

**Seismic Analysis of Structures**  
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**Module 1 – Seismology**

*Exercise Problems :*

- 1.4. Estimate the probabilities of surface rupture length, rupture area and maximum surface displacement exceeding 90km, 600km<sup>2</sup> and 15m respectively (use Equations. 1.14b-d) for  $M_w = 7.5$ . Assume the rupture parameters to be log normally distributed.
- 1.5. A site is surrounded by three line faults as shown in Figure 1.28. Determine the expected mean value of PGA at the site using the attenuation relationship given by Cornell et. al. (Equation 1.22)

**Figure 1.28**

- 1.6. A site is surrounded by two sources as shown in Figure 1.29. Determine the anticipated mean value of PGA and the probability of it exceeding the value of 0.2g using the same attenuation law used in exercise problem 1.2 with  $\sigma_{lnPGA} = 0.57$

**Figure 1.29**

- 1.7. A site has two earthquake sources, a point source (source 1) and segments of line sources (source 2). The distributions of magnitudes of earthquake for source 1 and source 2 are shown in Figure 1.30. The distribution of the source to site distance for source 2 is also shown in the same figure. Occurrences of earthquakes for sources 1 and 2 are given by

$$\log \lambda_{m_1} = 4 - 0.7m$$

$$\log \lambda_{m_2} = 3 - 0.75m$$

Develop a seismic hazard curve for peak acceleration at the site using the attenuation relationship same as that for example 1.2. Assume that magnitudes less than 4 do not contribute to the seismic hazard.

**Figure 1.30**

- 1.8. A region as shown in Figure 1.31 is to be microzoned with respect to PGA at the ground surface. Use the attenuation relationship same as that for example 1.2 and assume it to be valid for the rock bed. For microzination, the region is divided into 4 subareas. Determine the design PGA for the centre of each sub region given that  $M_{max}$  for point and line sources are 7 and 8 respectively.

**Figure 1.31**

- 1.9. A site is 100km away from a point source of earthquake. The seismicity of the region is represented by the following recurrence relationship

$$\log \lambda_m = 3.0 - 0.75M$$

The attenuation relationship for the region is given by

$$\ln PGA(g) = -4.141 + 0.868M - 1.09 \ln [R + 0.0606 \exp(0.7M)] \quad \sigma_{\ln PGA} = 0.37$$

- a) What are the probabilities that at least one earthquake which will produce PGA greater than 0.2g in 50 years and 150 years period?
- b) What is the probability that exactly one earthquake which will produce PGA greater than 0.2g in 100 years period?
- c) Determine the PGA that would have a 10% of probability of exceedance in 50 years.
- 1.10. The site shown in Figure 1.31 is assumed to have only the point source. The histogram for the magnitude of the earthquake is taken to be the same as that shown in Figure 1.30. Using the same attenuation (for rock bed) and seismic occurrence (source-1) relationships (for exercise problem 1.7), make a probabilistic microzonation of the area in terms of PGA for 10% probability of exceedance. Assume minimum magnitude of earthquake as 4.
- 1.11. Seismic risk of a region with its epicentral distance measured from its centre as 100km is given by Equation 1.40 in which  $F_{ms}(M_i)$  is given by Equation 1.37. Make a microzonation of the region for its 4 sub regions shown in Figure 1.32 for the probability of exceedance of PGA=0.3g at the ground surface. Given  $M_0 = 4$  and  $M_u = 8.5$ ;  $\beta = 1$ . Assume the empirical equations to be valid for the rock bed.

**Figure 1.32**

- 1.12. State True (T) or False (F) for the following statements
- (i) Earthquake may occur due to
- |                                    |                           |
|------------------------------------|---------------------------|
| (a) interplate movement            | (b) intraplate movement   |
| (c) rupture of plates at the fault | (d) underground explosion |
| (e) volcanic eruption              |                           |
- (ii) Movement of the plates during earthquake
- |                                       |                               |
|---------------------------------------|-------------------------------|
| (a) is necessarily a strike slip      | (b) is necessarily a dip slip |
| (c) could be a combination of the two |                               |
- (iii) Fastest wave is
- |               |                  |                   |            |
|---------------|------------------|-------------------|------------|
| (a) love wave | (b) primary wave | (c) rayleigh wave | (d) S-wave |
|---------------|------------------|-------------------|------------|

- (iv) Movement of soil particles is at right angles to the direction of wave propagation for  
(a) primary wave      (b) rayleigh wave      (c) polarized wave
- (v) Subduction is a process which is responsible for  
(a) Formation of ocean  
(b) Formation of mountains  
(c) Mid oceanic ridges
- (vi) Moho discontinuity is the name of  
(a) discontinuity between inner core and outer core of the earth  
(b) discontinuity between the outer core and the mantle  
(c) discontinuity between the crust and the mantle
- (vii) Surface waves are  
(a) largest for shallow focal depth  
(b) increased with depth  
(c) almost periodic at large epicentral distance  
(d) having short period
- (viii) Richter magnitude  
(a) is the same as moment magnitude  
(b) is the same as body magnitude  
(c) can be negative  
(d) is open ended
- (ix) Magnitude is  
(a) exponentially related to the PGA  
(b) linearly related to the PGA  
(c) directly related to the intensity  
(d) directly related to the energy release
- (x) Attenuation relationship depends upon  
(a) local geological condition      (b) path of wave travel  
(c) soil condition      (d) epicentral distance
- (xi) One dimensional wave propagation through soil from rock bed provides  
(a) amplified PGA  
(b) amplified ground displacement  
(c) changed frequency contents

- (d) lower value of PGA for strong rock bed motion in soft soil
- (xii) One dimensional wave propagation
  - (a) is valid for ridges
  - (b) is valid for boundaries of the valleys
  - (c) is valid for the centre of basins
  - (d) is valid for highly non homogenous soil mass with many disjointed layers in different directions
- (xiii) Both arrival of earthquake and magnitude of earthquake may be modeled as
  - (a) lognormal model
  - (b) poisson model
  - (c) exponentially decaying model
- (xiv) Seismic hazard of a site
  - (a) primarily depends upon its epicentral distances from sources
  - (b) primarily depends upon both epicentral distances and magnitudes of earthquakes that occurred at the sources
  - (c) depends largely on soil amplification
  - (d) does not depend on attenuation low
- (xv) Ground damage in an earthquake can be
  - (a) faulting
  - (b) fissures
  - (c) liquefaction
- (xvi) How many seismograph station are needed to locate the epicenter of an earthquake
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
- (xvii)  $M_b$  is also known as Richter's magnitude.
- (xviii) Moment magnitude can be estimated from fault area only.
- (xix) Attenuation relationships are developed based on theoretical analysis.
- (xx) Seismic hazard curve is used for deterministic seismic hazard assessment.
- (xxi) Love waves are slower than Raleigh waves and introduce retrograde elliptical motion.
- (xxii) Earthquake intensity is usually higher at the epicentre and in loosely consolidated soil.

**Take the relevant figures from the slides or from the reference book**

## Module 1 – Seismology

### Exercise Solution:

1.4. Refer to the exercise problem 1.4.

#### **ERRATA FOR THE TEXT BOOK**

pp 36, Exercise Problem 1.10:

(i) (for Exercise problem 1.4) should be (for Exercise problem 1.7)

(ii) “in terms of PGA for 10% probability of exceedance” should be “in terms of PGA for 10% probability of exceedance in 1year”.

Using equations 1.14 (b-d), the mean values of the log of surface rupture length ( $L$ ), rupture area ( $A$ ) and maximum surface displacement ( $D$ ) for  $M_w = 7.5$  are

$$\text{Log } \bar{L} = 0.69M_w - 3.22 = 1.955; \quad \bar{L} = 90.16\text{km}$$

$$\text{Log } \bar{A} = 0.91M_w - 3.49 = 3.335; \quad \bar{A} = 2162.72\text{km}^2$$

$$\text{Log } \bar{D} = 0.82M_w - 5.46 = 0.69; \quad \bar{D} = 4.897\text{m}$$

Standard normal variants for  $L = 90\text{km}$ ,  $A = 60\text{km}^2$  and  $D = 15\text{m}$  are

$$Z_L = \frac{\text{Log } 90 - \text{Log } \bar{L}}{\sigma_{\text{Log}L}} = \frac{\text{Log } 90 - 1.955}{0.22} = -3.44 \times 10^{-3}$$

$$Z_A = \frac{\text{Log } 60 - \text{Log } \bar{A}}{\sigma_{\text{Log}A}} = \frac{\text{Log } 60 - 3.335}{0.22} = -2.32 \quad 0.24$$

$$Z_D = \frac{\text{Log } 15 - \text{Log } \bar{D}}{\sigma_{\text{Log}D}} = \frac{\text{Log } 15 - 0.69}{0.42} = 1.157$$

From the table of standard normal variants, probabilities of exceedance of  $L = 90\text{km}$ ,  $A = 60\text{km}^2$  and  $D = 15\text{m}$  are 50.12%, 98.9% and 12.3% respectively. 99.9%

1.5. Refer to the exercise problem 1.5 and Figure 1.28.

Minimum distances of the line sources from the site and the corresponding  $M_{\text{max}}$  are shown in Table 1.5. In the same table, expected maximum PGAs at the site due to the earthquakes at the line sources are calculated using Equation 1.22

Table 1.5: Expected maximum value of PGA at the site

Line Source	Source to site distance (km)	$M_{\max}$	PGA (g)
1	30.41	6.8	0.216
2	11.03	5.5	0.153
3	16.74	7	0.426

The expected mean value of the PGA at the site corresponds to the maximum ((value)) of the PGAs as obtained above, and is equal to 0.426g.

1.6. Refer to the exercise problem 1.6 and Figure 1.29.

The source to site distance for the point source = 19.2km; minimum distance of the area source to site = 12.36km. Using Equation 1.22, the PGA values at the site for the point source and the area source are 0.25g and 0.8g respectively.

The anticipated mean value of the PGA at the site is therefore 0.8g.

The standard variant for PGA = 0.2g is

$$Z = \frac{\ln(0.8 \times 981) - \ln(196.2)}{0.57} = 2.43$$

From the standard normal variant table, probability of exceedance of PGA = 0.2g is 0.75%

1.7. Refer to the exercise problem 1.7 and Figure 1.30.

The mean rates of exceedance of magnitude 4 for each of the sources are:

$$\text{Source 1} \quad \lambda_{m1} = 10^{4-0.7(4)} = 15.85$$

$$\text{Source 2} \quad \lambda_{m2} = 10^{3-0.75(4)} = 1$$

The seismic hazard curve is developed by finding the mean annual rate of exceedance of PGAs (0.01g, 0.05g, 0.1g, 0.2g and 0.3g) and plotting them against the PGA values. Since probability histogram of the epicentral distance for the source 1 is unity, the computations are first shown for source 2. For source 2,  $P[M=4]=0.7$ ;  $P[R=10]=0.15$ . The mean value of PGA for this combination of M and R is given by

$$\begin{aligned}\ln PGA(gals) &= 6.74 + 0.859M - 1.80\ln(R + 25) \\ &= 6.74 + 0.859(4) - 1.80\ln(35) \\ PGA &= 43.65gals\end{aligned}$$

Standard normal variant Z for 0.01g (9.81 gals) is

$$Z = \frac{\ln(9.81) - \ln(43.65)}{\sigma_{\ln PGA}} = \frac{\ln(9.81) - \ln(43.65)}{0.57} = -2.62$$

From the standard variant table, the probability of exceedance of PGA=0.01g is

$$P_{0.01g} = 99.56\%$$

Annual rate of exceedance of PGA=0.01g is given by

$$\begin{aligned}\lambda_{0.01g} &= \lambda_{m2} P_{0.01g} P(M=4) P(R=10) \\ &= (1)(0.9956)(0.7)(0.15) = 0.1045\end{aligned}$$

Similarly calculations are carried out for Z corresponding to PGAs 0.05g, 0.1g, 0.2g and

0.3g and  $\lambda_{0.05g}$ ,  $\lambda_{0.1g}$ ,  $\lambda_{0.2g}$  and  $\lambda_{0.3g}$  are determined for M=4 and R=10

The values obtained are:

$$P_{0.05g} = 42.07\% ; P_{0.1g} = 7.78\% ; P_{0.2g} = 0.41\% ; P_{0.3g} = 0.041\% \quad \text{and} \quad \lambda_{0.05g} = 0.044 ;$$

$$\lambda_{0.1g} = 0.0082 ; \lambda_{0.2g} = 4.3 \times 10^{-4} ; \lambda_{0.3g} = 4.3 \times 10^{-5}$$

Values of  $\lambda$  for different combinations of M and R can be determined using the same procedure.

Values for  $\lambda_{0.05g}$  for different combinations of M and R for source zone-2 are shown in

Table 1.6 below.

Table 1.6: Value of  $\lambda_{0.05g}$  for different combinations of M and R

<b>R(km)</b> <b>M</b>	<b>10</b>	<b>15</b>	<b>20</b>	<b>25</b>	<b>30</b>
<b>4</b>	0.044	0.074	0.056	0.022	$3.6 \times 10^{-3}$
<b>5</b>	$9.75 \times 10^{-3}$	0.023	0.025	0.014	$3.25 \times 10^{-3}$
<b>6</b>	$6.73 \times 10^{-4}$	$1.78 \times 10^{-3}$	$2.2 \times 10^{-3}$	$1.5 \times 10^{-3}$	$4.12 \times 10^{-4}$
<b>7</b>	$1.68 \times 10^{-5}$	$4.48 \times 10^{-5}$	$5.6 \times 10^{-5}$	$3.93 \times 10^{-5}$	$1.12 \times 10^{-5}$

From the table,  $\sum \lambda_{0.05g} = 0.281$

In this manner,  $\sum \lambda_{0.01g}$ ,  $\sum \lambda_{0.1g}$ ,  $\sum \lambda_{0.2g}$  and  $\sum \lambda_{0.3g}$  are obtained as 1.12, 0.205, 0.048 and 0.012 respectively.

For source zone-1 and R=100km different values of  $\lambda_{0.05g}$  are obtained as shown in Table 1.7 below.

Table 1.7: Values of  $\lambda_{0.05g}$  for different combinations of M and R=100km

<b>M</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
$\lambda_{0.05g}$	$1.45 \times 10^{-4}$	0.015	0.03	$4.03 \times 10^{-3}$

Following values are thus obtained for source zone-1

$$\sum \lambda_{0.05g} = 0.281, \sum \lambda_{0.01g} = 1.89, \sum \lambda_{0.1g} = 0.03, \sum \lambda_{0.2g} = 5.05 \times 10^{-3}, \sum \lambda_{0.3g} = 1.5 \times 10^{-4}$$

Therefore,

$$\lambda_{0.01g} = \sum \lambda_{0.01g} \Big|_{source1} + \sum \lambda_{0.01g} \Big|_{source2} = 1.89 + 1.12 = 3.01$$

$$\lambda_{0.05g} = \sum \lambda_{0.05g} \Big|_{source1} + \sum \lambda_{0.05g} \Big|_{source2} = 0.023 + 0.273 = 0.296$$

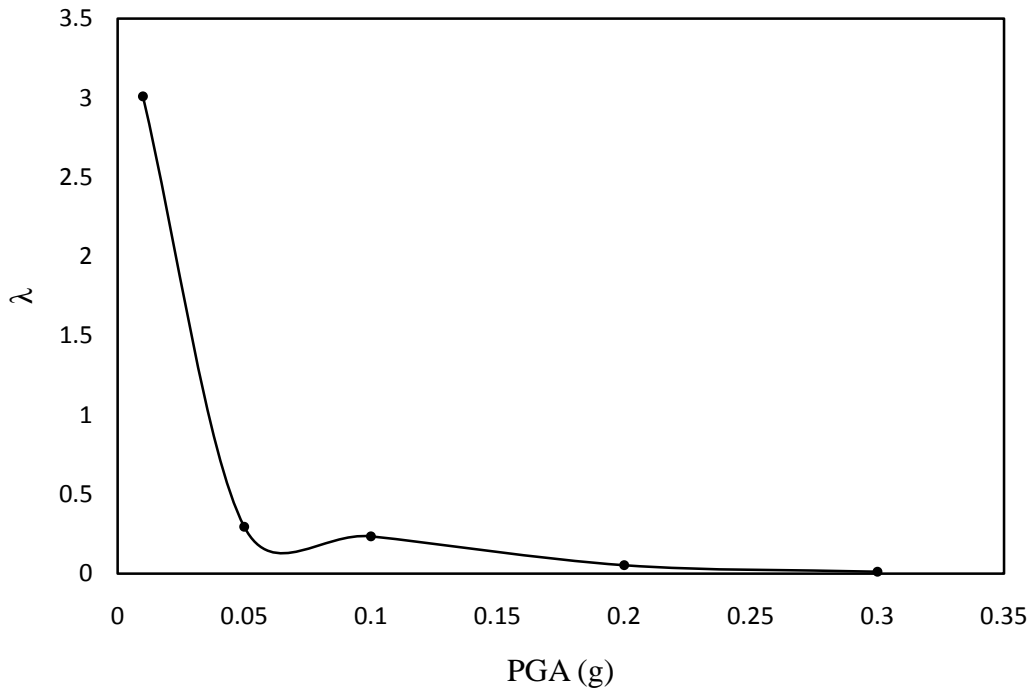
$$\lambda_{0.1g} = \sum \lambda_{0.1g} \Big|_{source1} + \sum \lambda_{0.1g} \Big|_{source2} = 0.03 + 0.205 = 0.235$$

$$\lambda_{0.2g} = \sum \lambda_{0.2g} \Big|_{source1} + \sum \lambda_{0.2g} \Big|_{source2} = 5.05 \times 10^{-3} + 0.048 = 0.053$$

$$\lambda_{0.3g} = \sum \lambda_{0.3g} \Big|_{source1} + \sum \lambda_{0.3g} \Big|_{source2} = 1.5 \times 10^{-4} + 0.012 = 0.012$$

The variation of  $\lambda$  with PGA is shown in Fig.1.33.





**Figure 1.33** Seismic hazard curve (variation of  $\lambda$  with PGA)

Refer to the exercise problem 1.8.

Sub regions with AF=3, 4, 3.5 and 2.5 are numbered as 1, 2, 3 and 4 respectively. PGAs at the bed rock for the centers of the sub regions are calculated using the minimum source to site distance (R) and the attenuation law given by Equation 1.22

Table 1.8: PGAs at the bed rock for sources 1 and 2

Sub Region	R (km) Source 1	R (km) Source 2	PGA Source 1 (g)	PGA Source 2 (g)
<b>1</b>	45.62	102.01	0.166	0.136
<b>2</b>	95.29	422.5	0.063	0.0141
<b>3</b>	55.5	897.5	0.131	0.0042
<b>4</b>	100.41	447.5	0.0589	0.0013

Corresponding PGAs at the ground surface and the design PGAs for the sub regions are shown below

Table 1.9: PGAs at the ground surface and the design PGA

Sub Region	PGA Source 1	PGA Source 2	PGA Design
---------------	-----------------	-----------------	---------------

	(g)	(g)	(g)
<b>1</b>	0.498	0.408	0.498
<b>2</b>	0.252	0.0564	0.252
<b>3</b>	0.4585	0.0147	0.4585
<b>4</b>	0.14725	0.0033	0.14725

Refer to the exercise problem 1.9.

Value of M corresponding to a PGA = 0.2g is obtained from the following equation:

$$\ln(0.2) = -4.141 + 0.868M - 1.09 \ln[100 + 0.0606 \exp(0.7M)]$$

M is determined by finding residue R to be zero for different trial values of M.

$$\text{Residue (R)} = \ln(0.2) + 4.141 - 0.868M + 1.09 \ln[100 + 0.0606 \exp(0.7M)]$$

R becomes almost zero for M=9.07

$$\log \lambda_m = 3.0 - 0.75M = -3.8; \quad \lambda_m = 1.576 \times 10^{-4}$$

$$P(N \geq 1) = 1 - e^{-50\lambda_m} = 7.85 \times 10^{-3}$$

$$P(N \geq 1) = 1 - e^{-150\lambda_m} = 0.0234$$

$$P(N = 1) = \lambda_m t e^{-\lambda_m t} = \lambda_m (100) e^{-100\lambda_m} = 0.0155$$

Annual rate of exceedance of PGA that would have 10% probability of exceedance in 50 yrs is

$$\lambda_m = \frac{\ln[1 - p(N \geq 1)]}{t} = \frac{\ln(1 - 0.1)}{50} = 0.0021$$

Corresponding value of M is given by  $\log(0.0021) = 3.0 - 0.75M$ ;  $M = 7.57$

The PGA value is obtained as

$$\ln PGA = -4.141 + 0.868(M) - 1.09 \ln[100 + 0.0606 \exp(0.7M)]$$

$$PGA = 0.066g$$

Refer to the exercise problem 1.10 and Figure 1.31.

From problem 1.7,  $\lambda_m$  for the point source is  $\log \lambda_m = 4 - 0.7m$ ;  $\lambda_m = 15.85$

From problem 1.8, epicentral distances for the sub regions are  $R_1 = 45.62\text{km}$ ,  $R_2 = 95.29\text{km}$ ,  
 $R_3 = 55.5\text{km}$ ,  $R_4 = 100.41\text{km}$ .

Table 1.10: Mean values of PGA (*gals*) for different combinations of M and R

<b>R \ M</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
$R_1$	12.3	29.08	68.7	162.39
$R_2$	4.71	11.13	26.31	62.18
$R_3$	9.78	23.10	54.05	127.79
$R_4$	4.39	10.38	24.53	57.97

Standard normal variant Z for 0.01g for PGA = 12.3 gals (R = 45.62; M=4) is

$$Z = \frac{\ln(9.81) - \ln(12.3)}{0.57} = -0.396$$

Probability of exceedance  $P_{0.01g} = 65.35\%$

Annual rate of exceedance of PGA = 0.01g is given by  $\lambda_{0.01g} = \lambda_m P_{0.01g} P(M=4) = 7.768$  in which P(M=4) is taken from Figure 1.30(b).

In a similar way,  $\lambda_{0.05g}$ ,  $\lambda_{0.1g}$ ,  $\lambda_{0.2g}$  and  $\lambda_{0.3g}$  are obtained for the PGA corresponding to R =45.62; M = 4).

$\lambda$  values corresponding to all values of the PGAs shown in Table 1.10 are computed. For example, the computed values of  $\lambda_{0.05g}$  are shown in the following table.

Table 1.11: Values of  $\lambda_{0.05g}$  for different combinations of R and M

<b>R \ M</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	$\sum \lambda_{0.05g}$
$R_1$	0.092	0.283	$9.1 \times 10^{-3}$	$6.14 \times 10^{-3}$	0.39
$R_2$	$2.35 \times 10^{-4}$	$7.37 \times 10^{-3}$	$1.74 \times 10^{-3}$	$4.14 \times 10^{-3}$	0.0135
$R_3$	0.0277	0.148	$7.15 \times 10^{-3}$	$3.35 \times 10^{-3}$	0.1862
$R_4$	1.389	$5.15 \times 10^{-3}$	$1.4 \times 10^{-3}$	$3.8 \times 10^{-3}$	1.4

Preparing such tables,  $\sum \lambda_{0.01g}$ ,  $\sum \lambda_{0.05g}$ ,  $\sum \lambda_{0.1g}$ ,  $\sum \lambda_{0.2g}$  and  $\sum \lambda_{0.3g}$  are determined for each sub region and the plots of  $\sum \lambda$  vs. PGA are shown in Figure 1.34. From the figures, PGAs corresponding to 10% probability of exceedance in one year can be obtained. These PGAs are multiplied by the corresponding values of AF to obtain the PGA values on the ground surface

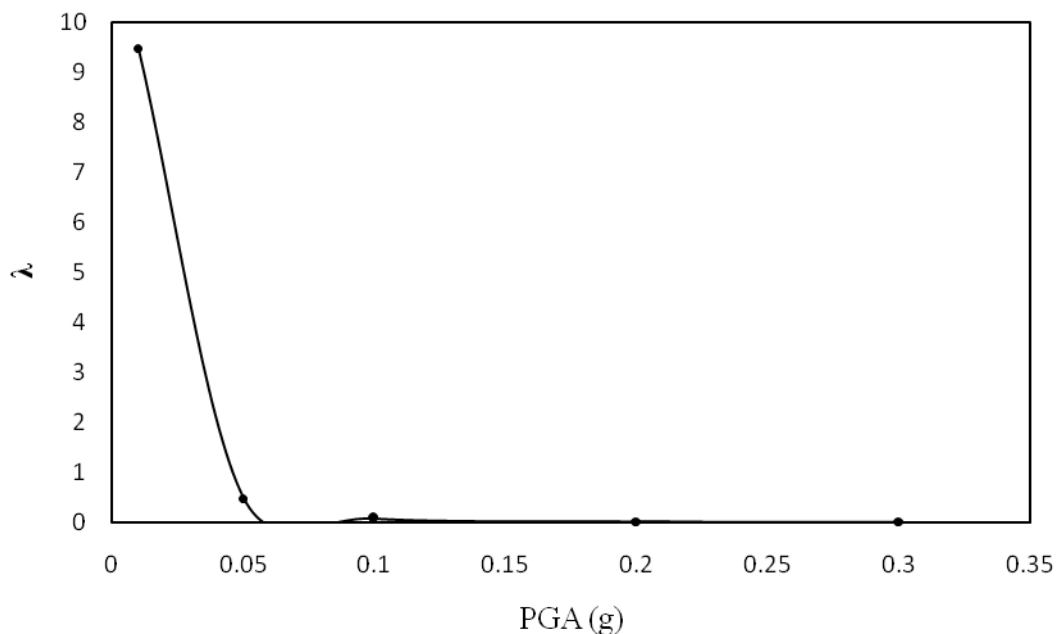
and are used to micro zone the area as shown below. Since the ordinates of the figures become extremely small at PGA= 0.05g and above, Table 1.12 provides the values of ordinates to plot the curves. Interpolated values from the table are used to obtain PGAs corresponding to  $\lambda = 0.1$ .

Table 1.12 Variation of  $\lambda$  with PGA for different regions

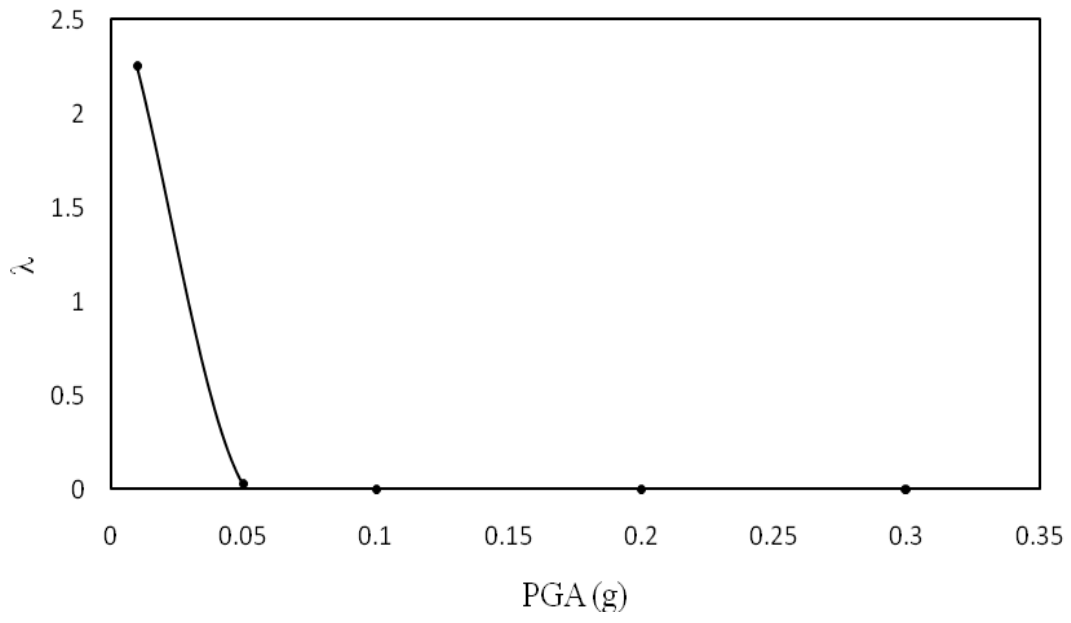
PGA	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>
0.01g	9.467	2.253	7.501	1.92
0.05g	0.475	0.029	0.25	0.023
0.1g	0.066	0.0028	0.0322	0.00217
0.2g	0.00713	0.000168	0.00312	0.000119
0.3g	0.00166	0.000022	0.000655	0.0000142

Table 1.13: Microzonation with respect to PGA (g) corresponding to 10% probability of exceedance in one year

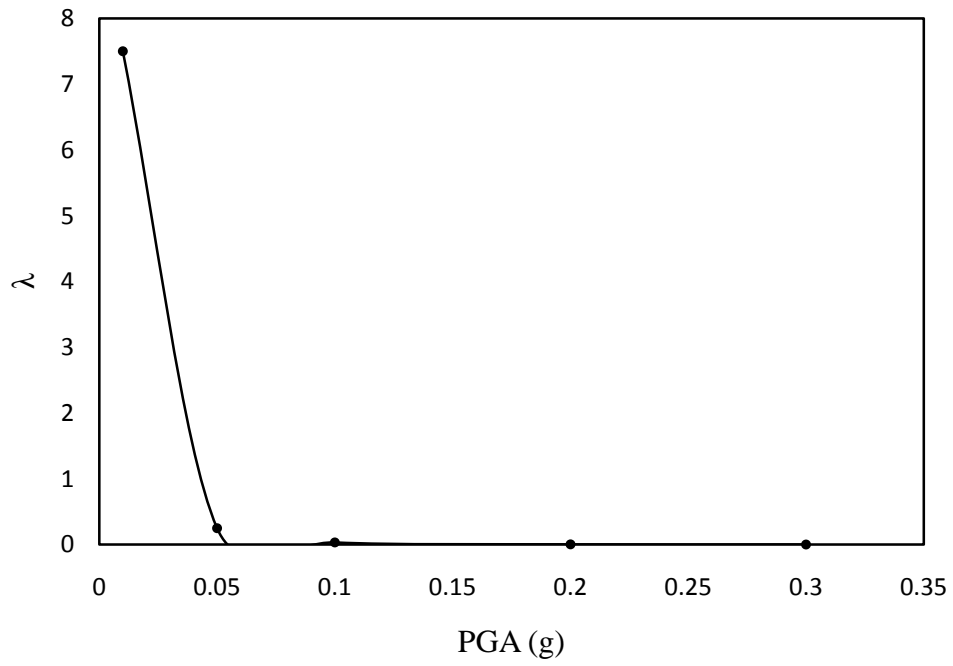
0.2685	0.1948
0.28	0.12



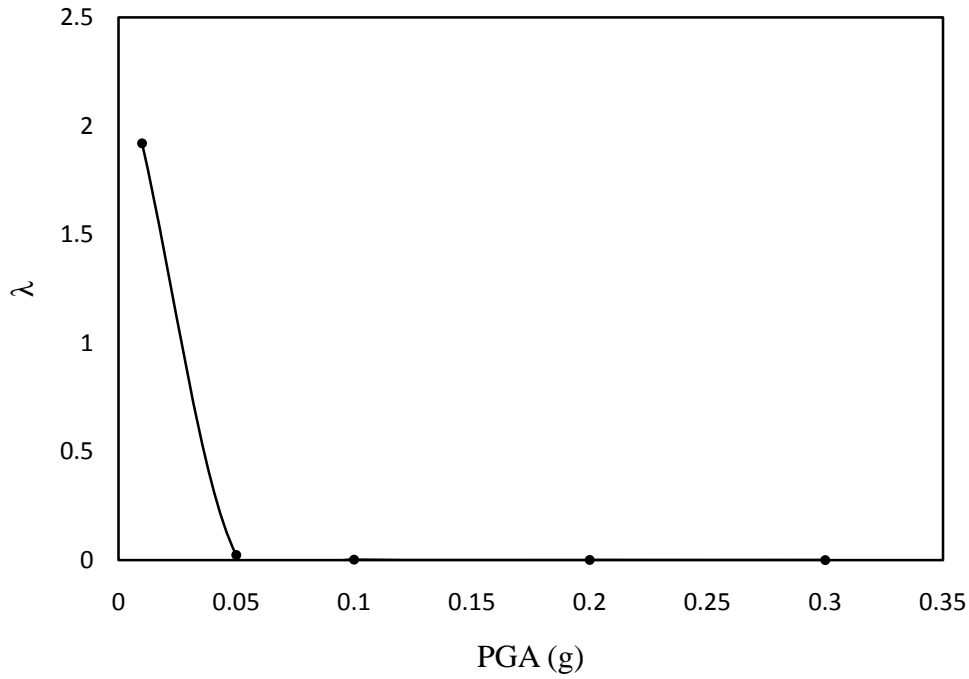
(a)



(b)



(c)



(d)

**Figure 1.34** Seismic hazard curves for the sites (a) site  $R_1$  (b) site  $R_2$  (c) site  $R_3$  (d) site  $R_4$

Refer to the exercise problem 1.11 and Figure 1.31.

Epicentral distance of the four sub regions are

$$R_1 (AF = 3) = 115.97\text{km}; \quad R_2 (AF = 3.5) = 115.97\text{km}$$

$$R_3 (AF = 4) = 86.3\text{km}; \quad R_4 (AF = 2.5) = 86.3\text{km}$$

The accelerations at the bed rock for the regions are

$$a_1 = \frac{0.3g}{3} = 0.1g; \quad a_2 = \frac{0.3g}{3.5} = 0.86g;$$

$$a_3 = \frac{0.3g}{4} = 0.075g; \quad a_4 = \frac{0.3g}{2.5} = 0.12g$$

Using equations 1.40 (a-c),

$$r_1 = a_1 R_1 = 11.597; \quad r_2 = a_2 R_2 = 99.73;$$

$$r_3 = a_3 R_3 = 6.47; \quad r_4 = a_4 R_4 = 10.356$$

$$\text{Accordingly, } m_1 = \frac{1}{1.15} \ln \frac{r_i}{1.83} = 1.605$$

Similarly,  $m_2 = 3.48$ ,  $m_3 = 1.098$ ,  $m_4 = 1.51$

$$F_{MS}(m_1) = -10.08 \quad F_{MS}(m_2) = -0.689$$

$$F_{MS}(m_3) = -17.4 \quad F_{MS}(m_4) = -11.185$$

Probability of exceedance of PGA=0.3g for the four regions are shown below.

Table 1.14: Microzonation in terms of the probabilities of exceedance for PGA=0.3g

11.08%	1.689%
18.4%	12.185%

