

Example Problem

Channel width (rectangular) = 2m, Depth = 1m, $Q = 3.0 \text{ m}^3/\text{s}$, Height above datum = 2m. **Compute specific and total energy**

Ans: $A = b \cdot y = 2.0 \cdot 1.0 = 2 \text{ m}^2$

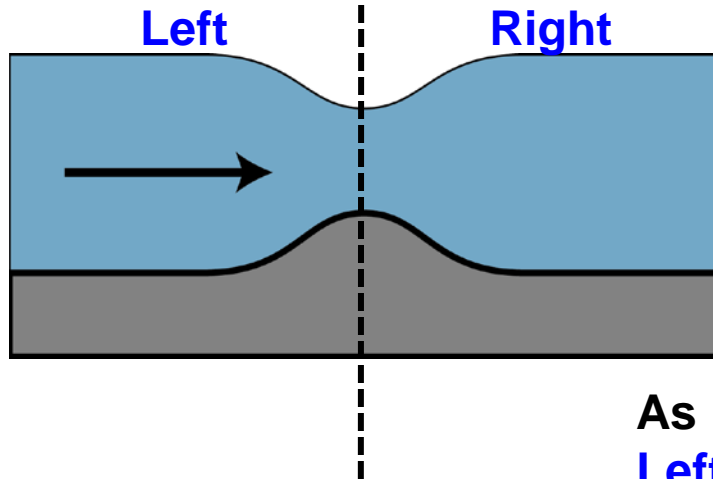
$$\text{Specific energy} = E = y + \frac{Q^2}{2gA^2}$$

$$\text{Total energy} = E = 1 + \frac{3^2}{2 * 9.81 * 2^2}$$

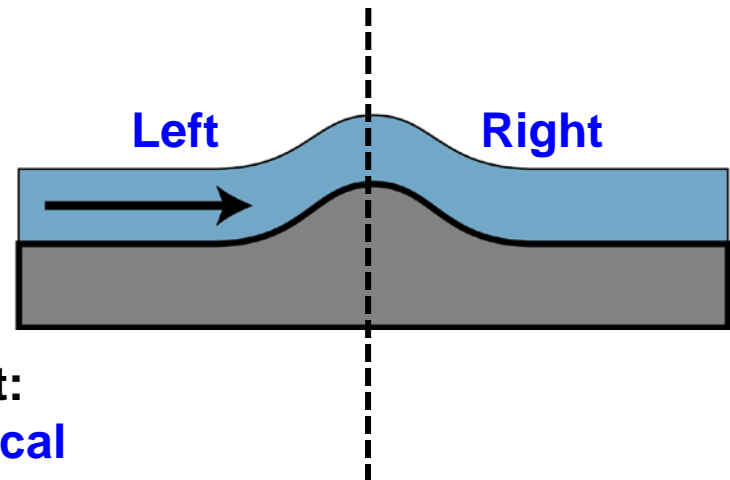
Datum height + specific energy = $2.0 + 1.20 = 3.20 \text{ m}$

Example Problem

Will the flow over a bump be supercritical or subcritical?



or



As it turns out:

Left = subcritical

Right = supercritical

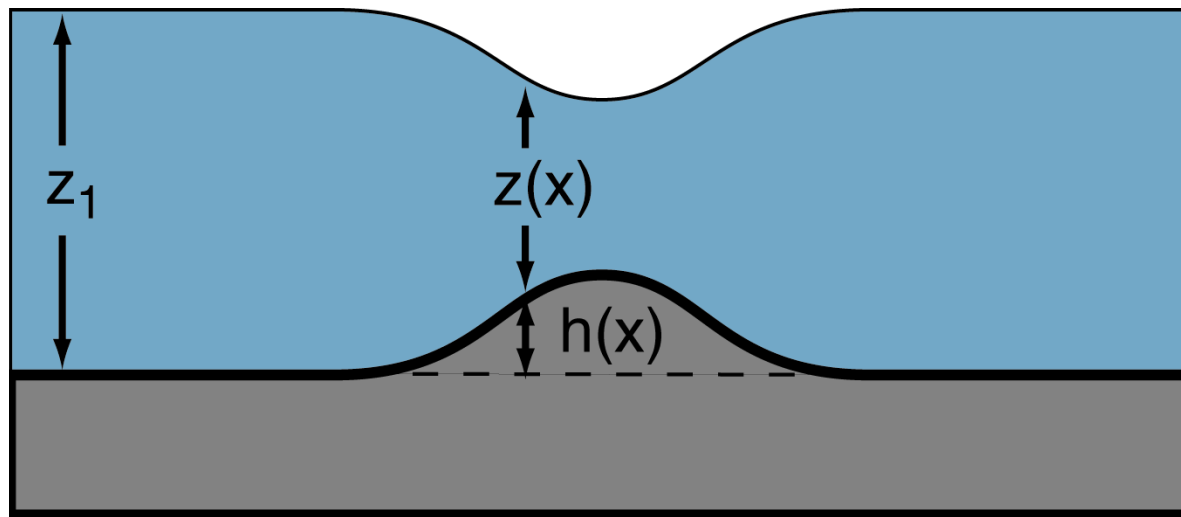
Using the Bernoulli equation for frictionless, steady, incompressible flow along a streamline:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Example Problem

Contd...

Apply Bernoulli equation along free surface streamline ($p=0$):



$$\frac{V_1^2}{2g} + z_1 = \frac{V(x)^2}{2g} + z(x) + h(x)$$

For a channel of rectangular cross-section,

$$Q = V b z$$

Example Problem

Contd...

Substitute $Q = V z b$ into Bernoulli equation:

$$\frac{Q^2}{2gb^2z_1^2} + z_1 = \frac{Q^2}{2gb^2z(x)^2} + z(x) + h(x)$$

To find the shape of the free surface, take the x-derivative:

$$0 = -\frac{Q^2}{wgb^2z(x)^3} \frac{dz}{dx} + \frac{dz}{dx} + \frac{dh}{dx}$$

Solve for dz / dx :

$$\frac{dz}{dx} = \frac{dh/dx}{\left[\frac{Q^2}{gb^2z^3} - 1\right]} = \frac{dh/dx}{\left[\frac{V^2}{gy} - 1\right]} = \frac{dh/dx}{Fr^2 - 1}$$

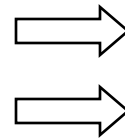
Example Problem

Contd...

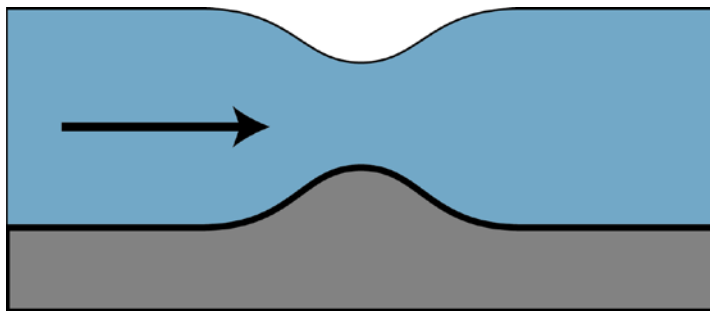
$$\frac{dz}{dx} = \frac{dh/dx}{Fr^2 - 1}$$

Since subcritical: $Fr < 1$
 supercritical: $Fr > 1$

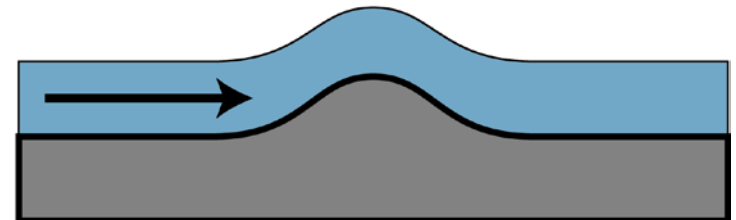
Subcritical flow with $dh/dx > 1$
Supercritical flow with $dh/dx > 1$



$dz/dx < 1$
 $dz/dx > 1$



if flow is subcritical



if flow is supercritical

Exercises

- 1) A rectangular channel 4m wide has a flow discharge of 10.0 m³/s and depth of flow as 2.5 m. Draw specific energy diagram and find critical and alternate depth.
- 2) A triangular channel with side slopes having ratio of 1:1.5 has a discharge capacity of 0.02 m³/s. Calculate:
 - a. critical depth
 - b. E_{min}
 - c. Plot specific energy curve
 - d. Determine energy for 0.25 ft and alternate depth
 - e. Velocity of flow and Froude number
 - f. Calculate required slopes if depths from d are to be normal depths for given flow.