### Module 4 : Deflection of Structures

### Lecture 7 : Tutorial Problems

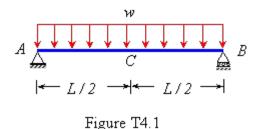
#### Objectives

In this course you will learn the following

• Some tutorial problems related to this module.

# **TUTORIAL PROBLEMS**

T4.1 Using moment area method, determine the end slope and deflection of the mid-span point *C* in the beam shown in Figure T4.1.



T4.2 Determine the slope and deflection at the internal hinge of the beam shown in the Figure T4.2.

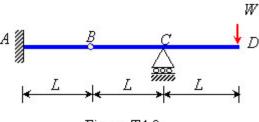
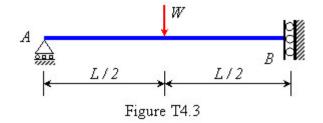
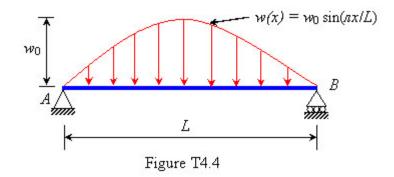


Figure T4.2

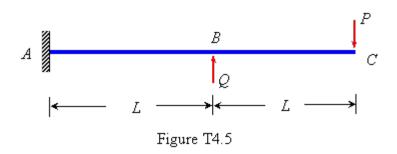
T4.3 Determine the slope at A and deflection of B of the beam shown in Figure T4.3 using the moment area method.



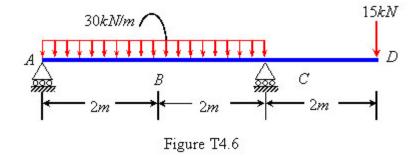
T4.4 Find the maximum slope and deflection of the simply supported beam shown in Figure T4.4 using moment area method.



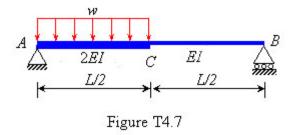
T4.5 Using conjugate beam method determine the ratio P / Q for the beam shown in Figure T4.5 if (i) slope at C is zero and (ii) deflection at C is zero.



T4.6 Determine the deflection at *B* for the beam using conjugate beam method. Take EI= $7 \times 10^4$  kNm<sup>2</sup>.



T4.7 Determine the mid-span deflection for the beam using conjugate beam method.



T4.8 Determine the expression for the slope and deflection of the free end of the cantilever beam shown in the Figure T4.8

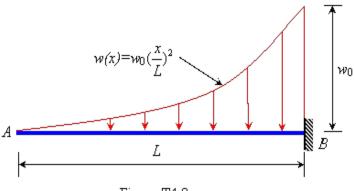


Figure T4.8

T4.9 Determine the horizontal and vertical displacement of joint *C* of the pin-jointed frame as shown in Figure T4.9. All the members of the frame have uniform axial rigidity *(AE)*. Use the unit load method and verify by the strain energy method.

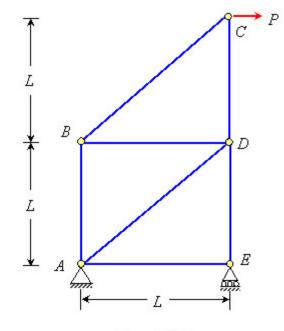


Figure T4.9

T4.10 Determine the vertical displacements of joint D and E of the pin-jointed frame as shown in Figure 4.10. All the members of the frame have uniform axial rigidity (AE). Use the unit load method and verify by the strain energy method.

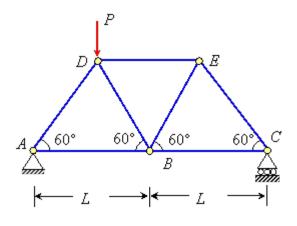


Figure T4.10

T4.11 Determine the vertical displacement of joint C and horizontal displacement E of the pinjointed frame as shown in Figure T4.11. All the members of the frame have uniform axial rigidity (AE). Use the unit load method and verify by the strain energy method.

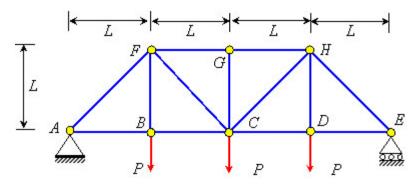


Figure T4.11

T4.12 Determine the horizontal and vertical displacements of joints C and D of the pin-jointed frame as shown in Figure T4.12. All the members of the frame have uniform axial rigidity (*AE*). Use the unit load method and verify by the strain energy method.

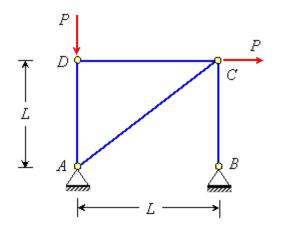


Figure T4.12

T4.13 Determine the vertical displacement of joint *C* and horizontal displacement of joint *D* of the pin-jointed frame as shown in Figure T4.13. All the members of the frame have uniform axial rigidity,  $AE=15\times10^3$  kN. Use the unit load method and verify by the strain energy method.

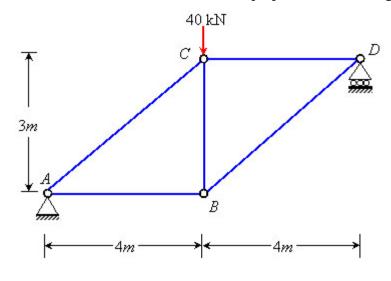
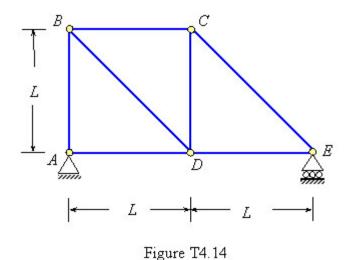


Figure T4.13

T4.14 Determine the vertical displacement of joint *D* if member *BC* of the pin-jointed frame as shown in Figure T4.14 is long by an amount  $\Delta$  from the original length *L*. All the members of the frame have same *AE* value.



T4.15 Using unit load method and strain energy method, determine the deflection and slope of point C of the uniform beam shown in Figure T4.15.

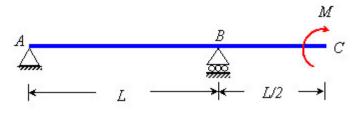


Figure T4.15

T4.16 Using unit load method and strain energy method, determine the deflection at the center of the beam shown in Figure T4.16 under the distributed load *w*.

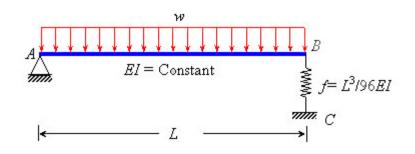
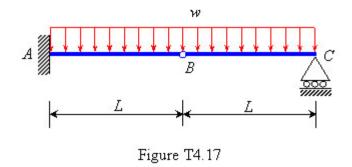


Figure T4.16

T4.17 Using unit load method and strain energy method, determine the deflection and rotation of the point B of the beam shown in Figure T4.17. The beam is carrying a uniformly distributed load, w over the entire length.



T4.18 Using unit load method and strain energy method, determine the deflection under the load *W* and horizontal displacement of roller at *D* of the rigid-jointed plane frame shown in Figure T4.18.

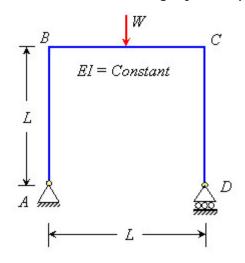


Figure T4.18

T4.19 Using unit load method and strain energy method, determine the deflection at the center of AB and horizontal displacement of roller at C of the rigid-jointed plane frame shown in Figure T4.19.

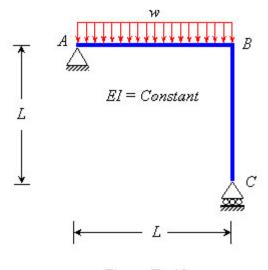
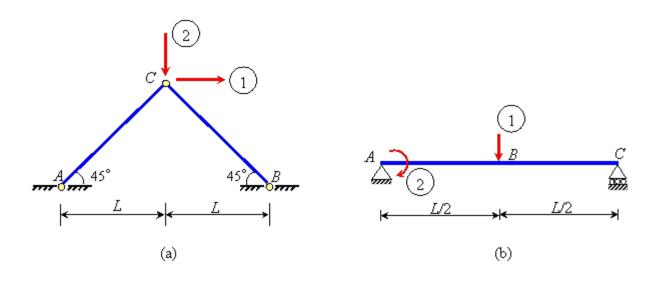


Figure T4.19

T4.20 Verify the Maxwell-Betti Law of reciprocal displacements for the structures in Figure T4.20. The required direction 1 and 2 are marked on the structures.



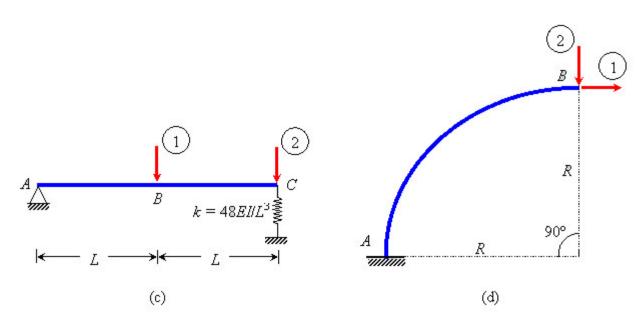


Figure T4.20

# Recap

In this course you have learnt the following

• You have learned some tutorial problems related to this module.

# Answers of tutorial problems

**T4.1** 
$$\theta_A = \frac{wL^3}{24EI}$$
 and  $\Delta_C = \frac{5wL^4}{384EI}$ 

**T4.2** 
$$\Delta_B = \frac{WL^3}{3EI}$$

**T4.3** 
$$\theta_A = \frac{3WL^2}{8EI}$$
 and  $\Delta_B = \frac{11WL^3}{48EI}$ 

**T4.4** 
$$\theta_A = \frac{w_0 L^3}{\pi^3 EI}$$
 and  $\Delta_{\text{max}} = \frac{w_0 L^4}{\pi^4 EI}$ 

**T4.5** (i) 
$$\frac{P}{Q} = \frac{1}{4}$$
 and (ii)  $\frac{P}{Q} = \frac{5}{16}$ 

**T4.6** 
$$\Delta_B = 1 mm (\downarrow)$$

$$\Delta_C = \frac{1536EI}{1536EI}$$

**T4.8** 
$$\Delta_A = \frac{w_0 L}{72EI} \quad \text{and} \quad \theta_A = \frac{w_0 L^3}{60EI}$$

**T4.9** 
$$\Delta_{CH} = \frac{(7 + 4\sqrt{2})PL}{AE} (\rightarrow)$$
$$\Delta_{CV} = \frac{3PL}{AE} (\downarrow)$$

**T4.16** 
$$\Delta = \frac{wL^4}{64EI}$$

**T4.17** 
$$\Delta_{B} = \frac{7wL^{4}}{24EI}$$
**T4.18** 
$$\Delta_{W} = \frac{WL^{3}}{48EI}$$

$$\Delta_{DH} = \frac{WL^{3}}{8EI}$$

**T4.19** 
$$\Delta = \frac{5wL^4}{384EI}$$
$$\Delta_{CH} = \frac{wL^4}{24EI}$$

**T4.20** (a)  $\Delta_{12} = \Delta_{21} = 0$  (b)  $\Delta_{12} = \Delta_{21} = \frac{L^2}{16EI}$ (c)  $\Delta_{12} = \Delta_{21} = \frac{L^3}{96EI}$  (d)  $\Delta_{12} = \Delta_{21} = \frac{R^3}{2EI}$