6 = 2.6$	$A_{14} = 0.08g\ \ \phi_{13} = 0.15$
$A_7 = 0.2g\ \ \phi_7 = -0.2$	$A_{15} = 0.04g\ \ \phi_{14} = 0.4$
$A_8 = 0.1g\ \ \phi_8 = -0.4$	$A_{16} = 0.02g\ \ \phi_{16} = 0.25$

ω_2 to ω_{16} is given by $\omega_n = n\omega_1$ n=2 to 16

The generated time history is modulated by modulating functions given by Equations 2.83, 2.84 and 2.86. Find the absolute peak and rms values of acceleration and compare them. Also, compare between the times at which (absolute) peaks occur. Take maximum value of the modulating function to be unity. Take c = 0.5; $t_1 = 5\,\text{sec}$; $t_2 = 10\,\text{sec}$ for Equation 2.83 and $b_1 = 0.412$; $b_2 = 0.8$ for Equation 2.84.

Take the relevant figures from the slides or from the reference book

Module 2 – Seismic Inputs

Exercise Solution:

ERRATA FOR THE TEXT BOOK

pp 79, Equation 2.43: PHA in cms^{-1} should be in cms^{-2}

pp 81, Equation 2.54: PGA in cms^{-1} should be in cms^{-2}

pp 89, Example 2.11: (i) Kanai and Tajimi (Equation 2.73 should be 2.72)

(ii) Clough and Penzien (Equation 2.75 should be 2.74)

(iii) given by Equations 2.70, 2.71, 2.73 and 2.76 should be 2.70, 2.71, 2.72 and 2.74

pp 91, Equation 2.83: $e^{c\;t-t_2}$ should be $e^{-c\;t-t_2}$

pp 92, Equation 2.92: (i) terms within **exp** should be with negative sign

(ii) $\omega_0 = 1.09$ should be $f_0 = 1.09$

pp 95, Exercise problem 2.18: Equation 2.69 should be 2.66

Figure 2.45 should be 2.46

Exercise problem 2.19: Equation 2.75 should be 2.74

Exercise problem 2.20: Equation 2.75 should be 2.74

Equation 2.94 should be 2.93

Equation 2.93 should be 2.92

Equation 2.95 should be 2.94

Exercise problem 2.21: Equations 2.84, 2.85 and 2.87 should be 2.83, 2.84

Refer to the exercise problem 2.12.

The earthquake acceleration record of Lomaprieta sampled at 0.005s is taken for analysis and is shown in Figure 2.51. A total time history of 23.3s is taken constituting 4660 discrete ordinates (N). Input to the MATLAB® is

$$YY = \frac{2}{N} fft\;\; y, N$$

$$d\omega = \frac{2\pi}{T} = 0.269\,\mathrm{rad\,s^{-1}}$$

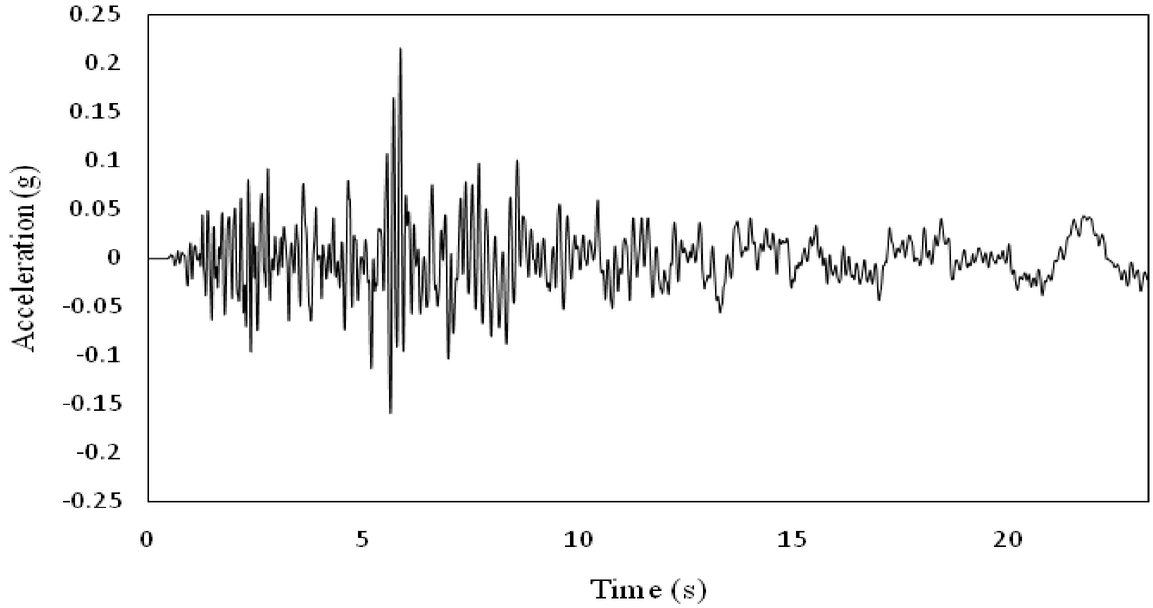
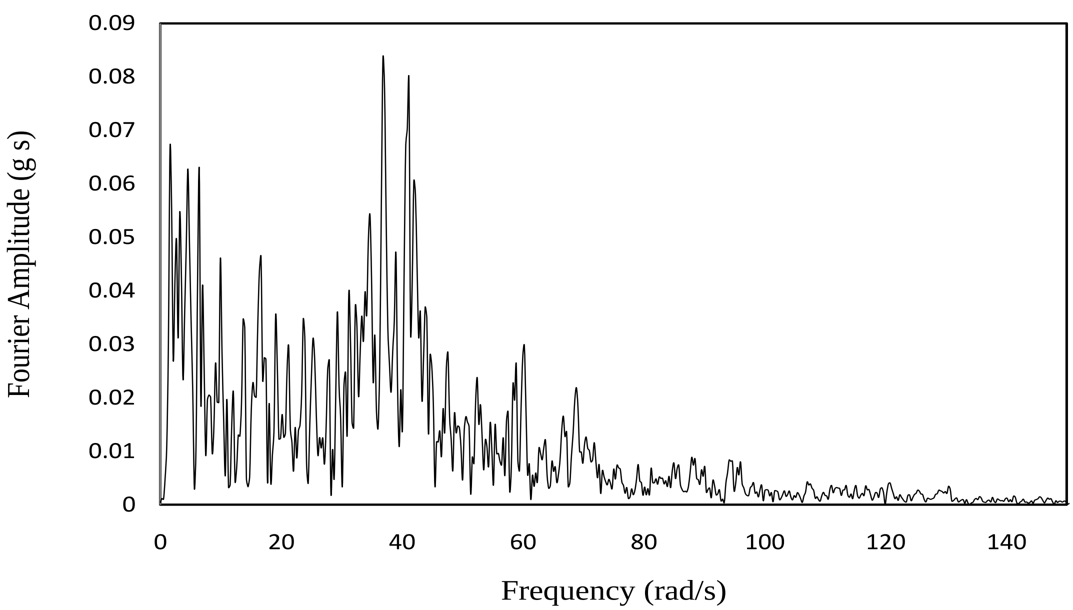
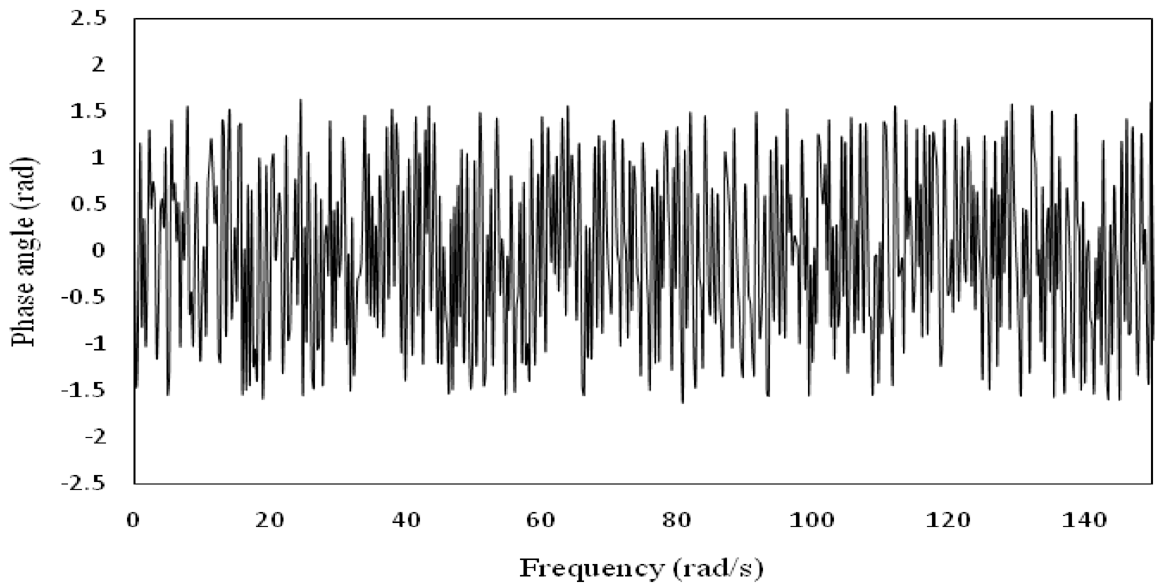


Figure 2.51 Time history of acceleration of Lomaprieta earthquake

The Fourier amplitude plot i.e., $(a_i^2 + b_i^2)^{1/2}$ vs. ω_i $\left(i = 0\cdots\cdots\frac{N}{2}\right)$ and phase plot i.e., $\tan^{-1}\frac{b_i}{a_i}$ vs. ω_i $\left(i = 0\cdots\cdots\frac{N}{2}\right)$ are shown in Figure 2.52. Note that output from FFT are divided by $\frac{N}{2}$.



(a)



(b)

Figure 2.52 Fourier spectrums (a) amplitude (b) phase angle

Fourier amplitude spectra before and after smoothing are compared in Figure 2.53

Peak (before smoothing) = $0.08339\,\mathrm{ms^{-2}}$

Frequency of occurrence = $36.85\,\mathrm{rad\,s^{-1}}$

Peak (after smoothing) = $0.06064\,\mathrm{ms^{-2}}$

Frequency of occurrence = $37.12\,\mathrm{rad\,s^{-1}}$

Nyquest frequency = $\dfrac{N\pi}{T}$ = $6.283\times10^{2}\ \mathrm{rad\,s^{-1}}$

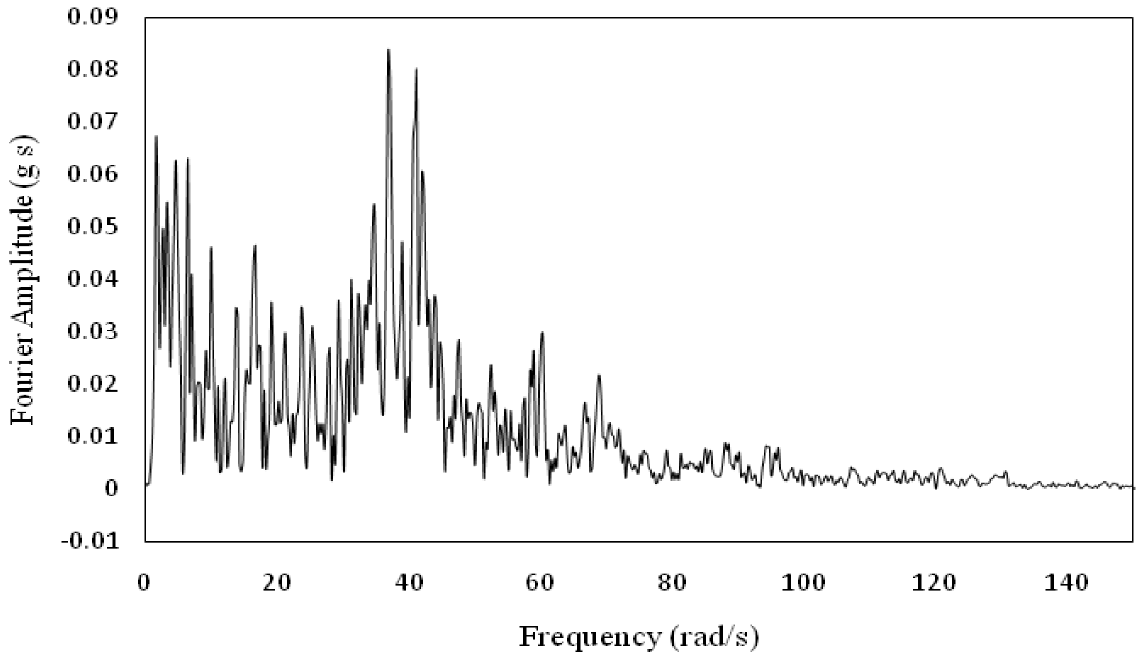
Peak (phase angle, smoothed) = $0.9062\,\mathrm{rad\,s^{-1}}$

rms (time history) = $0.3026\,\mathrm{ms^{-2}}$

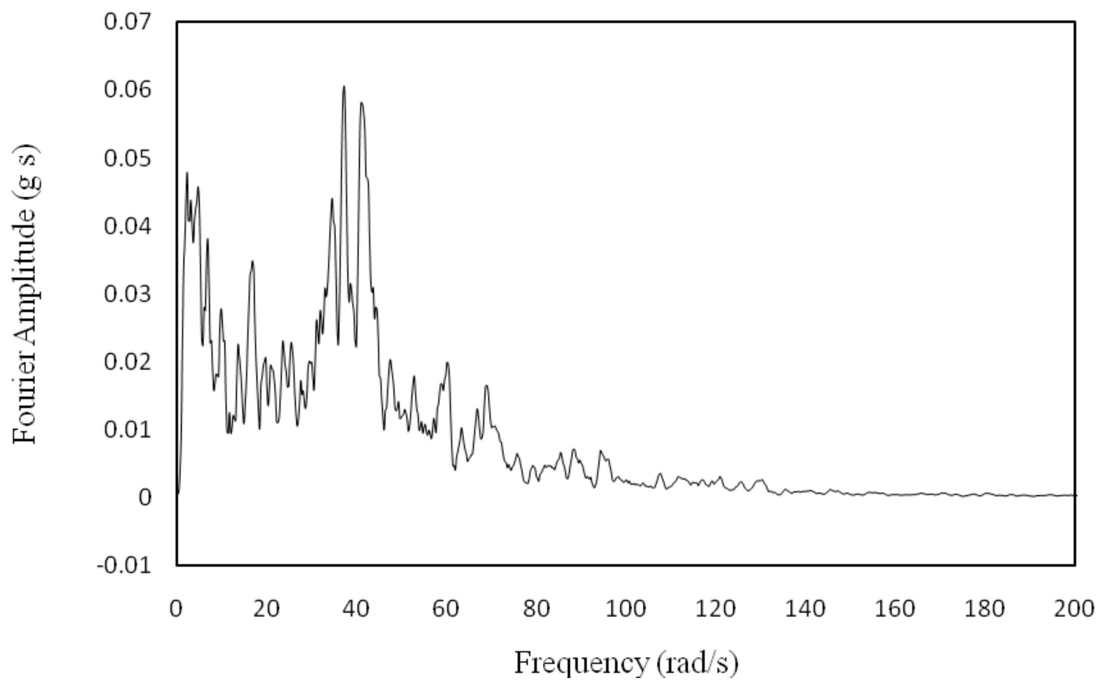
rms (Fourier components) = $0.3026\,\mathrm{ms^{-2}}$

Absolute maximum (original time history) = $2.119\,\mathrm{ms^{-2}}$

Absolute maximum (retrieved one) = $2.119\,\mathrm{ms^{-2}}$



(a)



(b)

Figure 2.53 Fourier amplitude spectrum (a) unsmoothed (b) smoothed

Refer to the exercise problem 2.13.

The spectral ordinates (raw) are obtained as $S(\omega_n) = \dfrac{c_n^2}{2d\omega}$. The plot of c_n vs. ω_n has been shown in Figure 2.53. $S(\omega_n)$ ordinates (raw) are shown in Figure 2.54. The smoothed spectrum is also shown in the same figure.

Area under the smoothed curve = $0.09106\,\mathrm{m^2s^{-4}}$

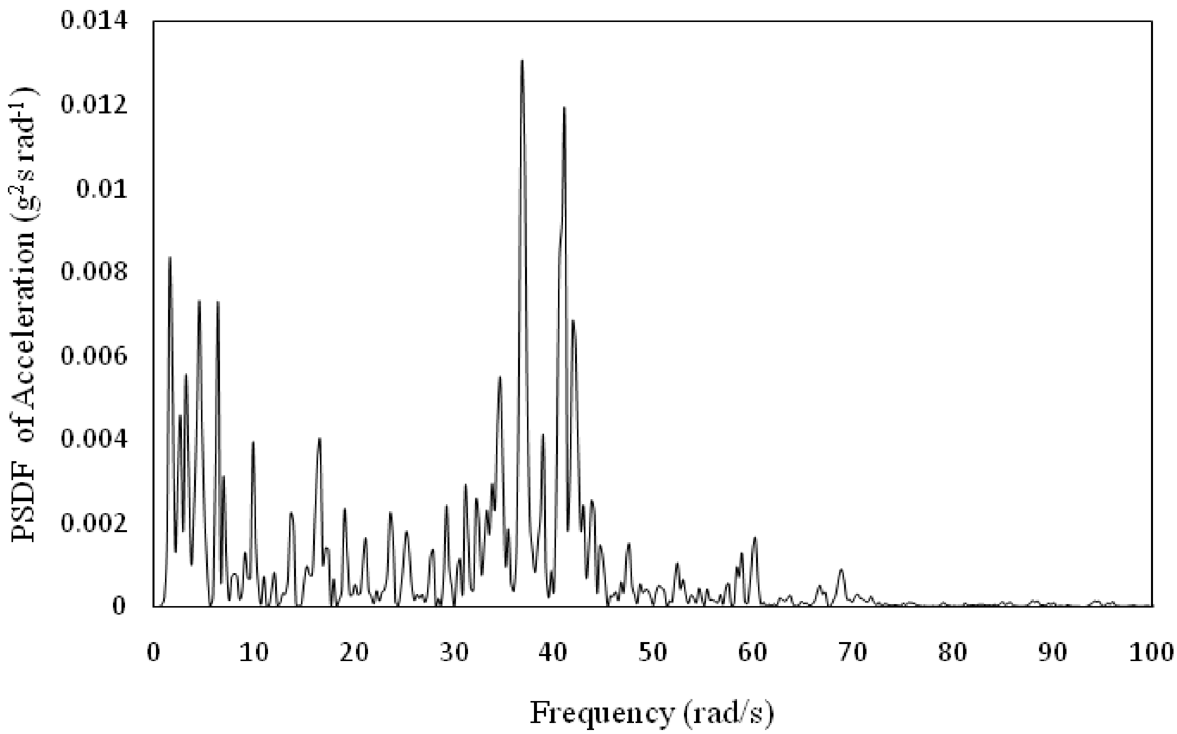
Mean square value of the time history = $0.09102\,\mathrm{m^2s^{-4}}$

λ_0 and λ_2 of the smoothed PSDF are: $\lambda_0 = 0.09016$, $\lambda_2 = 57.23$, $\Omega = \sqrt{\dfrac{\lambda_2}{\lambda_0}} = 25.07$

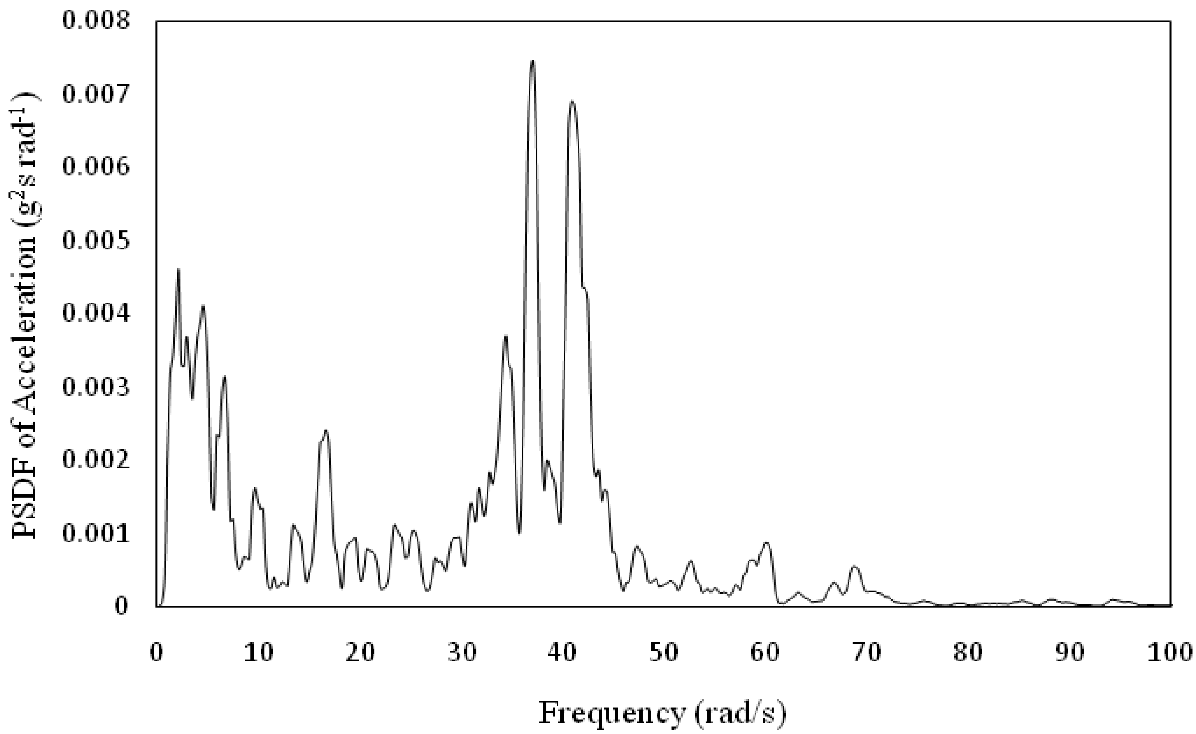
Mean peak = $\sqrt{2\lambda_0 \ln\left(\dfrac{2.8\Omega T_d}{2\pi}\right)} = 1.0\,\mathrm{ms^{-2}}$ (Equation 2.19c)

Absolute peak (time history) = $2.119\,\mathrm{ms^{-2}}$

rms (from PSDF) = $\sqrt{\lambda_0} = 0.302\,\mathrm{ms^{-2}}$; rms (time history) = $0.302\,\mathrm{ms^{-2}}$



(a)



(b)

Figure 2.54 PSDF of Lomaprieta earthquake (a) unsmoothed (b) smoothed

Note that mean peak value calculated by assuming the time history as an ergodise process is much less than the absolute peak of the time history. The reason for this is attributed to a few very high acceleration values clustered within 5.6 to 6.5 in the time history as shown in Figure 2.51.

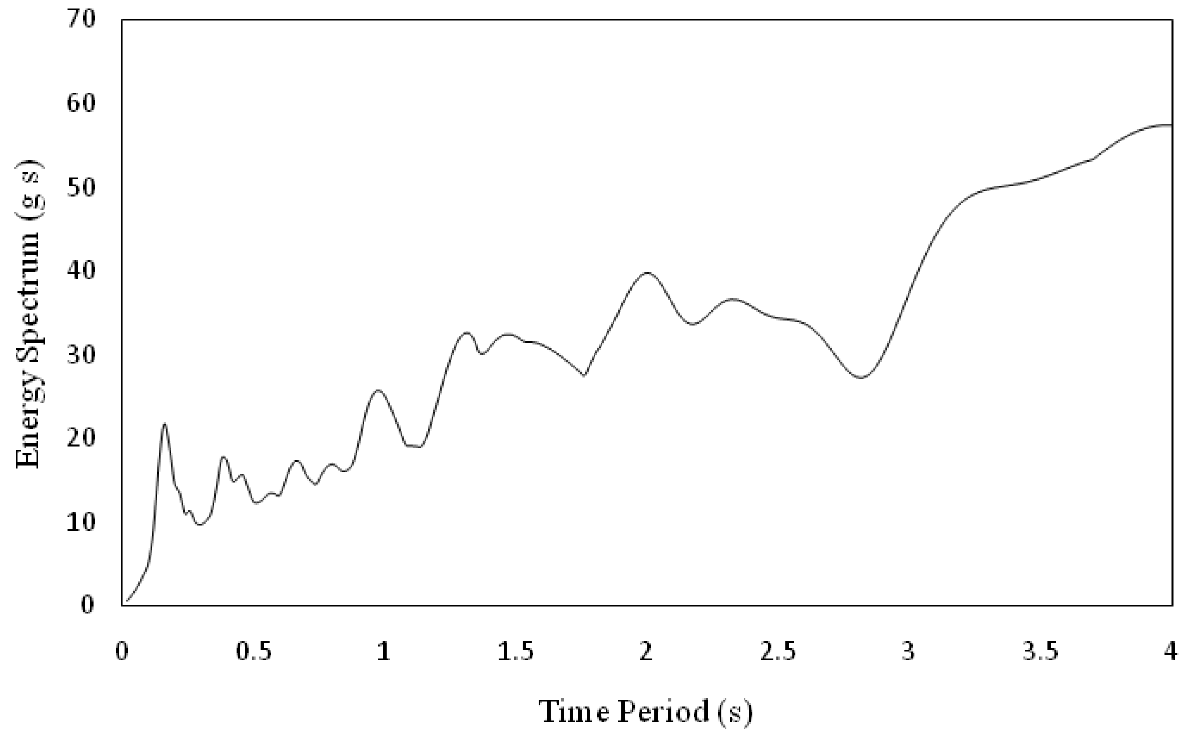
Refer to the exercise problem 2.14.

Using sesimo signal (www.seismosoft.com), the Fourier and pseudo acceleration spectrums are obtained. The energy spectrum is obtained using Equation 2.29. The spectrums are shown in Figure 2.55

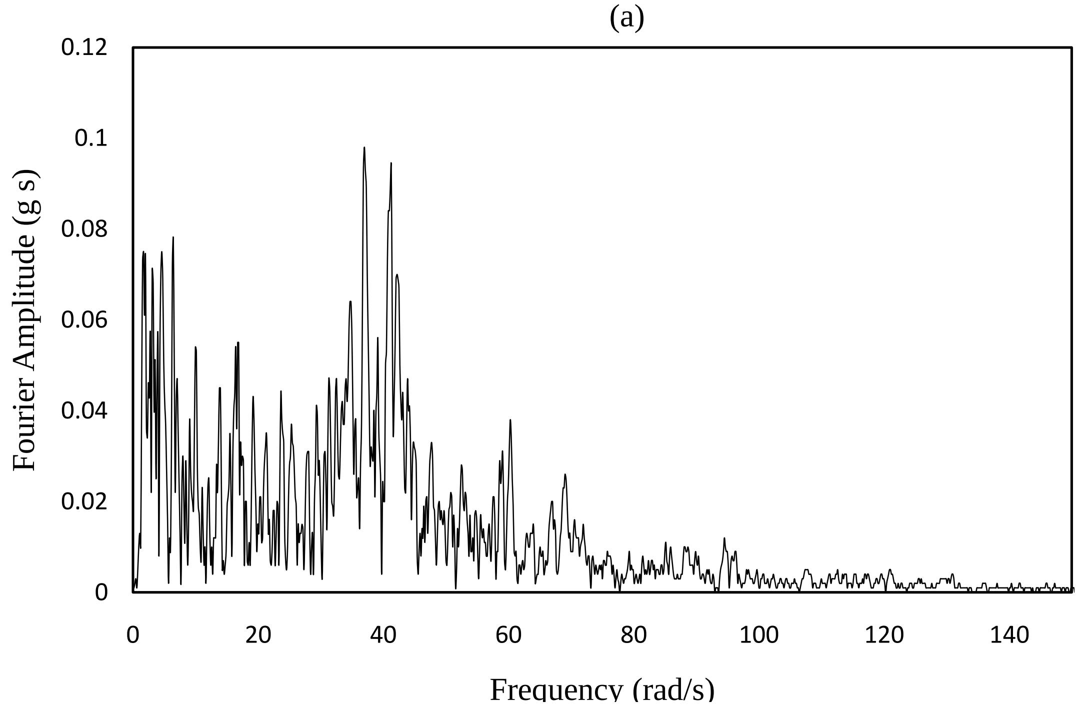
T_{peak} (Energy spectrum) = 0.16 s (first peak); 0.38 s (2^{nd} peak)

T_{peak} (Fourier spectrum) = 0.169 s

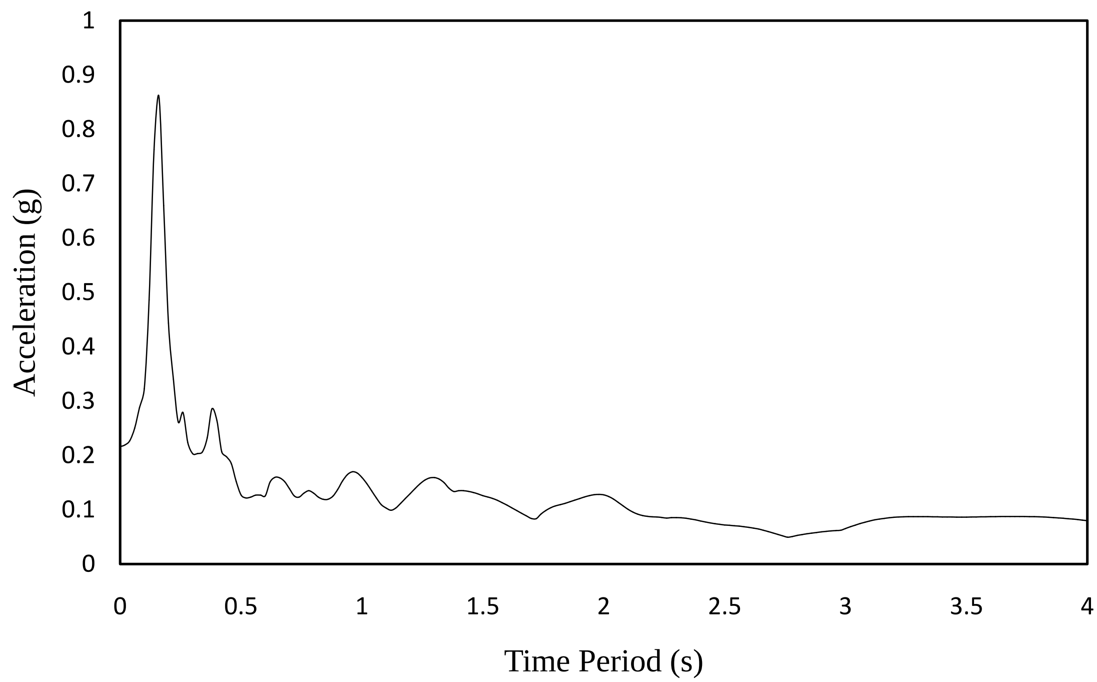
T_{peak} (Pseudo acceleration spectrum) = 0.16s


70
60
50
40
30
20
10
0
Energy Spectrum (g s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Time Period (s)

(a)


0.12
0.1
0.08
0.06
0.04
0.02
0
Fourier Amplitude (g s)
0
20
40
60
80
100
120
140
Frequency (rad/s)

(b)



(c)

Figure 2.55 Spectrums of San Fernando earthquake (a) energy spectrum (b) Fourier spectrum (c) acceleration spectrum

Refer to the exercise problem 2.15.

The displacement, pseudo velocity and pseudo acceleration response spectrums of the San Fernando earthquake are plotted in the tripartite plot for a duration of earthquake 28s. The plot is shown in Figure 2.56. In the same figure, the idealized spectrum by a series of straight lines is shown.

	T_a	T_b	T_c	T_d	T_e	$T_f\,(s)$
San Fernando	0.03	0.06	0.35	5	9	20
El-centro	0.035	0.125	0.5	3	10	15

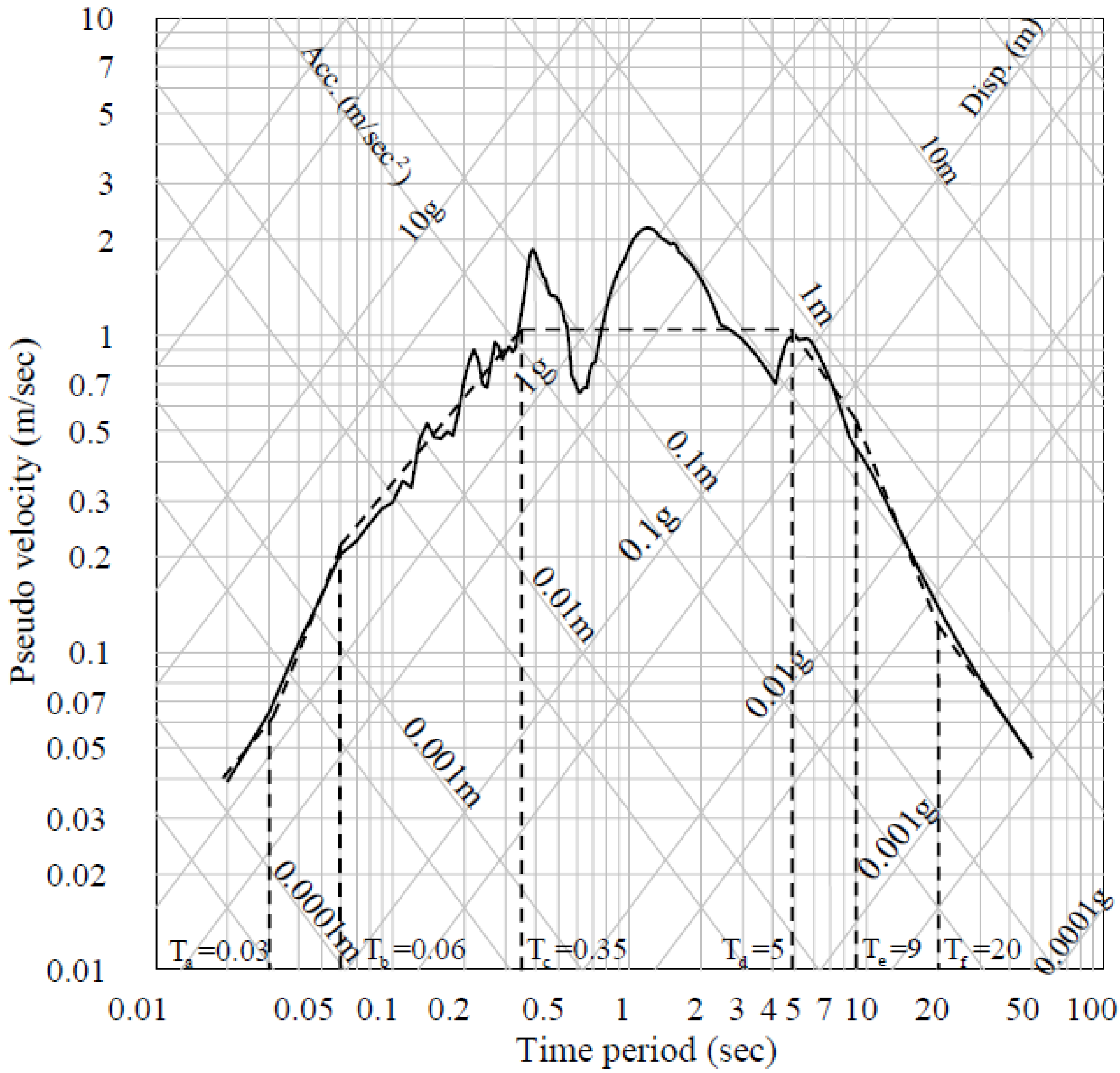


Figure 2.56 Tripartite plot of San Franando earthquake

Refer to the exercise problem 2.16.

PHA, PHV and PHD are calculated as

$$\ln PHA\ \ g\ = -4.141 + 0.868M - 1.09\ln\ \ R + 0.0606e^{0.7m}$$ [for M = 7.5 and R = 75km]

PHA = 0.0826g

Similarly, $PHV = 15e^{7.5}\ \ 75 + 0.17e^{0.59M}\ \ ^{-1.7} = 13.113\,\text{cms}^{-1}$

$$PHD = \frac{PHV^{\ 2}}{PHA}\left(1 + \frac{400}{75^{0.6}}\right) = 65.8\,\text{cm}$$

For 5% damping, $\alpha_A = 2.12$, $\alpha_V = 1.65$, $\alpha_D = 1.39$ (50th percentile values) and $\alpha_A = 2.71$, $\alpha_V = 2.30$, $\alpha_D = 2.01$ (84th percentile). The values are taken from the reference [2.4]

The 50th and 84th percentile idealized response spectrums drawn for the calculated peak values of ground acceleration (PHA), velocity (PHV) and displacement (PHD) and are shown in Figure 2.57

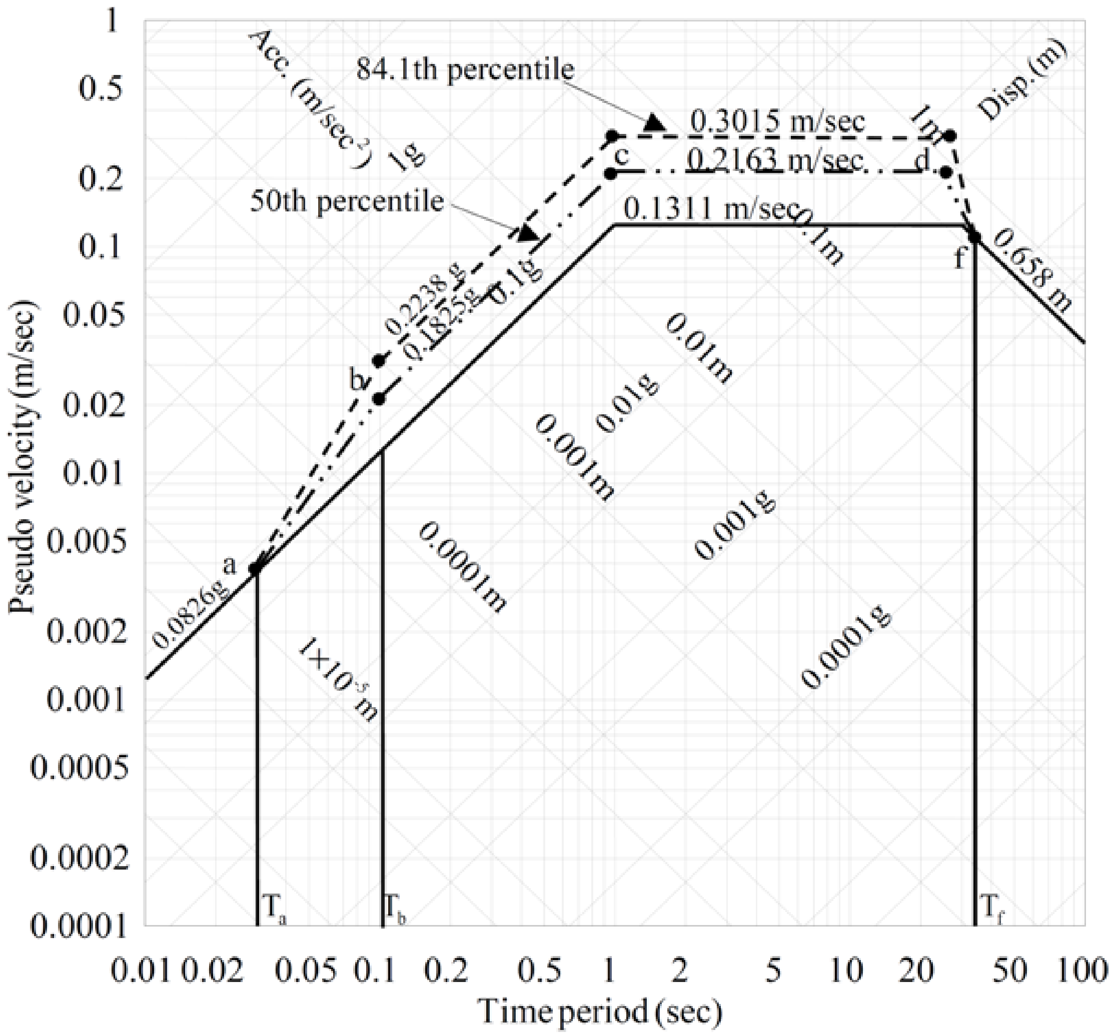


Figure 2.57 Design response spectra for 50th and 84th percentile in tripartite plot

Refer to the exercise problem 2.17.

10% probability of exceedance in 50 yrs corresponds to mean annual rate of exceedance as

$$a_r = \frac{\ln\left[1-P\ \ N\geq 1\right]}{50} = \frac{\ln\ 1-0.1}{50} = 0.0021 \approx 0.002$$

For the above value of a_r, the PGA value is estimated as 0.4g from Table 2.7. The normalized spectral ordinates for different values of T corresponding to the value of $a_r = 0.002$ are obtained from Table 2.6 and are shown below

$T\ \ s$	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2

$\overline{S}$	>3	>3	>3	>3	>3	2.25	2.15	2	1.75	1.5	0.05

Restricting the normalized response spectrum ordinate to a maximum of 3, the uniform hazard spectrum for 10% probability of exceedance in 50 yrs is determined by multiplying $\overline{S}$ with PGA and is shown in Figure 2.58.

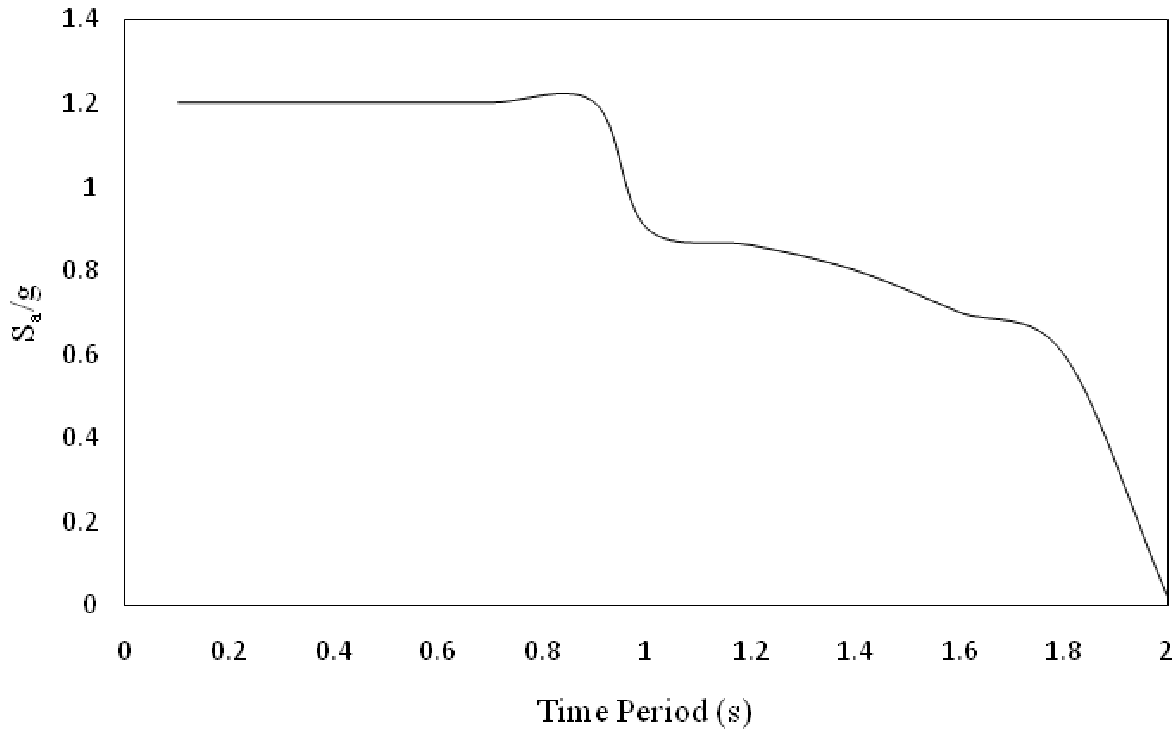


Figure 2.58 Uniform hazard spectrum with 10% probability of exceedance in 50 years

Refer to exercise problem 2.18.

For Equation 2.66, the values of the coefficients are taken from the reference [2.26].

Values of the coefficients for different values of T are given below

T s	b_1	b_2	b_3	b_5	b_6	b_7
0.1	1.653	0.327	-0.098	-0.934	0.046	0.136
0.3	1.974	0.334	-0.070	-0.893	0.239	0.356
0.5	1.881	0.384	-0.039	-0.846	0.279	0.439
0.7	1.797	0.418	-0.023	-0.818	0.297	0.483
0.9	1.742	0.442	-0.015	-0.802	0.309	0.508
1.0	1.724	0.450	-0.014	-0.798	0.314	0.517

1.2	1.701	0.462	-0.014	-0.794	0.324	0.528
1.4	1.695	0.469	-0.017	-0.794	0.333	0.535
1.6	1.706	0.473	-0.025	-0.801	0.347	0.539
1.8	1.715	0.472	-0.029	-0.804	0.351	0.539
2.0	1.737	0.471	-0.037	-0.812	0.360	0.537

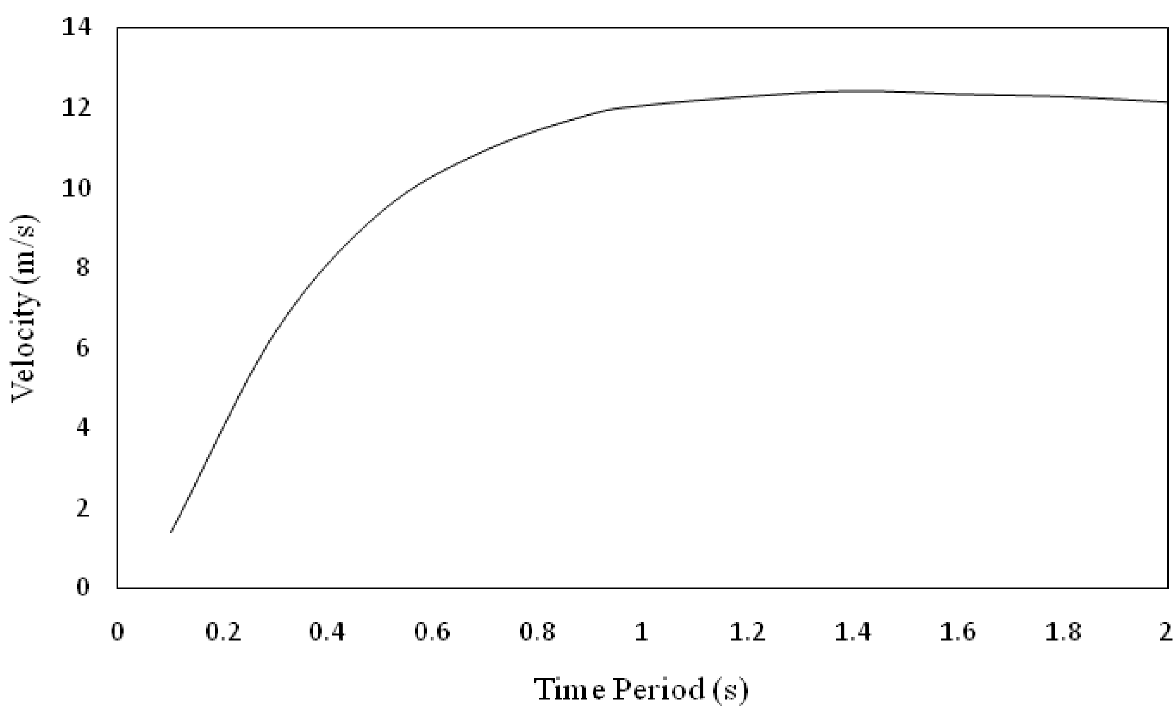
$b_4 = 0$

$$\log S_v(T) = b_1 + b_2(M_w - 6) + b_3(M_w - 6)^2 + b_4 R + b_5 \log R + b_6 G_B + b_7 G_C$$

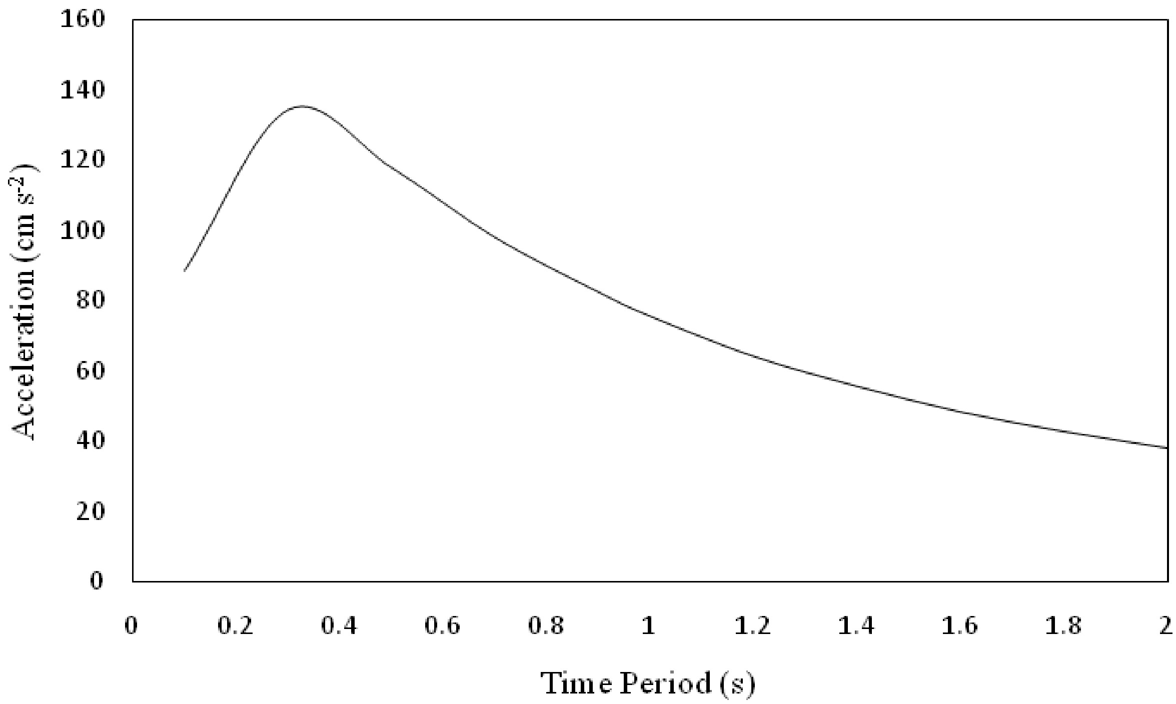
Assume $G_C = 1$; $G_B = 0$. For $M_\omega = 7$ and $R = 100\,\text{km}$, $S_v(T)$ for different periods are computed and the plot of $S_v(T)$ vs. T and the corresponding plot of $S_a(T)$ vs. T are shown in Figure 2.59.

Note that $S_a(0) = PGA$ is calculated using Equation 2.43 for $M_\omega = 7$ and R=100km. The normalized spectrum at the rock bed is shown in Figure 2.60.

The site specific normalized spectrum (after modification) is given in the same figure. The ordinates of the normalized spectrum are multiplied by 3PGA to obtain the site specific spectrum which is shown in Figure 2.61.

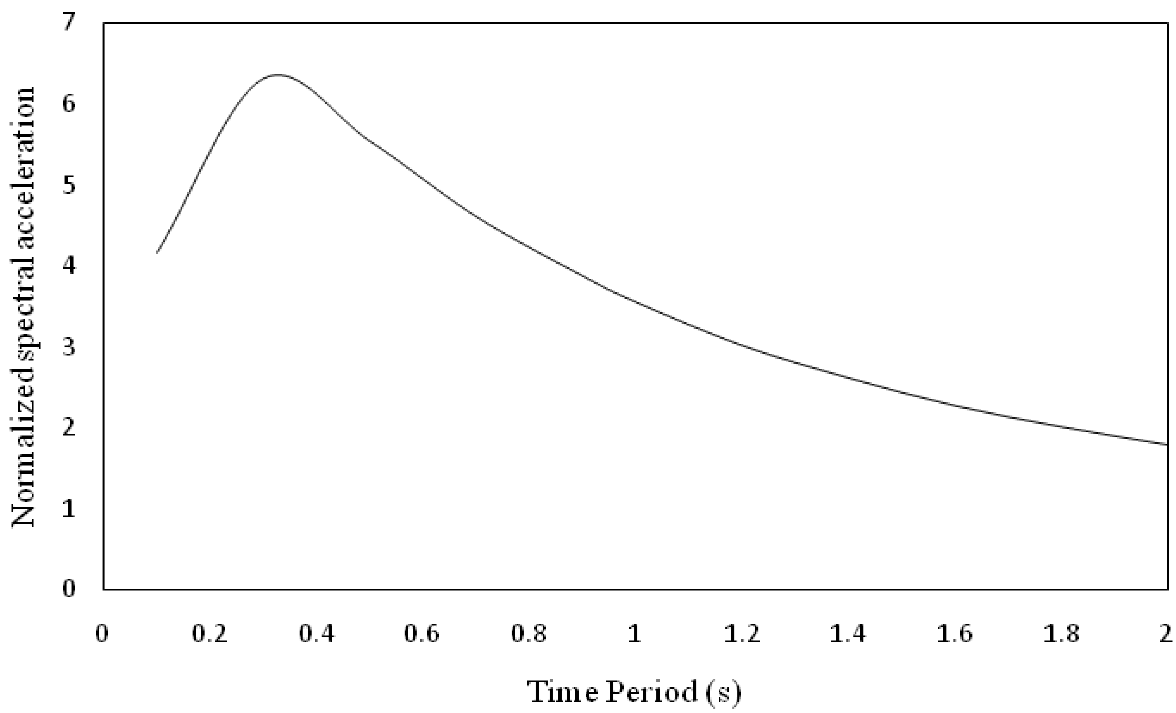


(a)

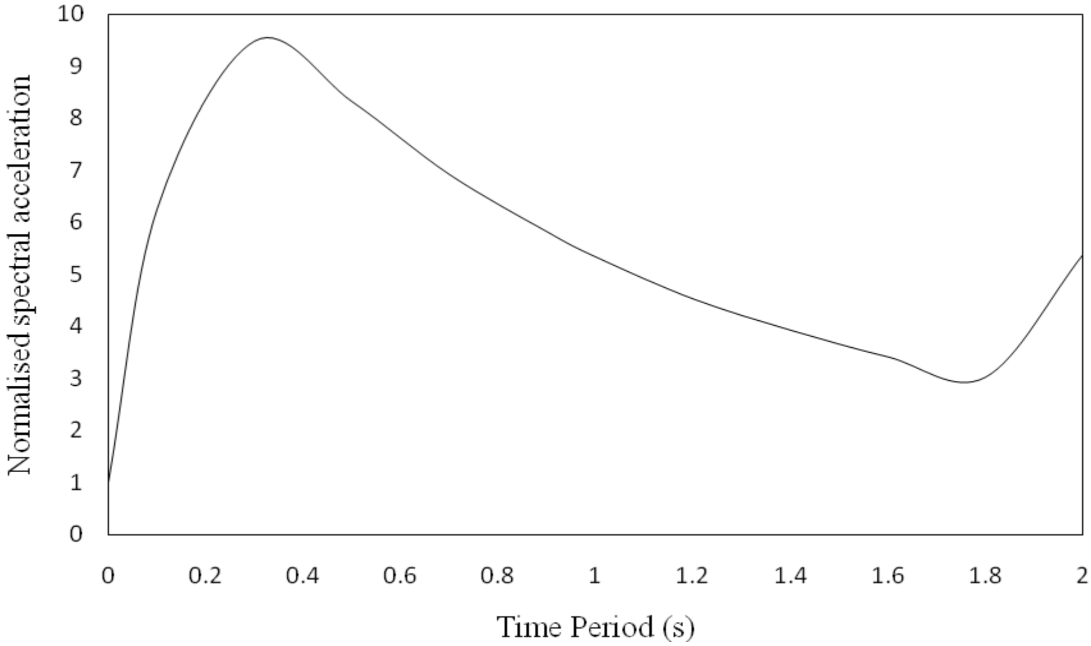


(b)

Figure 2.59 Spectrums at the rock bed (a) velocity (b) acceleration



(a)



(b)

Figure 2.60 Normalized spectrums at the rock bed (a) unmodified (b) modified for the site

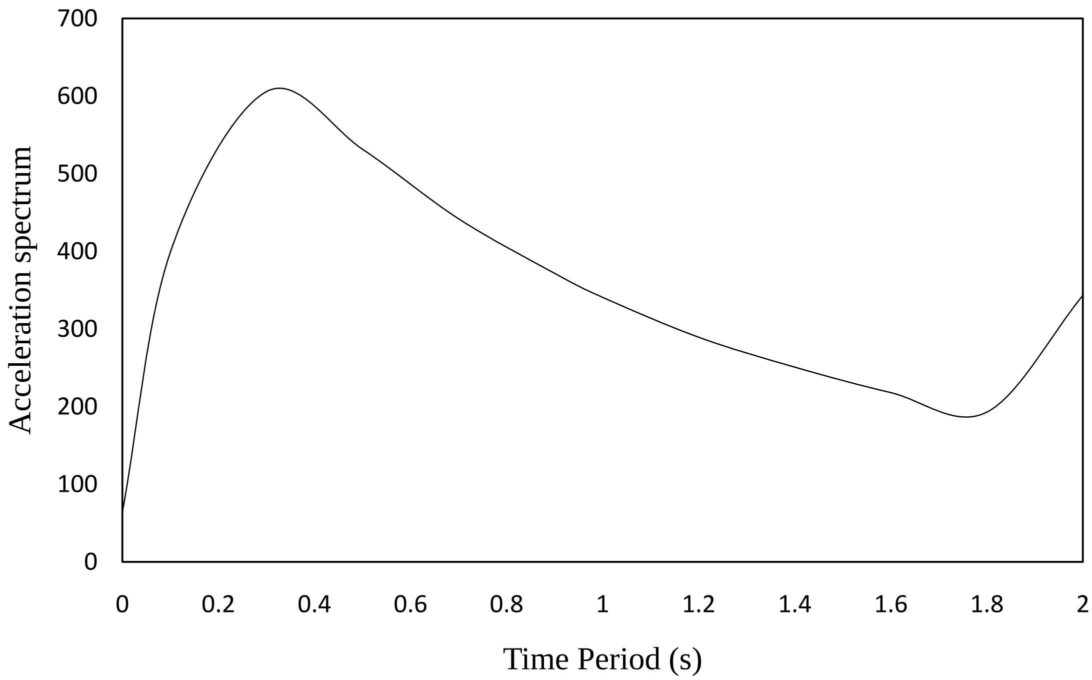


Figure 2.61 Site specific acceleration response spectrum

Refer to the exercise problem 2.19.

Using Equation 2.74 to find $S_{\ddot{u}_g}$, $\lambda_0 = \int_0^\infty S_{\ddot{u}g}\ \ \omega\ \ d\omega$; $\lambda_2 = \int_0^\infty \omega^2 S_{\ddot{u}g}\ \ \omega\ \ d\omega$

$$\Omega = \sqrt{\frac{\lambda_2}{\lambda_0}} = \frac{\int_0^\infty \omega^2 \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2 S_{\ddot{u}_0}(\omega)\, d\omega}{\int_0^\infty \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2 S_{\ddot{u}_0}(\omega)\, d\omega};$$

Since $S_{\ddot{u}_0}(\omega) = S_0 = \text{constant}$, $\Omega = \sqrt{\frac{I_2}{I_1}}$

in which $I_2 = \int_0^\infty \omega^2 \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2 d\omega$ and $I_1 = \int_0^\infty \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2 d\omega$

The integrations I_1 and I_2 are performed numerically for ω up to $30\,\text{rad}\,\text{s}^{-1}$ with $d\omega = 0.05\,\text{rad}\,\text{s}^{-1}$ and $\omega_g = 10\pi$, $\xi_g = 0.4$. The value of I_2 and I_1 are obtained as $I_1 = 1088.2$; $I_2 = 1633.6$ and $\Omega = 1.225$

$PGA = \sqrt{2I_1 \ln\left(\frac{2.8\Omega T_d}{2\pi}\right) S_0}$ in which T_d is the duration of earthquake

$$S_0 = \frac{(PGA)^2}{2I_1 \ln\left(\frac{2.8\Omega T_d}{2\pi}\right)}$$

$$\begin{aligned}\ln S_0 &= 2\ln PGA - \ln\left[2I_1 \ln\left(\frac{2.8\Omega T_d}{2\pi}\right)\right] \\ &= 2\left[-4.141 + 0.868M - 1.09\ln\left(R + 0.0606e^{0.7M}\right)\right] - \ln\left[2I_1 \ln\left(\frac{2.8\Omega T_d}{2\pi}\right)\right]\end{aligned}$$

For a given set of values of M and R, S_0 may be calculated from the above equation and put in the expression for $S_{\ddot{u}_g}(\omega)$ i.e.,

$$S_{\ddot{u}_g}(\omega) = \left[\left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2 S_0\right]$$

For M = 7, R = 50km and T_d = 30s, $S_0 = 1.122 \times 10^{-6}$

For M = 7, R = 100km and $T_d = 30s$, $S_0 = 2.92 \times 10^{-7}$ (PSDF will be obtained consistent with PGA in g unit)

For these two cases, plots of the PSDFs are shown in Figure 2.62. Peak values for the two cases are 3.12×10^{-6} and 8.137×10^{-7} respectively. The frequencies at which the peaks occur are the same and are equal to $27.95\,\text{rad}\,\text{s}^{-1}$.

If Equations 2.54 and 2.51 are used to obtain PGA, then

$$\log\ PGA\ = 0.3\ 1.667M - 2.167\ - 0.014 = 0.5M - 0.664$$

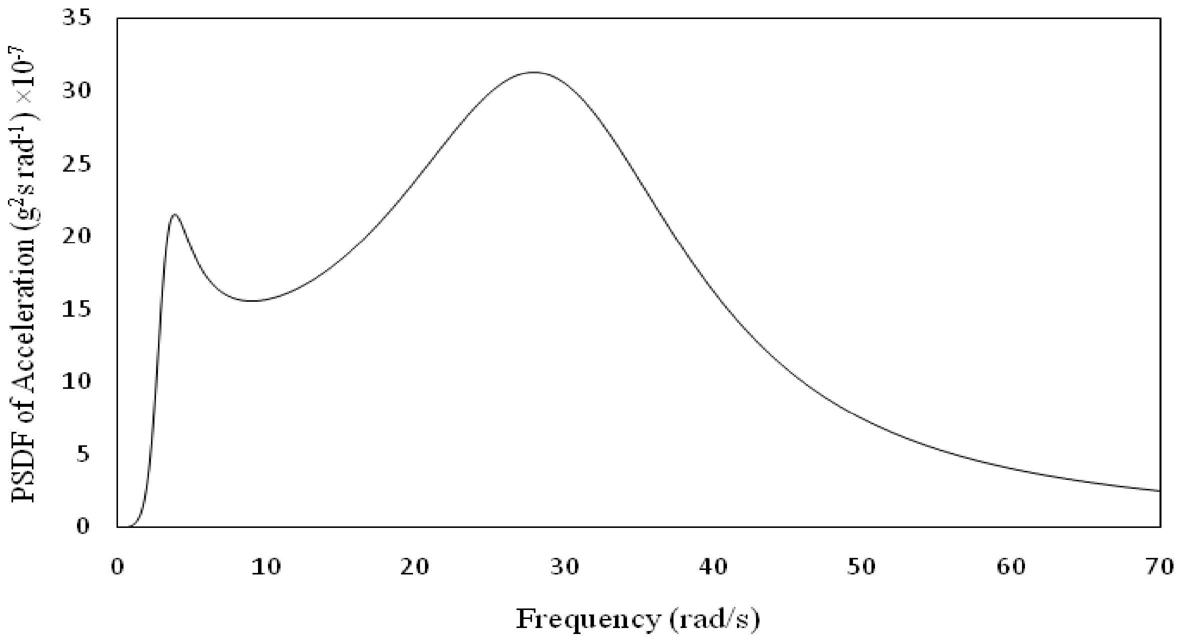
Expression for S_0 becomes

$$\log S_0 = 2\ 0.5M - 0.664\ - \log\left[2I_1 \ln\left(\frac{2.8\Omega T_d}{2\pi}\right)\right]$$

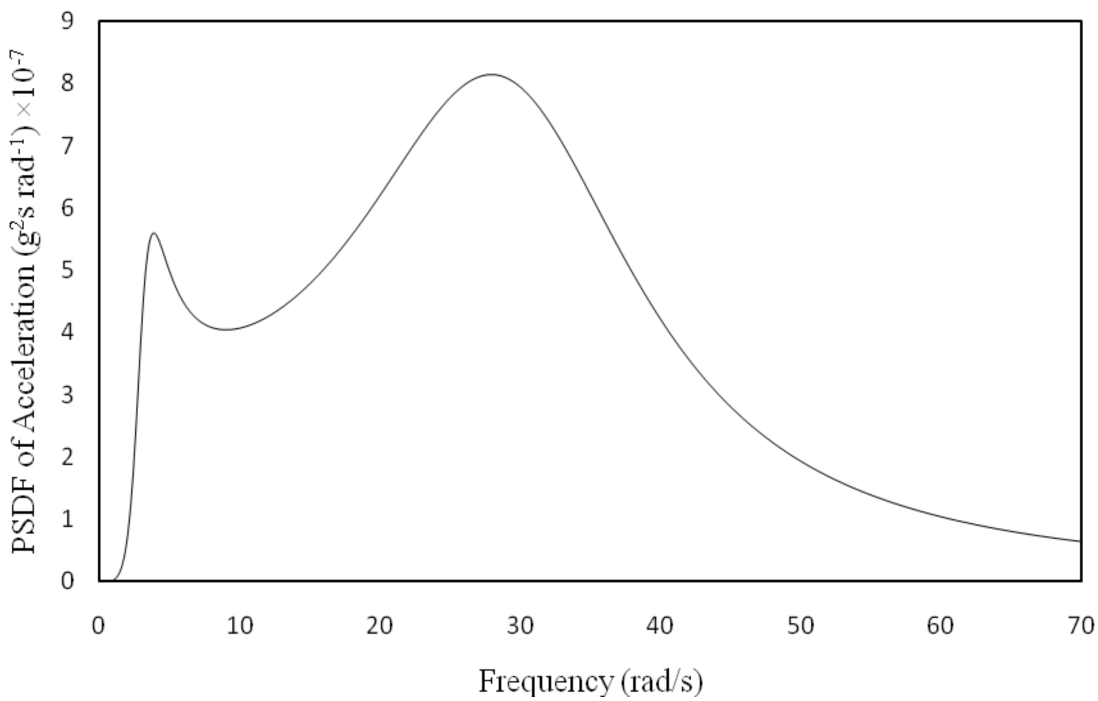
$$= 5.672 - 3.784 = 1.888$$

For M = 7 and T_d = 30s, $S_0 = 77.26$ (PSDF will be obtained in unit consistent with PGA in cms^{-2})

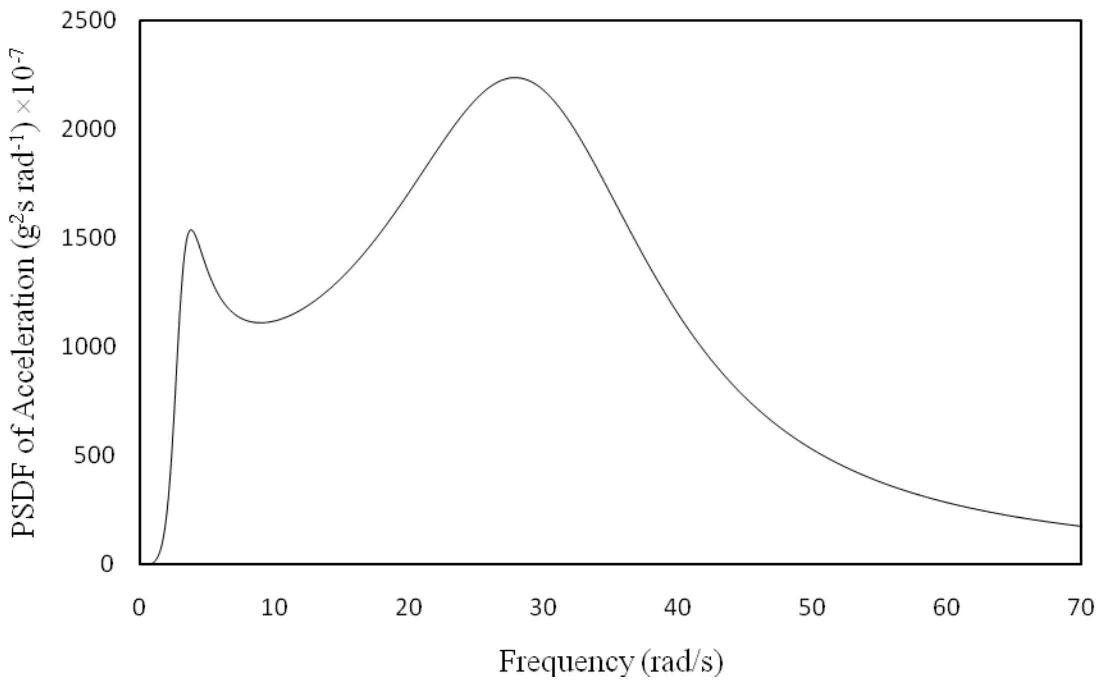
The shape of the PSDF curve remains the same as that shown in Figure 2.63. The peak value of the PSDF is 220×10^{-6}



(a)



(b)



(c)

Figure 2.62 PSDF of ground acceleration (a) R = 50 km (b) R = 100 km (c) R independent (Eq. 2.54)

Refer to the exercise problem 2.20.

Component of $\ddot{x}_g$ along the line joining the piers is $\ddot{x}_g \cos 20^0$

PSDF of earthquake along the line of piers = $S_{\ddot{x}_g} \cos^2 20^0$

Spatial correlation length $l_{AB} = 400 \cos 20 = 375.88\text{m}$

$l_{AC} = 751.75\text{m}$, $\quad l_{BC} = l_{AB}$

PSDF matrix = $0.883 \begin{bmatrix} 1 & & \\ \rho_{AB} & 1 & \\ \rho_{AC} & \rho_{BC} & 1 \end{bmatrix} S_0 \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2$; $S_0 = 2.53 \times 10^{-3}$

$$= 2.23 \times 10^{-3} \begin{bmatrix} 1 & & \\ \rho_{AB} & 1 & \\ \rho_{AC} & \rho_{BC} & 1 \end{bmatrix} \left|H_1(\omega)\right|^2 \left|H_2(\omega)\right|^2$$

Note that S_0 is computed from PGA=0.4g as described in the previous problem and upper triangle of the matrix is symmetric if real; complex conjugate if complex.

For $\omega = 3 \,\text{rad}\,\text{s}^{-1}$.

$$\left|H_1(\omega)\right|^2 = \frac{1 + (2\xi_g p_g)^2}{(1 - p_g^2)^2 + (2\xi_g p_g)^2} = \frac{1 + \left(\frac{2.4}{10\pi}\right)^2}{\left[1 - \left(\frac{3}{10\pi}\right)^2\right]^2 + \left(\frac{2.4}{10\pi}\right)^2} = 1.018;$$

$$p_g = \frac{\omega}{\omega_g} \text{ and } p_f = \frac{\omega}{\omega_f}$$

$$\left|H_2(\omega)\right|^2 = \frac{(p_f)^4}{(1 - p_f^2)^2 + (2\xi_f p_f)^2} = \frac{\left(\frac{3}{\pi}\right)^4}{\left[1 - \left(\frac{3}{\pi}\right)^2\right]^2 + \left(\frac{2.4}{\pi}\right)^2} = \frac{0.3315}{0.98768} = 0.8419$$

Hindy and Novak (Equation 2.93)

C is assumed as o.5

$$\rho_{AB} = \exp\left[\frac{-0.5 \times 375.88}{2\pi \times 150}\right] = 0.8192$$

$$\rho_{AC} = \exp\left[\frac{-0.5 \times 375.88}{\pi \times 150}\right] = 0.6711$$

$$\rho_{AB} = \rho_{AC}$$

Clough and Penzien (Equation 2.99)

$$\rho_{AB} = \exp\left[-\frac{3\times 375.88}{150} i\right] = 0.33 - 0.944i$$

$$\rho_{AC} = \exp\left[-\frac{6\times 375.88}{150} i\right] = -0.782 - 0.623i$$

$$\rho_{BC} = \rho_{AB}$$

Harichandran and Vannarke (Equation 2.92)

$$1 - A + \alpha A \;=\; 1 - 0.736 + 0.147\times 0.736 \;= 0.3722$$

$$1 - A \;=\; 1 - 0.736 \;= 0.264$$

$$\theta \;\; \omega \;= 5210\left[1+\left(\frac{3}{1.09\times 2\pi}\right)^{2.278}\right]^{-1/2} = 4853$$

$$\rho_{AB} = 0.736\exp\left[-\frac{2\times 375.88}{0.147\times 4853}\times 0.3722\right] + \exp\left[-\frac{2\times 375.88}{4853}\times 0.3722\right]\times 0.264$$
$$= 0.497 + 0.249 = 0.746$$

$$\rho_{AC} = 0.736\exp\left[-\frac{4\times 375.88}{0.147\times 4853}\times 0.3722\right] + \exp\left[-\frac{4\times 375.88}{4853}\times 0.3722\right]\times 0.264$$
$$= 0.33 + 0.23 = 0.56$$

$$\rho_{BC} = \rho_{AB}$$

Loh (Equation 2.94)

$\left|D_{ij}\right|$ is to be taken in km

Assume $K_0 = 4.769$, a = 2.756

$$\rho_{AB} = \exp\;\; -2.756\times 0.376\;\; \cos\;\; 2\pi\;\; 4.769\;\; \times 0.376\;\; = 0.259$$

$$\rho_{AC} = \exp\;\; -2.756\times 2\times 0.376\;\; \cos\;\; 2\pi\;\; 4.769\;\; \times 2\times 0.376\;\; = -0.108$$

$$\rho_{BC} = \rho_{AB}$$

It is seen that the correlation coefficients significantly vary with the expressions adopted for the correlation functions.

2.21. Refer to the exercise problem 2.21.

The generated time history is shown in Figure 2.63.The modulating functions given by Equations 2.83, 2.84 and 2.86 are shown in Figures 2.64. The three time histories obtained by multiplying the original time histories with the modulating functions are plotted in Figures 2.65

Table 2.8 Characteristics of Modulated time histories

Modulating Function	Absolute peak (g)	rms (g)	T_{Peak} (s)
Equation 2.83	1.4	0.239	7.38
Equation 2.84	0.4886	0.1209	2.56
Equation 2.86	1.368	0.2793	7.38

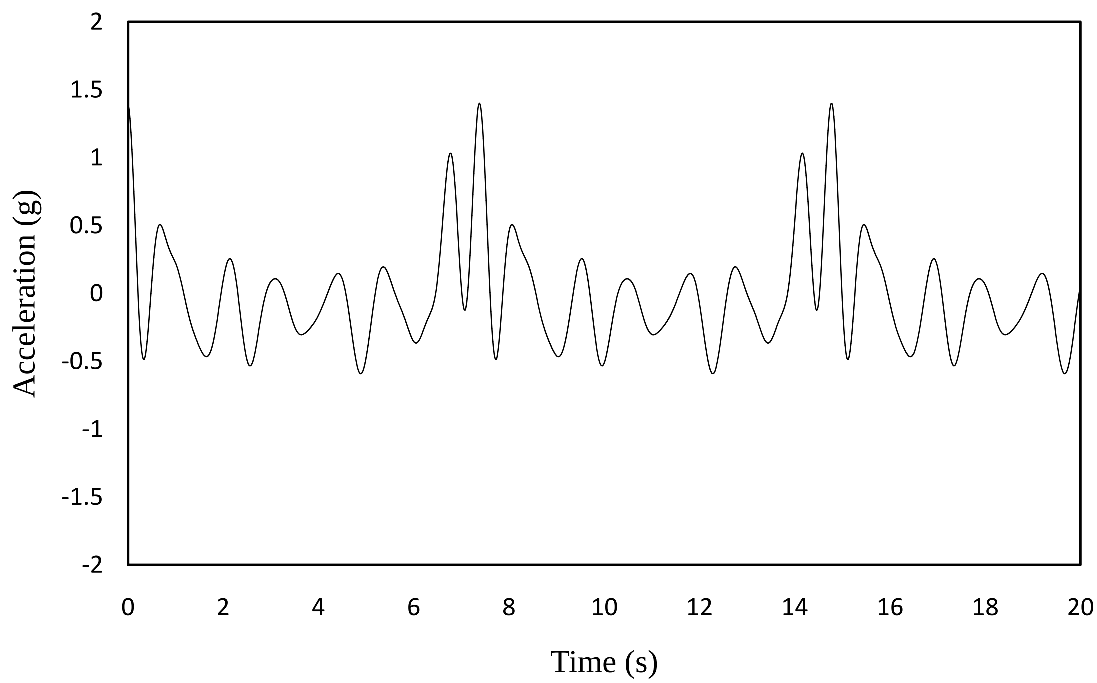
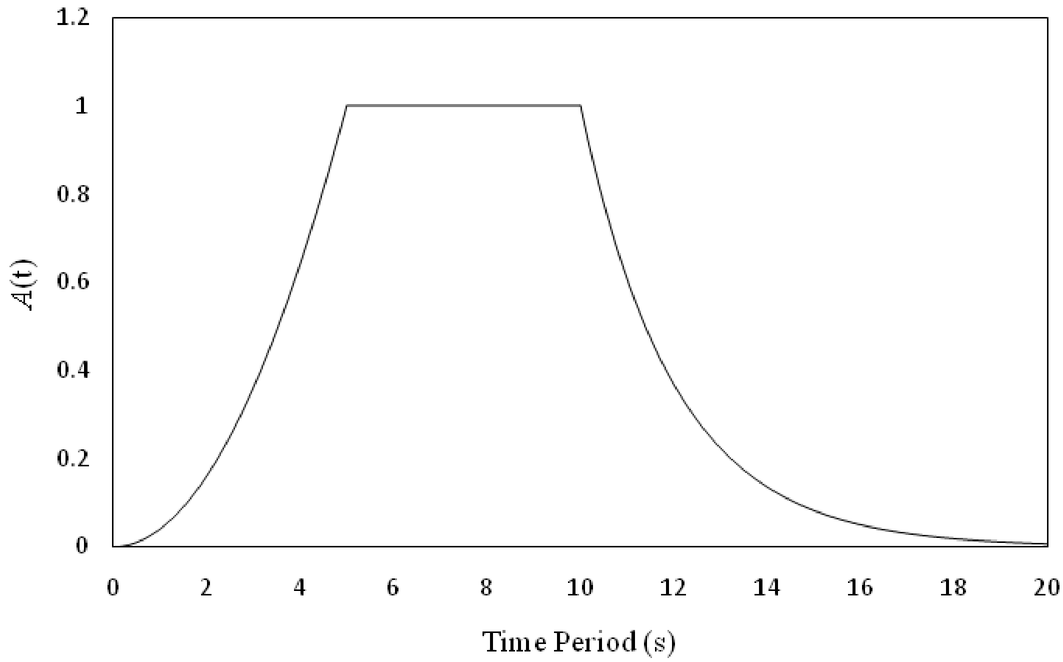
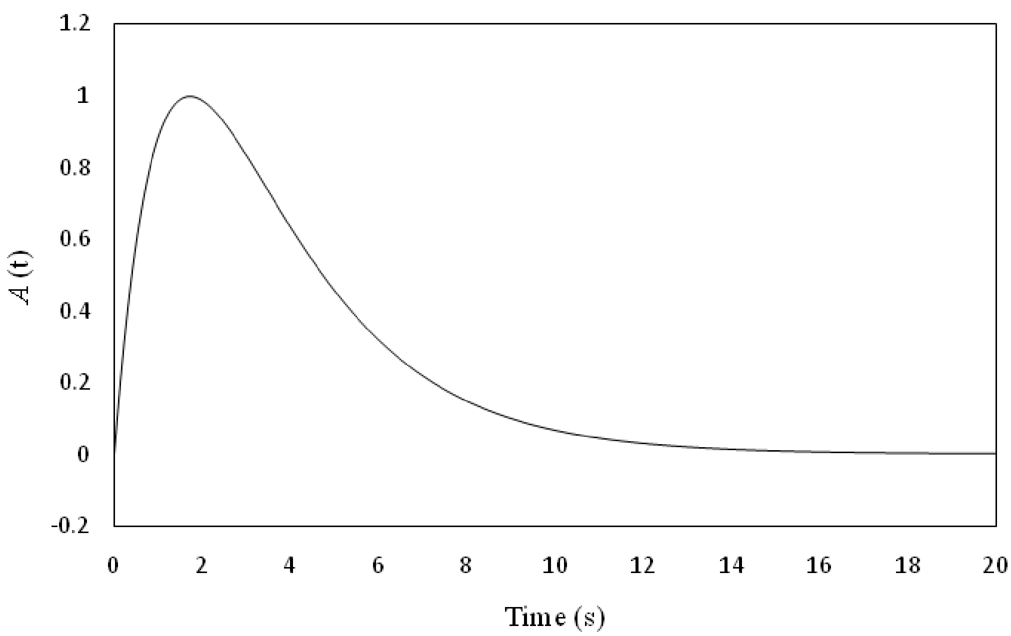


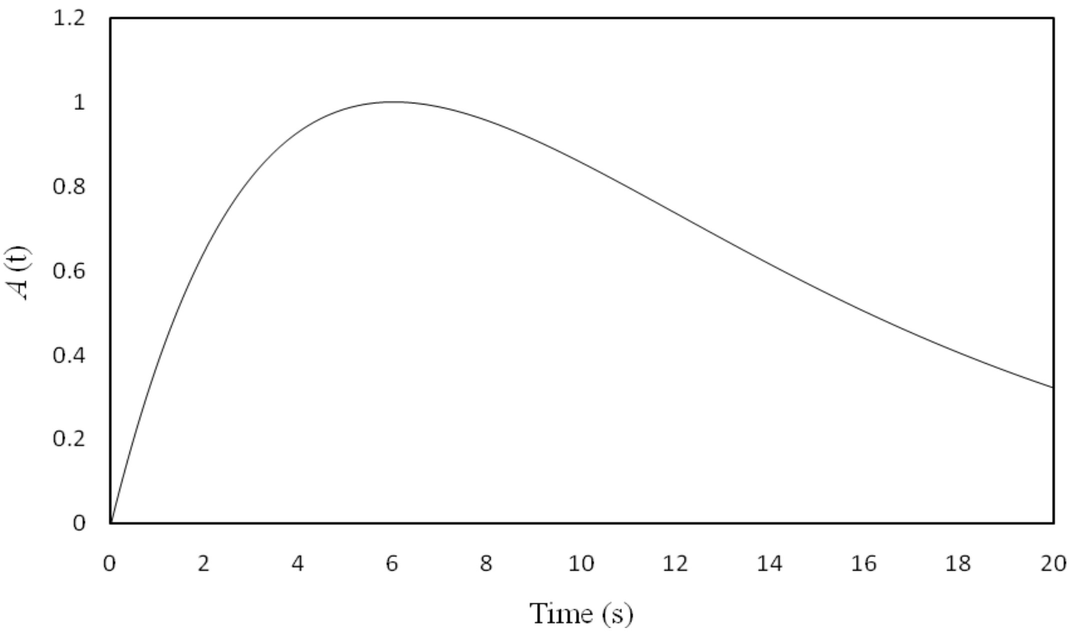
Figure 2.63 Generated time history


1.2
1
0.8
0.6
0.4
0.2
0
A(t)
0
2
4
6
8
10
12
14
16
18
20
Time Period (s)

(a)

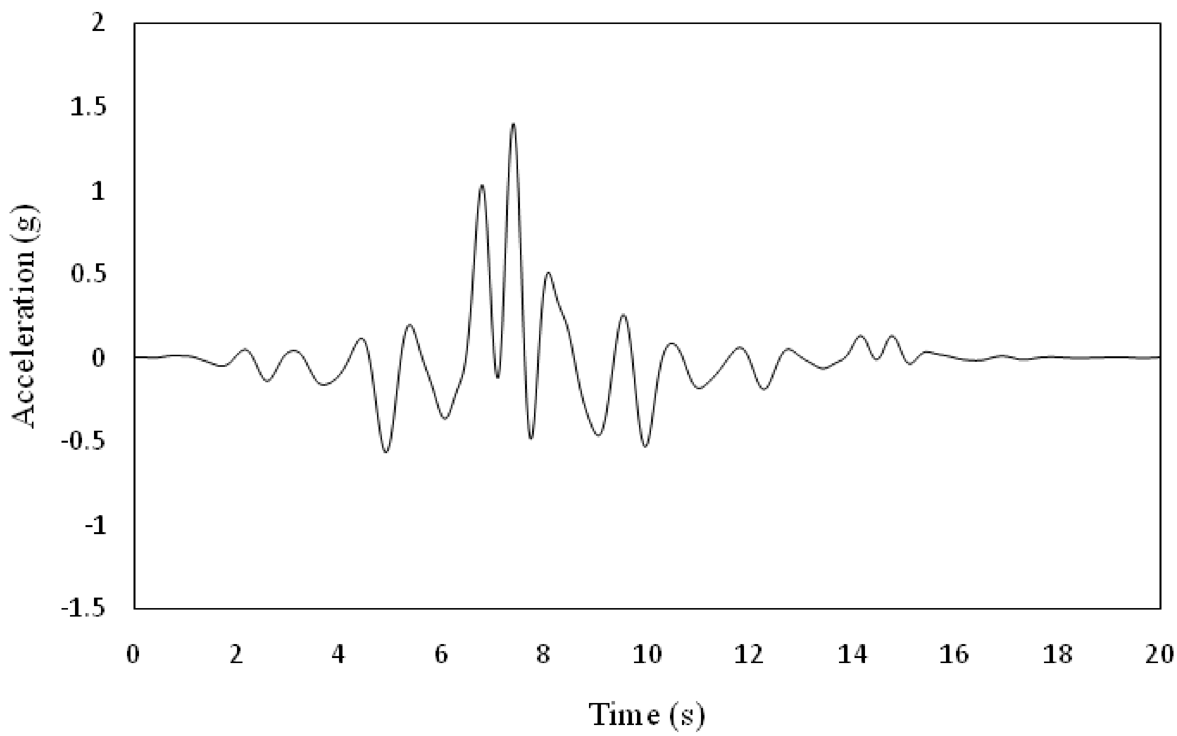

1.2
1
0.8
0.6
0.4
0.2
0
-0.2
A (t)
0
2
4
6
8
10
12
14
16
18
20
Time (s)

(b)

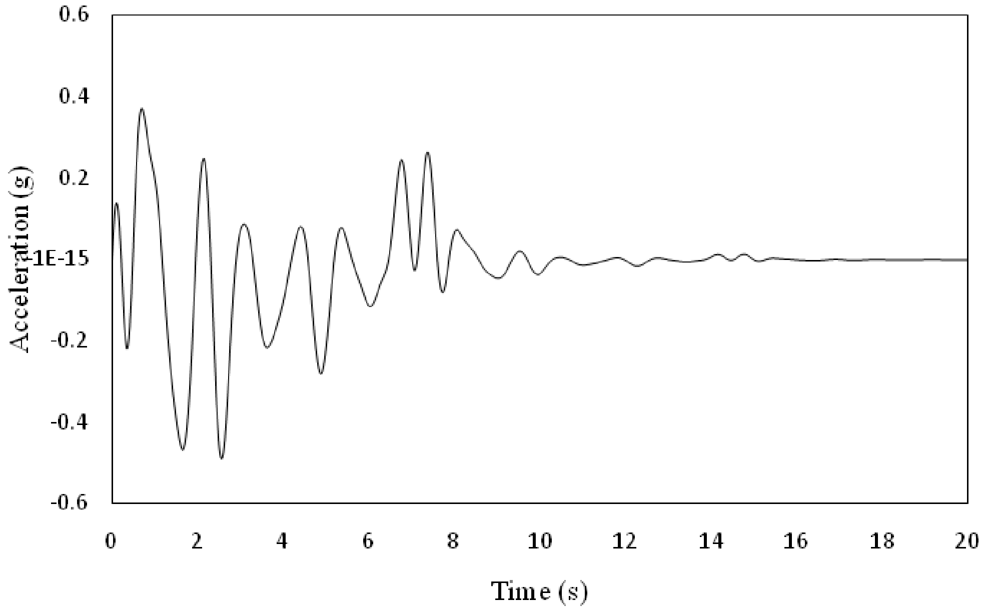


(c)

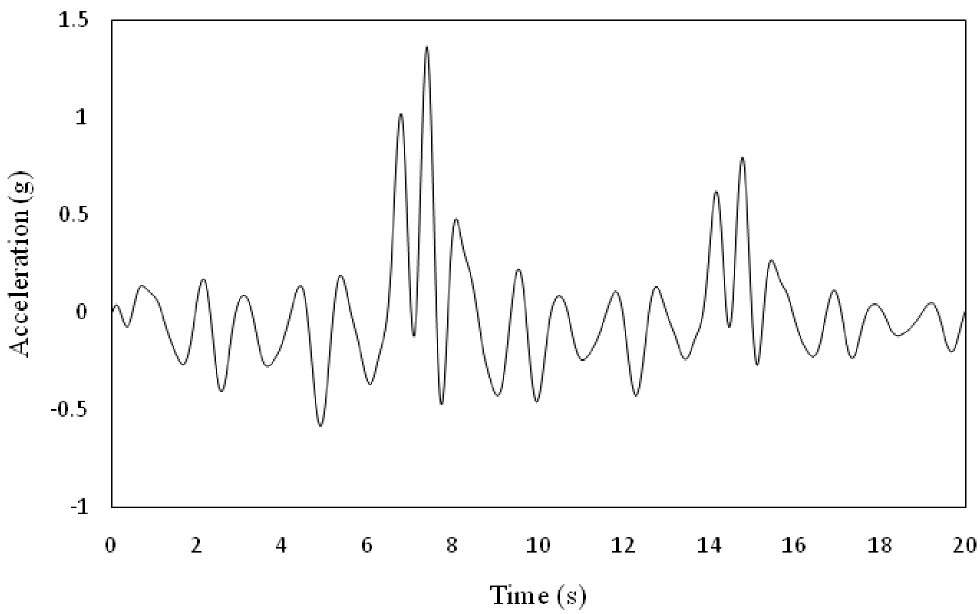
Figure 2.64 Modulating functions (a) Eq. 2.83 (b) Eq. 2.84 (c) Eq. 2.86



(a)



(b)



(c)

Figure 2.65 modulated time histories of accelerations (a) for modulating function 'a' (b) for modulating function 'b' (c) for modulating function 'c'

It is seen that the absolute peak value, rms value and the occurrences of the peaks differ if different types of modulating functions are used to modulate a time history.