

Example Problem

V is wind velocity with a pdf $f(v) = 1/10$; $0 \leq v \leq 10$. The pressure ω at point is given by $\omega = 0.003 v^2$. Find the expected value of pressure.

Solⁿ :

$$f(v) = \frac{1}{10}; 0 \leq v \leq 10; \omega = 0.003 v^2$$

$$E(\omega) = ?$$

$$E(\omega) = \int_{-\alpha}^{+\alpha} \omega g(\omega) d\omega, \text{ given } \omega = 0.003 v^2 \text{ or } d\omega = 0.006 v dv$$

given, $f(v) = \frac{1}{10}$ & $\omega = 0.003 v^2 \therefore g(\omega)$ will be monotonically increasing function.

$$\begin{aligned} \text{Therefore, } g(\omega) &= f(v) \left| \frac{dv}{d\omega} \right| = \frac{1}{10} \times \frac{0.0274}{\sqrt{\omega}} \\ &= 0.00274(\omega)^{-1/2} \text{ as } \omega = 0.003 v^2 \end{aligned}$$

Example Problem

Contd...

$$\text{or } v = \sqrt{\frac{w}{0.003}} = 18.257\sqrt{w} \text{ or } \frac{dv}{dw} = 18.257 \times \frac{1}{2} \times (w)^{-1/2}$$

$$\text{or } g(w) = \frac{1}{10} \times 18.257 \times \frac{1}{2} \times (w)^{-1/2} = 0.9129 (w)^{-1/2}$$

$$\text{Now, } E(w) = \int_0^{0.3} \left[w \times 0.9129 (w)^{-1/2} \right] dw = 0.9129 \times \left[\frac{w^{3/2}}{3/2} \right]_0^{0.3}$$

$$= 0.9129 \times \frac{2}{3} \times [0.3]^{3/2} \quad \left[\begin{array}{l} 0 \leq v \leq 10 \\ \text{or } 0 \leq 18.257\sqrt{w} \leq 10 \\ \text{or } 0 \leq w \leq 0.3 \end{array} \right]$$

$$= 0.1 \text{ (Ans.)}$$