Multiphase Flow

Questions with Answers

Chapter 2

1) With the help of neat sketches discuss the flow patterns observed in vertical and horizontal heated tubes.

   Ans: Refer to Fig 2.3 & Fig 2.4 along with the associated discussion.

2) Also discuss the probable reasons for the differences in flow patterns in a horizontal heated tube as compared to (i) vertical heated tube (ii) horizontal unheated tube.

   Ans: (i) Influence of gravity which leads for asymmetric phase distribution and stratification of the two phases.

   (ii) (a) Departure from hydrodynamic and thermal equilibrium
   (b) Presence of radial temperature profile

3) Gas-liquid stratified flow in a horizontal pipe encounters a vertical T junction. How do you anticipate the flow pattern to change after the T?

   Ans : As the stratified gas-liquid mixture encounters a vertical T, some amount of gas will enter the side branch. This reduces the relative proportion of gas in the main arm after the T and is expected to cause gas-liquid bubbly flow after the two phase mixture encounters the junction.

4) What is film inversion? When is it encountered in two phase flow?

   Ans : The phenomena of film inversion occurs when a two phase mixture under stratified flow encounters a return bend. This is illustrated in Fig. 2.11 (a). Under this condition, the film on the inner walls while travelling through the 180^0 bend flows along the outer wall after the bend. The film inversion of blue kerosene is evident in Fig. 2.11 (a).

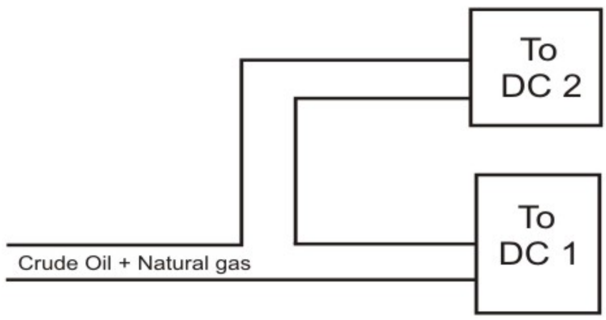
5) With the help of neat sketches describe a mixed flow pattern (neither separated nor dispersed) for (a) gas-liquid flow (b) liquid-liquid flow.

   Ans : Gas-liquid flow – Slug Flow
   Liquid-liquid flow – Three layer flow with lighter liquid on the top, heavier liquid on the bottom and a mixture of droplets of the two liquids in the central portion. Refer to Table 2.1.

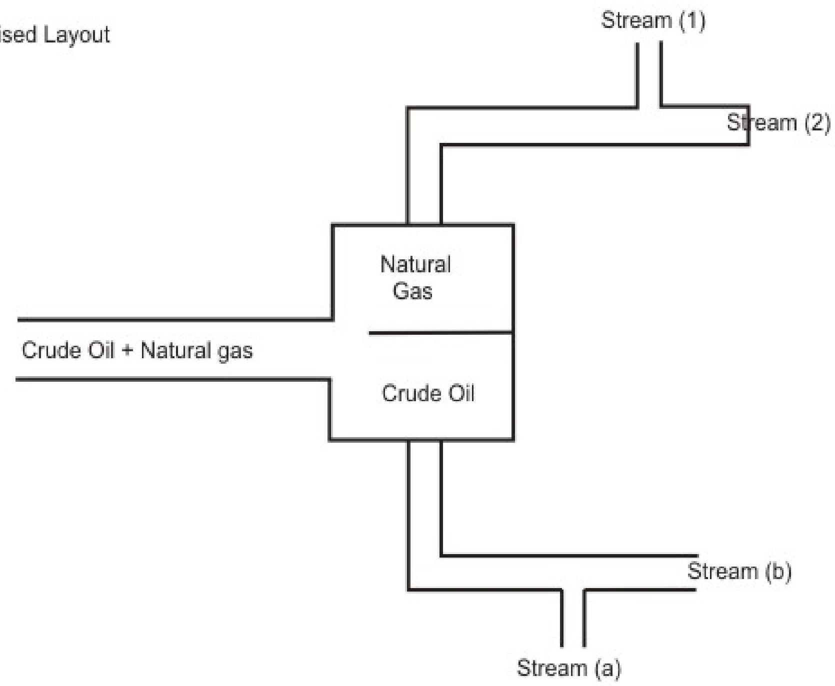
6) A distillation column in a petroleum refinery plant operates with a mixture of crude oil and natural gas as feed which is fed by a pipeline into the feed tray of the column. Due to some reasons, the refinery is provided with an increased supply of raw materials (crude and gas). In order to handle the increased supply, the plant manager had two options – (a) t install a distillation column with a capacity to handle large quantity feed and (b) to install another column along with the existing one, introduce the previous quantity of the feed in the old column and divert the extra quantity to the new column. From cost considerations, the management decided to go for the second option. With that option, they just needed to required quantity of feed into the new column. These installations would cost less and require less time to start the operation. Accordingly, the necessary changes were made. The new column was installed in the plant site and the total feed (crude+gas) was divided into two parts by a T junction and each part was fed into the respective distillation column. It was expected that the plant would run smoothly after this. However, after a few days it was noticed that neither of the two columns were operating property and there was a drastic fall in efficiency of the existing column which was operating fine before the new venture was taken up. You as a new management trainee of the company was called upon to look into the problem since you had a course of multiphase flow in your college. You were asked to investigate the problem and propose a possible solution to it. You found out that the columns were not getting the composition of the feed they were used to handle and this occurs because there is misdistribution of the two phases at the T. You were aware of this feature of the T junction from your knowledge of multiphase flow. Accordingly, you suggested a revised design of the pipeline so that the problem can be alleviated. Present schematics of the original and revised pipeline layout in order to ensure trouble free operation of the plant.

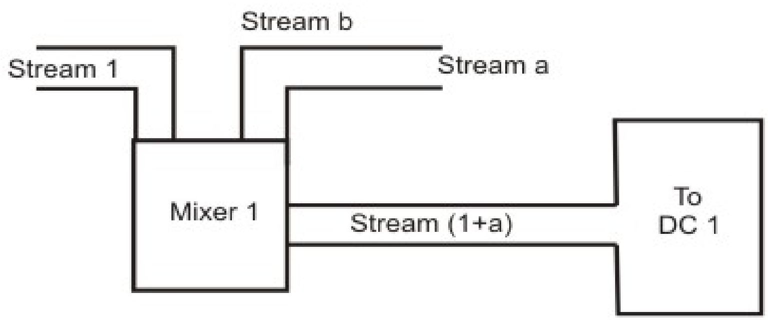
Ans :

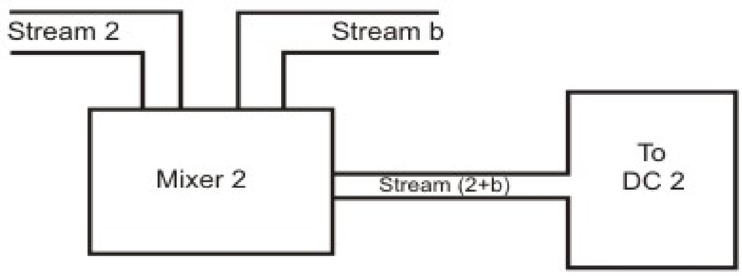
Original Layout



Revised Layout







7. Using relevant flow pattern maps (Baker et al for horizontal tube and Hewitt and Roberts for vertical tube) evaluate the most likely flow pattern occurring for a steam-water system flowing in a 2.54cm diameter (vertical (b) horizontal pipe where the system pressure is 70 Bar, the mass qualities are 1% and 50 % and the mass fluxes are (a) 500 Kg/m2 –sec and (b) 2000 Kg/m2 –sec respectively.

Solution:

<u>For Vertical tube:</u>

<u>**For 1st Case**</u>

$$G = \frac{W}{A} = \frac{W_1 + W_2}{A} = G_1 + G_2$$

Here $G_1 + G_2 = G = 500 kg/m^2 - \sec$

$$\frac{G_2}{G} = x = 1\% = \frac{1}{100} = 0.01$$

$$\therefore G_2 = 0.01G$$

$$\therefore G_2 = 0.01 \times 500 = 5 kg/m^2 - \sec$$

$$\therefore G_1 = 500 - 5 = 495 kg/m^2 - \sec$$

In 70 Bar, we get saturation temperature from stem table = 255.9^0 c and v_g = 0.027382 m^3/kg and v_f = 0.0013514 m3/kg and $\rho_g = 36.520\,\text{kg/m}^3$ and $\rho_g = 739.98\,\text{kg/m}^3$

$$G_1 = \frac{W_1}{A} = \frac{\rho_1 u_1 A_1}{A}$$

$$= \rho_1 u_1 (1-\alpha) = \rho_1 u_{1s} = \rho_1 j_1$$

$$\therefore G_1^2 = \rho_1^2 j_1^2$$

$$\therefore \rho_1 j_1^2 = \frac{G_1^2}{\rho_1}$$

$$or\ \rho_f j_f^2 = \frac{G_f^2}{\rho_f}$$

$$= \frac{(495)^2 (kg/m^2 - \sec)^2}{739.98\, kg/m^3} = 331.1238142\, kg/\sec - m$$

Similarly

$$\rho_g j_g^2 = \frac{G_g^2}{\rho_g} = \frac{(5)^2 \left(kg/m^2 - \text{sec}\right)^2}{36.520\, kg/m^3}$$
$$= 0.684556407\ kg/\text{sec}^2 - m$$

Therefore from figure we get flow pattern is Bubble – slug.

For 2nd Case:

For G = 2000 kg/m² –sec and x = 50% = 0.5 we get

G_2 = xG = 0.5 ×2000 = 1000kg/m²-sec

and

G_1 = (1-x)G = 0.5 ×2000 = 1000 kg/m²-sec

Now
$$\rho_g j_g^2 = \rho_2 j_2^2 = \frac{G_2^2}{\rho_2} = \frac{G_g^2}{\rho}$$
$$= \frac{(1000)^2}{36.520}$$
$$= 27382.2563\, kg/Sec^2 - m$$
$$\text{and}\ \rho_f j_f^2 = \frac{G_f^2}{\rho_f^2} = \frac{(1000)^2}{739.98}$$
$$= 1351.387875\, kg/\text{sec}^2 - m$$

Therefore from figure we get flow pattern is Annular

For Horizontal tube:-

1st case

For pressure 70 bar from figure we get $\lambda = 4.8$ and $\psi = 2.3$ for G = 500 kg/m²-sec and x = 1% = 0.01

$G_2 = G_g$ = 495 kg/m²-sec

$$\therefore G_f \psi = 495 \times 2.3$$
$$= 1138.5\, kg/m^2 - \text{sec}$$

and $\frac{G_g}{\lambda} = \frac{G_2}{\lambda} = \frac{5}{4.8} = 1.04166\, kg/m^2 - \sec$

There from figure we get flow pattern is slug

2nd Case:

For $G = 2000\ kg/m^2$-sec and $x = 50\% = 0.5$

We get $G_1 = G_f = 1000\ kg/m^2$ and

$G_2 = G_g = 1000\ kg/m^2$ –sec

$$\therefore G_f \psi = 1000 \times 2.3 \ = 2300\, kg/m^2 - \sec$$

$$G_f \psi = \frac{1000}{4.8} = 208.333\, kg/m^2 - \sec$$

Therefore from figure we get flow is Dispersed Bubbly flow.

Chapter-3

1.) Derive a relation to express (a) x, the mass quality in terms of volumetric quality β and the density ρ_1 and ρ_2 of the phases.(b) slip ratio in terms of α and β only (c) j, the overall volumetric flux in terms of inlet velocity of saturated liquid (u_{lo}) to a evaporating tube, the mass quality x and the phase densities ρ_l and ρ_v.

(1a)

$$\frac{1-x}{x} = \frac{1-\beta}{\beta}\ \frac{v_G}{v_L}\ = \frac{1-\beta}{\beta}\ \frac{\rho_L}{\rho_G}$$

$$1 - x = \frac{(1-\beta)\rho_L x}{\beta \rho_G}$$

$$\beta\rho_G - x\beta\rho_g = \rho_L x - \beta\rho_L x \qquad x = \frac{\rho_G \beta}{\rho_L + \beta(\rho_G - \rho_L)}$$

(1b) $$\frac{U_g}{U_L} = \frac{\frac{Q_g}{A\alpha}}{\frac{Q_L}{A(1-\alpha)}} = \frac{Q_g}{Q_L}\frac{1-\alpha}{\alpha} = \frac{\beta}{1-\beta}\frac{1-\alpha}{\alpha}$$

(1c)

$$j = j_L + j_G$$
$$= \frac{Q_L}{A} + \frac{Q_G}{A} = \frac{W_L}{\rho_L A} + \frac{W_G}{\rho_G A} = \frac{W_L v_L}{A} + \frac{W_G v_G}{A}$$
$$= \frac{W}{A}\left[\frac{(1-x)v_L}{A} + \frac{x v_G}{A}\right]$$
$$= \frac{W}{A}\left[(1-x)v_L + x v_G\right]$$
$$= \frac{W}{A}\left[(1-x)v_L + x v_G\right]$$
$$= \frac{W v_L}{A}\left[(1-x) + \frac{x v_G}{v_L}\right]$$
$$= U_{F0}\left[1 - x + \frac{v_G}{v_L}\right]$$
$$= U_{F0}\left[1 + x\left(\frac{v_G}{v_L}\right)\right]$$
$$= U_{F0}\left[1 + x\left(\frac{v_G}{v_L} - 1\right)\right]$$

(2) Estimate α in a heated tube in which evaporation os occurring in terms of x (mass quality), ulo (inlet velocity of liquid to be evaporated), $\overline{u_{gj}}$ (drift velocity), phase densities ρ_l and ρ_v and C_0 the distribution coefficient. Assume the drift flux model to be applicable for the vapor-liquid mixture.

$$\frac{\beta}{1-\beta}=\frac{Q_G}{Q_L}=\frac{W_G v_G}{W_L v_L}=\frac{x}{1-x}\frac{v_G}{v_L}$$

$$\therefore \beta=\frac{x}{1-x}\;\frac{v_G}{v_L}\;(1-\beta)=\left[\frac{x}{1-x}.\frac{v_G}{v_L}\right]-\left[\frac{x}{1-x}\;\frac{v_G}{v_L}\beta\right]$$

$$or\;\left[1+\frac{x}{1-x}\;\frac{v_G}{v_L}\right]=\frac{x}{1-x}\;\frac{v_G}{v_L}$$

$$\therefore \beta\frac{\frac{x}{1-x}\;\frac{v_G}{v_L}}{1+\frac{x}{1-x}\frac{v_G}{v_L}}=\frac{\frac{xv_G}{\cancel{(1-x)}v_G}}{\frac{(1-x)v_L+xv_G}{\cancel{(1-x)}v_L}}=\frac{xv_G}{(1-x)v_L+xv_G}$$

$$=\frac{1}{\left[\frac{(1-x)}{x}\frac{v_L}{v_G}+1\right]}$$

(3) Find α in a heated tube in which evaporation is occurring in terms of x (mass quality), u_{L0} (inlet velocity of liquid to be evaporated), $\bar{u}_{gj}$ (drift velocity), ρ_L,ρ_G (phase densities) and C_0 (distribution coefficient) assuming drift flux model to be applicable for the two phase vapor – liquid mixture.

Solution : From drift flux model , $\bar{u}_g=\frac{\langle j_g\rangle}{\langle\alpha\rangle}=C_0\langle j\rangle+\bar{u}_{gj}$

Dividing by j throughput , $\frac{\bar{u}_g}{\langle j\rangle}=\frac{\langle j_g\rangle}{\langle\alpha\rangle\langle j\rangle}=C_0+\frac{\bar{u}_{gj}}{\langle j\rangle}$

Or, $\frac{\langle\beta\rangle}{\langle\alpha\rangle}=C_0+\frac{\bar{u}_{gj}}{\langle j\rangle}\quad\left(\because \beta=\frac{\langle j_g\rangle}{\langle j\rangle}\right)$

Or, $\alpha=\frac{\beta}{C_0+\frac{\bar{u}_{gj}}{\langle j\rangle}}$ (1)

In equation (1)

$$\beta=\frac{Q_G}{Q_G+Q_L}=\frac{\frac{W_G}{\rho_G}}{\frac{W_G}{\rho_G}+\frac{W_L}{\rho_L}}=\frac{\frac{W(x)}{\rho_G}}{\frac{W_x}{\rho_G}+\frac{W(1-x)}{\rho_L}}=\frac{\frac{x}{\rho_G}}{\frac{x}{\rho_G}+\frac{1-x}{\rho_L}}=\frac{x\rho_L}{x\rho_L+(1-x)\rho_G}=\frac{x\rho_L}{\rho_G+x(\rho_L-\rho_G)}$$

$$j=j_L+j_G=\frac{Q_L}{A}+\frac{Q_G}{A}=\frac{W_L}{\rho_L A}+\frac{W_G}{\rho_G A}=\frac{W(1-x)}{\rho_L A}+\frac{Wx}{\rho_G A}=\frac{W}{A}\left[\frac{1-x}{\rho_L}+\frac{x}{\rho_G}\right]=\frac{Q\rho_L}{A}\left[\frac{1-x}{\rho_L}+\frac{x}{\rho_G}\right]$$

At inlet, mixture is all liquid with $W=W_L$, $Q=Q_L$

$$\frac{Au_{LO}\rho_L}{A\rho_L}\left[1-x+x\frac{\rho_L}{\rho_G}\right]=u_{LO}\left[1-x+\frac{x\rho_L}{\rho_G}\right]$$

Substituting individual terms in equation (1) we get :

$$\alpha=\frac{\frac{x\rho_L}{\rho_G+x(\rho_L-\rho_G)}}{C_0+\frac{u_{gj}}{u_{LO}\left[1-x+x\frac{\rho_L}{\rho_G}\right]}}=\frac{\frac{x\rho_L}{\rho_G(1-x)+x\rho_L}}{\frac{C_0\left[1-x+\frac{x\rho_L}{\rho_G}\right]+\frac{u_{gj}}{u_{LO}}}{1-x+x\frac{\rho_L}{\rho_G}}}$$

$$=\frac{\frac{x\left(\frac{\rho_L}{\rho_G}\right)}{1-x+x\left(\frac{\rho_L}{\rho_G}\right)}}{\frac{C_0-C_0x+C_0x\frac{\rho_L}{\rho_G}+\frac{u_{gj}}{u_{LO}}}{1-x+x\left(\frac{\rho_L}{\rho_G}\right)}}$$

$$= \frac{x\left(\frac{\rho_L}{\rho_G}\right)}{C_0 x\left[\frac{\rho_L}{\rho_G}-1\right]+\left[C_0+\frac{u_{gj}}{u_{LO}}\right]}$$

$$= \frac{x}{C_0 x\left[\frac{\frac{\rho_L}{\rho_G}-1}{\frac{\rho_L}{\rho_G}}\right]+\left(C_0+\frac{u_{gj}}{u_{LO}}\right)}\frac{\rho_G}{\rho_L}$$

$$= \frac{x}{C_0 x\left[1-\frac{\rho_G}{\rho_L}\right]+\left(C_0+\frac{u_{gj}}{u_{Lo}}\right)\frac{\rho_G}{\rho_L}}$$

$$= \frac{x}{C_0 x\left[\frac{\rho_L-\rho_G}{\rho_L}\right]+\left(C_0+\frac{u_{gj}}{u_{Lo}}\right)\frac{\rho_G}{\rho_L}}$$

4) Water at 10 atm enters a straight evaporator tube. If the velocity ratio of water to vapor is constant at 2.0 and mass flux is $2X10^5$ kg/hr (m^2), estimate the void fraction and momentum flux at a quality of 0.1. Assume ρ_{water} = 903.18 kg/m^3 and ρ_{vapor} = 3.6142 kg/m^3 .

$$P = 10\, atm\ ,\ \frac{u_2}{u_1} = 2.0\, G_{TP} = 2\times 10^5\ kg/hr(m^2)$$

$$x = 0.1$$

$$\rho_{water} = 903.2\, kg/m^3 \qquad \rho_{steam} = 3.6\ kg/m^3$$

$$\frac{u_2}{u_1} = \frac{Q_2/A_2}{Q_1/A_1} = \frac{\frac{G_{TP}x}{\rho_2\alpha}}{\frac{G_{TP}(1-x)}{\rho_1(1-\alpha)}}$$

$$= \frac{x}{1-x}\left(\frac{\rho_1}{\rho_2}\right)\frac{1-\alpha}{\alpha}$$

Substituting the values of $\frac{u_2}{u_1}, x$ and $\frac{\rho_1}{\rho_2}$, we get α from the following equation

$$2.0 = \frac{0.1}{0.5} \times \frac{903.2}{3.6} \frac{1-\alpha}{\alpha}$$

Momentum flux = $G_1 u_1 + G_2 u_2 = G\left[(1-x)u_1 + xu_2\right]$

$$u_1 = \frac{Q_1}{A_1} = \frac{g(1-x)}{\rho(1-\alpha)}$$

$$u_2 = \frac{Gx}{\rho_2 \alpha}$$

$\therefore$ Momentum flux $= G^2\left[\frac{(1-x)^2}{\rho_1(1-\alpha)} + \frac{x^2}{\rho_2 \alpha}\right]$

Chapter 5

1) A vertical tubular test section is installed in an experiment high pressure water loop. The tube is 1.016 cm ID and 2.134m long and is uniformly heated with 100 KW power. Saturated water enters at the base at (400) psia with a flow rate of 450 kg/hr. Calculate the total pressure drop and compare with the measured value of 8 psia.

Heat flux $\phi = \frac{100}{\pi DL}$

$P = 1000\,psia = 68.0\,pqv$

From steam table $v_1 = 0.001347 m^3/kg$

$$v_2 = 0.02795 m^3/kg$$

$$h_{12} = 1531.34 KJ/kg$$

$$v_{12} = 0.026603 m^3/kg$$

Mass flow rate = $450 kg/hr$

$$W\frac{dh}{dz}=\pi D\phi \Rightarrow \frac{dh}{dz}=\frac{\pi D\phi}{W}$$

$$\frac{dx}{dz}=\frac{dx}{dh}.\frac{dh}{dz}=\frac{\pi D\phi}{Wh_{12}}=\frac{\pi D}{Wh_{12}}\times\frac{100}{\pi DL}=\frac{100}{\frac{453}{3600}\times 1531.34\times 2.134}=0.2432m^{-1}$$

$$G=\frac{W}{A}=\frac{453}{\frac{\pi}{4}\left(1.016\times10^{-2}\right)^2\times3600}=1552.095kg/m^2s$$

$$\phi\pi DL=W\times e^{h_{12}}\Rightarrow x_e=\frac{\phi\pi DL}{Wh_{12}}=\frac{\pi DL}{Wh_{12}}\times\frac{100}{\pi DL}$$

$$=\frac{100}{Wh_{12}}=\frac{100}{\frac{453}{3600}\times1531.34}=0.5189\left(x_e<1.0\right)$$

$$\left(-\frac{dp}{dz}\right)_F=\frac{2C_fG^2}{D}\left(v_1+xv_{12}\right)$$

$$C_f=0.005\quad \mathrm{Re}_{LO}=\frac{DG}{\mu_l}=87.1231.1$$

$$\therefore\left(-\Delta p\right)_F=\frac{2\times0.005\times\left(1552.095\right)^2}{1.016\times10^{-2}}\left[0.001347\left(2.134\right)\right]$$

$$=41745.397kg/m.s^2+\frac{\left(2.134\right)^2}{2}\times0.2432\times0.0266$$

$$=6.0621psi$$

$$\left(-\frac{dp}{dz}\right)_g=\frac{g}{v_1+xv_{12}}=\frac{9.81}{0.001347+0.24322\left(0.026603\right)}$$

$$=\frac{9.81}{0.006469}\ln\left[\frac{0.001347+0.006469\times2.134}{0.001347}\right]$$

$$=3670.208\frac{kg}{ms^2}=0.5329psi$$

$$\left(-\frac{dp}{dz}\right)_{ace}=G^2v_{12}\frac{dx}{dz}$$

$$(-\Delta p)_{ace} = (1552.095)^2 \times 0.026603 \times 0.2432 \times 2.134$$

$$= 33260.2 \frac{kg}{ms^2} = 4.829\,psi$$

$$(\Delta p)_{total} = 11.424\,psi$$

Since $(\Delta p)_{total}$ is small compared to $1000\,psia$ assumption of constant fluid properties are justified. Check that exit steam is not superheated by calculating x_e. Check for compressibility and flashing effects by evaluating the terms $\frac{dv_g}{dp}, \frac{dv_f}{dp}, \left(\frac{\partial x}{\partial p}\right)_n$

$$\frac{dv_g}{dp} = -3.8 \times 10^{-3}\, ft^3 / lb\, psi \qquad \frac{dv_f}{dp} \approx 0$$

$\left(\frac{\partial x}{\partial p}\right)_h \approx 0$ at $x = 1$

$= -3.9 \times 10^{-4}\, psi^{-1}$ at $x = 0$

$= -2 \times 10^{-4}\, psi^{-1}$ at $x = 0.5$

2. Derive the expression of pressure gradient for a homogeneous two phase mixture for appreciable effect of kinetic energy.

Considering unit mass of fluid, the energy balance equation for single phase flow is as follows.

$$d\left(\frac{1}{2}u^2\right) + dU + d(zg\sin\theta) + d(pv) = \bar{d}q - \bar{d}w \qquad \text{(i)}$$

Kinetic energy internal energy gravitational pressure energy heat work content energy where u, U, z, p, v and θ refer to the velocity, internal energy, length of flow, pressure, volume and angle of elevation respectively.

Now, there is no work content $dw = 0$ and internal energy change can be given by

$$dU = dq + dF - pdv \qquad \text{(ii)}$$

Where dF is the irreversible frictional energy loss per unit mass of fluid.

Substituting in eqn (ii) in eqn (i) we get,

$$d\left(\frac{1}{2}u^2\right) + d\cancel{q} + dF - Pd\cancel{v} + d(g\sin\theta z) + Pd\cancel{v} + vdP = d\cancel{q}$$

(iii)

$$-vdp = d\left(\frac{1}{2}u^2\right) + dF + d(g\sin\theta z)$$

Dividing both sides by dz we get,

$$-v\frac{dp}{dz} = \frac{d}{dz}\left(\frac{1}{2}u^2\right) + \frac{dF}{dz} + \frac{d}{dz}(g\sin\theta z)$$

(iv)

$$-\frac{dp}{dz} = \frac{u}{v}\frac{du}{dz} + \frac{1}{v}\frac{dF}{dz} + \frac{g}{v}\sin\theta$$

Now, $\rho = \frac{1}{v}$

Thus substituting $\frac{1}{v}$ we get,

$$-\frac{dp}{dz} = u\rho\frac{du}{dz} + \rho\frac{dF}{dz} + \rho g\sin\theta \qquad \text{(v)}$$

This corresponds to

$$-\frac{dp}{dz} = \left(\frac{-dp}{dz}\right)_{acc} + \left(\frac{-dp}{dz}\right)_{f} + \left(\frac{-dp}{dz}\right)_{g}$$

In two phase flow according to HOMOGENEOUS MODEL u and ρ in eqn (v) should be substituted with u_M and ρ_M.

$$\therefore\ -\frac{dp}{dz} = u_M\rho_M\frac{du}{dz} + \rho_M\frac{dF}{dz} + \rho_M g\sin\theta \qquad \text{(vi)}$$

Where gravitational pressure gradient is

$$\left(\frac{-dp}{dz}\right)_g = \rho_M g\sin\theta = g\sin\theta / [xv_2 + (1-x)v_1]$$

$$= g\sin\theta / (v_1 + xv_{12}) = \frac{g\sin\theta}{(v_1 + xv_{12})}$$

and v_1 and v_2 are the specific volume of phase 1 and 2 respectively

Frictional pressure gradient

$$\left(\frac{-dp}{dz}\right)_f = \rho_M \frac{dF_M}{dz}$$

According to Fanning, head loss due to friction h_f

$$h_f = 4f\frac{L}{D}\frac{u^2}{2g} = \frac{-\Delta P_f}{\rho g} = \frac{F}{g}$$

Where F is energy loss due to friction per unit mass of fluid.

$$\therefore F = \frac{4fL}{D}\frac{u^2}{2}$$

For unit length of pipe

$$F = \frac{4f}{D}\frac{u^2}{2}$$

Now, for two phase flow $u = u_M$

$$\therefore F_M = \frac{4f_M}{D}\frac{u_M^2}{2}$$

Now, $u_M = \frac{G_M}{\rho_M}$

$$\therefore F_M = \frac{4f_M}{D}\frac{G_M^2}{2\rho_M^2} = \frac{4f_M}{D}G^2\left[xv_2 + (1-x)v_1\right]^2 \quad \text{(vii)}$$

Differentiating both sides with respect to z we get,

$$\frac{dF_M}{dz} = \frac{4f_M}{D}\frac{G_M^2}{2}\frac{d}{dz}\left[xv_2 + (1-x)v_1\right]^2$$

$$= \frac{4f_M}{D}\frac{G_M^2}{\cancel{2}\rho_M^2}\cancel{2}\left[xv_2 + (1-x)v_1\right]\left[x\frac{dv_2}{dz} + (1-x)\frac{dv_1}{dz} + v_{12}\frac{dx}{dz}\right]$$

$$= \frac{4f_M G_M^2}{D}(xv_1 + xv_{12})\left[x\frac{dv_2}{dz} + (1-x)\frac{dv_1}{dz} + v_{12}\frac{dx}{dz}\right]$$

$$\therefore \rho_M\frac{dF_M}{dz} = \frac{4f_M G_M^2}{D}\frac{(v_1 + xv_{12})}{(v_1 + xv_{12})}\left[x\frac{dv_2}{dz} + (1-x)\frac{dv_1}{dz} + v_{12}\frac{dx}{dz}\right] \quad \text{(viii)}$$

Now, variation of specific volume v with respect to z is due to pressure.

Therefore,

$$\rho_M \frac{dF_M}{dz} = \frac{4 f_M G_M^2}{D}\left[x\frac{dv_2}{dP}\frac{dP}{dz} + (1-x)\frac{dv_1}{dP}\frac{dP}{dz} + v_{12}\frac{dx}{dz}\right]$$

Pressure gradient due to acceleration

$$\left(\frac{-dP}{dz}\right)_{acc} = u_M \rho_M \frac{du_M}{dz} \qquad u_M = \frac{W_M}{\rho_M A}$$

$$\therefore \frac{du_M}{dz} = \frac{d}{dz}\left(\frac{W_M}{\rho_M A}\right) = \left[\frac{W_M}{A}\frac{d}{dz}\left(\frac{1}{\rho_M}\right) - \frac{1}{A^2}\frac{W_M}{\rho_M}\frac{dA}{dz}\right]$$

$$\therefore u_M \rho_M \frac{du_M}{dz} = \frac{W_M}{A}\left[\frac{W_M}{A}\frac{d}{dz}\left\{xv_2 + (1-x)v_1\right\} - \frac{W_M}{A^2 \rho_M}\frac{dA}{dz}\right] \qquad \text{(ix)}$$

$$= G_M^2\left[x\frac{dv_2}{dz} + (1-x)\frac{dv_1}{dz} + v_{12}\frac{dx}{dz}\right] - \frac{G_M^2}{A}(v_1 + xv_{12})\frac{dA}{dz}$$

$$= G_M^2\left[x\frac{dv_2}{dp}\frac{dp}{dz} + (1-x)\frac{dv_1}{dp}\frac{dp}{dz} + v_{12}\frac{dx}{dz}\right] - \frac{G_M^2}{A}(v_1 + xv_{12})\frac{dA}{dz}$$

Now combining three pressure gradients terms we get,

$$\left(\frac{-dp}{dz}\right) = \frac{g\sin\theta}{v_1 + xv_{12}} + \frac{4 f_M G_M^2}{D}\left[x\frac{dv_2}{dp}\frac{dp}{dz} + (1-x)\frac{dv_1}{dp}\frac{dp}{dz} + v_{12}\frac{dx}{dz}\right] + G_M^2\left[x\frac{dv_2}{dp}\frac{d}{dz} + (1-x)\frac{dv_1}{dPp}\frac{dp}{dz} + v_{12}\frac{dX}{dz}\right]$$
$$- \frac{G_M^2}{A}(v_1 + xv_{12})\frac{dA}{dz}$$

$$\left(\frac{-dp}{dz}\right)_{total} = \frac{\dfrac{g\sin\theta}{v_1 + xv_{12}} + \dfrac{4 f_M G_M^2}{D} v_{12}\dfrac{dx}{dz} + G^2 v_{12}\dfrac{dx}{dz} - \dfrac{G^2}{A}(v_1 + xv_{12})\dfrac{dA}{dz}}{\left[1 + \dfrac{4 f_M G_M^2}{D}\left\{x\dfrac{dv_2}{dp} + (1-x)\dfrac{dv_1}{dp}\right\} + G_M^2\left\{x\dfrac{dv_2}{dp} + (1-x)\dfrac{dv_1}{dp}\right\}\right]}$$

When there is too much flashing then

$$x = x(h, p)$$
$$dx = \frac{dx}{dh}|_P\, dh + \frac{\partial x}{\partial P}|_h\, dp$$
$$\frac{dx}{dz} = \frac{dx}{dh}|_P \frac{dh}{dz} + \frac{dx}{dp}|_h \frac{dp}{dz}$$

So , by substituting we get,

$$\left(\frac{-dp}{dz}\right)_{total} = \frac{\dfrac{g\sin\theta}{v_1 + xv_{12}} + \dfrac{4 f_M G_M^2}{D} v_{12}\dfrac{dx}{dh}|_P \dfrac{dh}{dz} + G_M^2 v_{12}\dfrac{dx}{dh}|_P \dfrac{dh}{dz} - \dfrac{G^2}{A}(v_1 + xv_{12})\dfrac{dA}{dz}}{\left[1 + \dfrac{4 f_M G_M^2}{D}\left\{x\dfrac{dv_2}{dp} + (1-x)\dfrac{dv_1}{dP} + \dfrac{4 f G_M^2 v_{12}}{D}\dfrac{dx}{dp}|_h\right\} + G_M^2\left\{x\dfrac{dv_2}{dp} + (1-x)\dfrac{dv_1}{dp} + v_{12}\dfrac{dx}{dp}|_h\right\}\right]}$$

Now, $\frac{dx}{dh} = \frac{1}{h_{12}}$ where $h_{12} \rightarrow$ molal enthalpy of vapourisation

For horizontal evaporator gravity pressure gradient is absent, then,

$$\left(\frac{-dP}{dz}\right)_{total} = \frac{\frac{4 f_M G_M^2}{D}\frac{v_{12}}{h_{12}}\frac{dh}{dz} + \frac{dh}{dz} + G_M^2 \frac{v_{12}}{h_{12}}\frac{dh}{dz} - \frac{G_M^2}{A}(v_1 + x v_{12})\frac{dA}{dz}}{\left[1 + \frac{4 f_M G_M^2}{D}\left\{x\frac{dv_2}{dp} + (1-x)\frac{dv_1}{dp} + \frac{4 f_M G_M^2 v_{12}}{D}\frac{dx}{dp}|_h\right\} + G_M^2\left\{x\frac{dv_2}{dP} + (1-x)\frac{dv_1}{dP} + v_{12}\frac{dx}{dP}|_h\right\}\right]}$$

3. Liquid evaporated from an inlet conduction at saturated temperature $(x=0)$ to a vapour liquid mixture having a mass quality x. For a linear change of x over length $L\left(\frac{dx}{dz} = const\right)$, derive an expression of the pressure drop over length L.

As derived in chapter-5,

$$-\frac{dp}{dz} = \frac{2 f_{TP} G^2}{D}\left(v_1 + x v_{12}\right) + G^2 v_{12}\frac{dx}{dz} + \frac{g\sin\theta}{v_1 + x v_{12}}$$

or

$$\int_0^L\left(-\frac{dp}{dz}\right)dz = \frac{2 f_{TP} G^2}{D}\int_0^L\left(v_1 + x v_{12}\right)dz + G^2 v_{12} k dz + \frac{g\sin\theta}{v_1 + x v_{12}}dz$$

$$= \frac{2 f_{TP} G^2}{D} v_1 L + \frac{2 f_{TP} G^2}{D} v_{12} k z dz + G^2 v_{12} k dz + \frac{g\sin\theta dz}{v_1 + x v_{12}}$$

$$= \frac{2 f_{TP} G^2}{D} v_1 L + \frac{2 f_{TP} G^2}{D} v_{12} L\frac{x}{2} + G^2 v_n k L + \frac{g\sin\theta L}{x v_{12}}\ln\left[1 + \frac{v_{12}}{v_1}x\right]$$

4. Saturated water at a rate of 300 kg/hr (m2) enters the bottom of a vertical evaporator tube 2.5 cm diameter and 2.0 m long. The tube receives a heat flux of 2x105 BTU/hr ft2 and there are no heat losses. Calculate the pressure drop for inlet pressures of 350 psia. Assume homogeneous flow with a constant friction factor of 0.005.

Assume homogeneous flow theory, proceed similarly as problem 1

Equating rate of heat addition /unit length to heat flux,

$\frac{dq_e}{dz} = \pi D\phi \longrightarrow$ heat flux

$$\frac{dq_e}{dz} = W\frac{d}{dz}h \qquad \frac{dh}{dz} = \frac{\pi D\phi}{w} = \frac{\not{\pi} D\phi}{G\frac{\not{\pi} D^2}{4}} = \frac{4\phi}{GD} \quad \text{(For constant W)}$$

For small pressure changes $h = h_1 + xh_{12}$
$= xh_2 + (1-x)h_1$

$$\frac{h-h_1}{h_{12}} = x \qquad \frac{dx}{dz} = \frac{1}{h_{12}}\frac{dh}{dz} = \frac{1}{h_{12}}\frac{4\phi}{GD} \qquad or, dx = \frac{4\phi}{h_{12}GD}dz$$

$$\left(-\frac{dP}{dz}\right) = \frac{2fG^2}{D}(v_1 + xv_{12}) + G^2 v_{12}\frac{dx}{dz} + \frac{g\cos\theta}{v_1 + xv_{12}}$$

Neglecting $\frac{dp}{dz}$, area change, flashing, compressibility effects

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right)dz = \frac{2f_{TP}G^2}{D}\int_0^L (v_1 + xv_{12})dz + \int_0^L \frac{g\cos\theta}{v_1 + xv_{12}} + G^2 v_{12}\int_0^L \left(\frac{dx}{dz}\right)dz$$

$$= \frac{2f_{TP}G^2 v_1}{D}L + \frac{2f_{TP}G^2 v_{12}}{D}\int_0^L xdz + g\cos\theta\int_0^L \frac{dz}{v_1 + xv_{12}} + G^2 v_{12}\int_0^L \left(\frac{dx}{dz}\right)dz$$

$$x = \frac{4\phi}{hG^2GD}z + c \qquad x = \frac{4\phi}{h_{12}GD}z$$

C=0 from B Cs

$$\Delta P = \frac{2f_{TP}G^2 v_1}{D} + \frac{2f_{TP}G^2}{D}v_{12}\frac{4\phi}{h_{12}GD}\frac{L^2}{2} + g\cos\theta\int_0^L \frac{dz}{v_1 + \frac{4\phi}{h_{12}GD}z} + \frac{G^2 v_{12}4\phi}{h_{12}GD}\int_0^L dz$$

$$= \frac{2f_{TP}G^2 v_1 L}{D} + \left(\frac{2L}{D}\right)^2 f_{TP}G\phi\frac{v_{12}}{h_{12}} + G\phi\frac{v_{12}}{h_{12}}\frac{4L}{D} + \frac{g\cos\theta G h_{12}}{4\phi v_{12}}\ln\left(1 + \frac{\phi v_{12}\rho_L}{G h_{12}}\frac{4L}{D}\right)$$

$v_{12} = 1.3064\ ft^2/lb \quad v_l = 0.01912\ ft^2/lb \quad h_{12} = 794.7\frac{B\tau u}{lb} \qquad \Delta p = 1.02$ psi

$\Delta\, p$ small compared to pressure , so assumption of constant fluid properties justified.

Check for compressibility effects and confirm exit steam is not superheated.

$$x_e = \frac{4z\phi}{Gh_{12}D} = 0.605$$

For compressibility & flashing effects $x = x(h, p)$

$$\left(\frac{\partial x}{\partial p}\right)_h = 0 \qquad x = 0$$

$$-3.9\times10^{-4} \qquad -2\times10^{4}\ psi^{-1}$$ at x=0.5

At $x = 0.6$ $\left(\frac{\partial x}{\partial p}\right) =$

$$v = xv_g + (1-x)v_l$$
$$= x(v_g - v_l) + v_l$$
$$= xv_{lg} + v_l$$

$$v - v_l = x(v_g - v_l)$$
$$\left(\frac{\partial v}{\partial p}\right)_h - \left(\frac{\partial v_l}{\partial p}\right)_h = \left(\frac{\partial x}{\partial p}\right)_h (v_g - v_l) + x\frac{\partial}{\partial p}(v_g - v_l)$$

$$\left(\frac{\partial v}{\partial p}\right)_h = v_{lg}\left(\frac{\partial x}{\partial p}\right)_h + x\left(\frac{\partial v_g}{\partial p}\right)_h$$

5 An air-water mixture flows through a circular pipe of cross-section A1. It has a nozzle of cross-section A2 at the centre. The pressure at the upstream, throat and downstream sections are p1, p2 and p3 respectively. Assuming incompressible homogeneous flow, derive the following equations where G is the mass flux and ρ1 and ρ2 are the densities of water and air respectively:

$$p_1 - p_2 = \frac{G^2}{\rho_1}\frac{A_1}{A_2}\left(\frac{A_1}{A_2} - 1\right)\left[1 + x\left(\frac{\rho_1}{\rho_2} - 1\right)\right]$$

$$p_1 - p_3 = \frac{G^2}{2\rho_1}\left(\left(\frac{A_1}{A_2} - 1\right)\right)^2\left[1 + x\left(\frac{\rho_1}{\rho_2} - 1\right)\right]$$

Clearly state any additional assumptions made

Chapter 6

Problem 1 Air-water at individual mass flow rates of 50kg/hr flow through a 3 cm diameter pipe at 27^0 C and 1.2 atm pressure. What is the overall volumetric flux. If the drift flux is 3m/s, what are the average velocities of the phases ?

Solution :

$$W_1 = W_2 = 50\,kg/hr \qquad T = 27^0 C$$

$$A = \frac{\pi}{4}(0.03)^2 \qquad P = 1.2\,atm$$

$$\rho_1 = 1000\,kg/m^3$$

$$j_{21} = 3 = \alpha(u_2 - j) \quad \text{.... (1)}$$

$$u_1 = \frac{W_1}{\rho_1 A(1-\alpha)} \quad \text{........(2)}$$

$$u_2 = \frac{W_2}{\rho_2 A\alpha} \quad \text{......(3)}$$

$$j = j_1 + j_2 = \frac{Q_1}{A} + \frac{Q_2}{A} = \frac{W_1}{\rho_1 A} + \frac{W_2}{\rho_2 A}$$

$$\therefore 3 = \alpha(u_2 - j\;) \quad \text{....(4)}$$

From equations (3) and (4), find α and u_2. Substitute in equation (2) to find u_1

Problem 2 Show that drift flux is independent of the motion of the observer during concurrent flow of two fluids .

Solution :

Let fluids 1 and 2 move at velocities u_1 and u_2. Let the observer move at velocity u in direction of fluid motion.

Therefore, fluid velocity is observed by observer = $u_2' = u_2 - u$

$$u_1' = u_1 - u$$

$$j_{21} = j_2(1-\alpha) - \alpha j_1 = u_2'\alpha(1-\alpha) - \alpha(1-\alpha)u_1'$$

$$= \alpha(1-\alpha)(u_2' - u_1') = \alpha(1-\alpha)(u_2 - u_1)$$

Similarly for observer moving at velocity u opposite to direction of fluid motion, $u_2' = u_2 + u$ and $u_1' = u_1 + u$. In this case also $j_{21} = \alpha(1-\alpha)u_{21}$

Problem 3

Sketch j_2 vs. α as a function of j_1 and j_1 vs. α as a function of j_2 and identify the condition for flooding , co-current & counter current flows

Given $\frac{j_{12}}{u_\infty}=\alpha(1-\alpha)^n$.

Solution :

$$\alpha=\frac{j_2}{j}\left(1-\frac{j_{21}}{j_2}\right)=\frac{j_2-j_{21}}{j}$$

$j_{21}=j_2-\alpha j$ ....(1)

$$\frac{dj_{21}}{d\alpha}=-j$$

Again, $j_{21}=u_\infty\,\alpha.(1-\alpha)^n$ ....(2)

$$\frac{dj_{21}}{d\alpha}=u_\infty\alpha.n.(1-\alpha)^{n-1}+u_\infty.(1-\alpha)^n$$
$$=u_\infty\left[\frac{n\alpha}{1-\alpha}+1\right](1-\alpha)^n$$

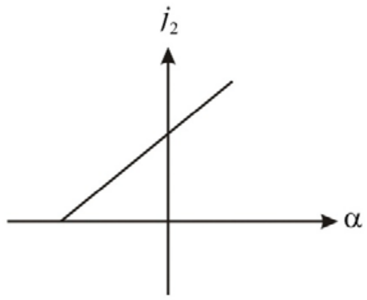
For flooding, curve of eqn(1) is tangent to curve of eqn (2).

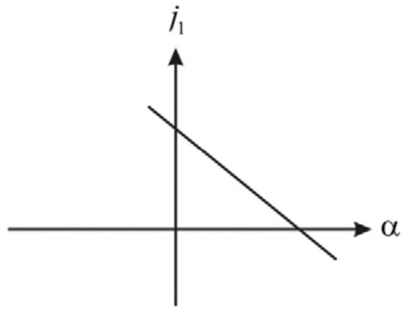
$$\therefore\left(\frac{dj_{21}}{d\alpha}\right)_1.\left(\frac{dj_{21}}{d\alpha}\right)_2=-1$$

Or, $$\boxed{ju_\alpha(1-\alpha)^n.\left[\frac{n\alpha}{1-\alpha}+1\right]=1}$$

Condition for flooding

$$j_2=\frac{\alpha j_1}{1-\alpha}+\frac{j_{21}}{1-\alpha}\equiv y=mx+c$$
$$j_1=-\alpha j+(j-j_{21})$$





Problem 4

For what value of n will $\frac{j_2}{u_\infty} = \alpha(1-\alpha)^n$ be linear? How does drift velocity of '2' depends on α ?

Solution:

For $n = 0$ $\frac{j_2}{u_\infty} = \alpha(1-\alpha)^0 \Rightarrow j_2 = \alpha u_\infty$

Now,

$$u_2 = \frac{j_2}{\alpha} = u_\infty$$

Drift velocity of component 2 = terminal velocity .

Thus, $u_2 = const \neq f(\alpha)$

Or, $\frac{\partial u_2}{\partial \alpha} = 0$

$$C_0 = \frac{\langle j_2 \rangle - \langle j_{21} \rangle}{\alpha \langle j \rangle} = \frac{\alpha u_\infty - \alpha u_\infty}{\alpha \langle j \rangle} = 0$$

Chapter 7

Problem1: Derive the expression to estimate pressure drop for flow of boiling water in straight pipes (no area change in pipe) where the water enters the pipe under saturated conditions and water and steam flows under stratified conditions.

Solution: From eqn (7.27), the equation for pressure gradient is:

$$-\frac{dp}{dz} = g\sin\theta\left[(1-\alpha)\rho_1 + \alpha\rho_2\right] + \left[F_{W1} + F_{W2}\right] + \frac{1}{A}\frac{d}{dz}\left[W_1 u_1 + W_2 u_2\right]$$

$$= \left(-\frac{dp}{dz}\right)_g + \left(-\frac{dp}{dz}\right)_f + \left(-\frac{dp}{dz}\right)_{acc} \qquad \text{..... (7.27)}$$

The frictional pressure gradient can be expressed in terms of two phase multiplier as:

$$\phi^2{}_{lo} = \frac{\left(-\frac{dp}{dz}\right)_{f\,TP}}{\left(-\frac{dp}{dz}\right)_{f\,lo}}$$

$$or, \left(-\frac{dp}{dz}\right)_{F_{TP}} = \frac{4}{D}\tau_w$$

$$= \phi^2{}_{lo} \cdot \left(-\frac{dp}{dz}\right)_{F_{lo}}$$

$$= \phi^2{}_{lo} \cdot \left[\frac{3}{A}\tau_{w_{lo}}\right]$$

$$= \phi^2{}_{lo} \cdot \left[\frac{4}{D}\right]\left[\frac{1}{2} f_{lo} \cdot \frac{G^2}{\rho_l}\right]$$

$$= \frac{2}{D}\phi^2{}_{lo}\left[f_{lo} \cdot \frac{G^2}{\rho_l}\right]$$

$$= \frac{2}{D}\frac{G^2}{\rho_l} \cdot f_{lo} . \phi^2{}_{lo}$$

$$\therefore \int_0^L \left(-\frac{dp}{dz}\right)_{F_{TP}} dz = \int_0^L \left(\frac{2}{D}\frac{G^2}{\rho_l} f_{lo} . \phi^2{}_{lo}\right) dz$$

$$= \frac{2}{D}\frac{f_{lo} G^2 L}{\rho_l}\left[\frac{1}{L}\int_0^L \phi^2{}_{lo} dz\right]$$

Since quality increases linearly with distance (derivation below), x=0 at z=0 and

$$\int_0^L \left(-\frac{dp}{dz}\right)_{f_{TP}} = \frac{2}{D}\frac{f_{lo} G^2 L}{\rho_l}\left[\frac{1}{x}\int_0^x \phi^2{}_{lo}\, dz\right]$$

The relationship between length and quality can be expressed by considering the enthalpy balance of the flow situation.

Let ϕ be the heat flux. Therefore the rate of heat addition per unit length is

$$\frac{dq_e}{dz} = \pi D \phi$$

From energy equation

$$\frac{dq_e}{dz} = W\frac{d}{dz}(h)$$
$$or, \frac{dh}{dz} = \frac{\pi D\phi}{W}$$

And $h = h_1 + xh_{12}$ Or, $\left(\frac{dh}{dx}\right)_p = h_{12}$

Now,
$$\frac{dx}{dz} = \frac{dx}{dh}.\frac{dh}{dz}$$
$$= \frac{1}{\frac{dh}{dx}}.\frac{dh}{dz}$$
$$= \frac{\pi D\phi}{Wh_{12}}$$
$$= \frac{\pi D\phi}{\frac{\pi}{4}D^2Gh_{12}} = \frac{4\phi}{DGh_{12}}$$

From the aforementioned expression of $\frac{dx}{dz}$, it is clear that quality increases linearly with distance.

The acceleration pressure gradient can be expressed a:

$$\frac{1}{A}\frac{d}{dz}(W_1u_1 + W_2u_2)$$
$$= \frac{1}{A}\frac{d}{dz}\left[\frac{W_1^2}{\rho_1 A_1} + \frac{W_2^2}{\rho_2 A_2}\right]$$
$$= \frac{1}{A}\frac{d}{dz}\left[\frac{W^2(1-x)^2}{\rho_1(1-\alpha)A} + \frac{W^2x^2}{\rho_2\alpha A}\right]$$

Now since A is constant, G is constant and

$$\frac{1}{A}\frac{d}{dz}\left(W_1u_1+W_2u_2\right)$$

$$=\frac{W^2}{A^2}\frac{d}{dz}\left[\frac{(1-x)^2}{\rho_1(1-\alpha)}+\frac{x^2}{\rho_2\alpha}\right]$$

$$=G^2\frac{d}{dz}\left[\frac{(1-x)^2}{\rho_1(1-\alpha)}+\frac{x^2}{\rho_2\alpha}\right]$$

$$\therefore\int_0^L\frac{1}{A}\frac{d}{dz}\left(W_1u_1+W_2u_2\right).dz$$

$$=\int_0^L G^2\left\{\frac{d}{dz}\left[\frac{(1-x)^2}{\rho_1(1-\alpha)}+\frac{x^2}{\rho_2\alpha}\right]\right\}dz$$

$$=G^2\left[\frac{(1-x)^2}{\rho_1(1-\alpha)}+\frac{x^2}{\rho_2\alpha}\right]_0^L$$

Substituting the expressions for individual components of pressure gradient, the pressure drop for length L is

$$\Delta P=\int_0^L\left(-\frac{dp}{dz}\right)dz=\int_0^L\frac{\tau_w.S}{A}+\int_0^L\frac{1}{A}\frac{d}{dz}\left(W_1u_1+W_2u_2\right)+\int_0^L\left[(1-\alpha)\rho_1+\alpha\rho_2\right]g\sin\theta$$

$$=\frac{2}{D}\frac{f_{lo}.G^2L}{\rho_l}\left[\frac{1}{x}\int_0^x\phi^2{}_{lo}dz\right]+G^2\left[\frac{(1-x)^2}{\rho_1(1-\alpha)}+\frac{x^2}{\rho_2\alpha}\right]_0^L+g\sin\theta\left[\int_0^L(1-\alpha)\rho_1+\alpha\rho_2\right]dz$$

Problem 2. Two incompressible fluids are flowing under separated flow through a nozzle in horizontal orientation. Express the pressure drop of the two phase system $\Delta P_{T.P}$ in terms of ΔP_1 and ΔP_2 , the pressure drop encountered by either of the fluids if they would be flowing alone through the nozzle.

Solution: For separated flow the momentum balance equation for flow of the individual components per unit volume under steady state condition can be written as:

$$\rho_1u_1\frac{du_1}{dz}=b_1+f_1-\frac{dp}{dz}...........(i)$$

$$\rho_2u_2\frac{du_2}{dz}=b_2+f_2-\frac{dp}{dz}...........(ii)$$

Since the two component separated flow is accelerated rapidly through nozzle, body force and frictional components can be neglected as compared to inertia term. Thus

From eqn (1)

$$-\frac{dp}{dz}=\rho_1 u_1\frac{du_1}{dz}$$

And from eqn (2)

$$-\frac{dp}{dz}=\rho_2 u_2\frac{du_2}{dz}$$

$$or \quad -\frac{dp}{dz}=\rho_1 u_1\frac{du_1}{dz}=\rho_2 u_2\frac{du_2}{dz}$$

Or on integration,

$$or \ \Delta P_{T.P}=\frac{\rho_1^2 u_1^2}{2\rho_1}=\frac{\rho_2^2 u_2^2}{2\rho_2}$$

$$or \ \Delta P_{T.P}=\frac{1}{2}\frac{\rho_1^2 u_1^2 A_1^2}{A_1^2\rho_1}=\frac{1}{2}\frac{\rho_2^2 u_2^2 A_2^2}{A_2^2\rho_2}$$

$$or \ \Delta P_{T.P}=\frac{1}{2}\ \frac{W_1^2}{A^2(1-\alpha)^2\rho_1}=\frac{W_2^2}{A^2\alpha^2\rho_2}$$

$$=\frac{1}{2}\ \frac{(W_1/A)^2}{(1-\alpha)^2\rho_1}=\frac{1}{2}\ \frac{(W_2/A)^2}{\alpha^2\rho_2}$$

$$=\frac{1}{2}\ \frac{G_1^{\ 2}}{(1-\alpha)^2\rho_1}=\frac{1}{2}\ \frac{G_2^{\ 2}}{\alpha^2\rho_2}$$

Now if either of fluid flows alone through the nozzle,

$$\Delta P_1=\frac{1}{2}\frac{G_1^2}{\rho_1} \ \text{And} \ \ \Delta P_2=\frac{1}{2}\frac{G_2^2}{\rho_2}$$

$$\therefore \frac{\Delta P_1}{\Delta P_{T.P}} = \frac{\frac{1}{2}\frac{G_1^2}{\rho_1}}{\frac{1}{2}\frac{G_1^2}{(1-\alpha)^2 \rho_1}} = (1-\alpha)^2 \text{..........}(iii)$$

$$\frac{\Delta P_2}{\Delta P_{T.P}} = \frac{\frac{1}{2}\frac{G_2^2}{\rho_2}}{\frac{1}{2}\frac{G_2^2}{\alpha^2 \rho_2}} = \alpha^2 \text{..........}(iv)$$

From equation (iii) we get

$$(1-\alpha)^2 = \frac{\Delta P_1}{\Delta P_{T.P}}$$

$$or\ (1-\alpha) = \left(\frac{\Delta P_1}{\Delta P_{T.P}}\right)^{1/2} \text{..........}(v)$$

And from equation (iv) we get

$$\alpha^2 = \frac{\Delta P_2}{\Delta P_{T.P}}$$

$$or\ \alpha = \left(\frac{\Delta P_2}{\Delta P_{T.P}}\right)^{1/2} \text{..........}(vi)$$

Adding equation (v) and equation (vi) we get

$$(1-\alpha) + \alpha = \left(\frac{\Delta P_1}{\Delta P_{T.P}}\right)^{1/2} + \left(\frac{\Delta P_1}{\Delta P_{T.P}}\right)^{1/2}$$

$$or\ 1 = \frac{(\Delta P_1)^{1/2} + (\Delta P_2)^{1/2}}{(\Delta P_{T.P})^{1/2}}$$

$$or\ (\Delta P_1)^{1/2} + (\Delta P_2)^{1/2} = \left(\Delta P_{T.P}\right)^{1/2}$$

$$or\ \left(\Delta P_{T.P}\right) = [(\Delta P_1)^{1/2} + (\Delta P_2)^{1/2}]^2$$

Problem 3 Develop the separate cylinders model for stratified gas-liquid flow assuming turbulent flow and a constant friction factor for both phases.

Solution: Let gas flow in a cylinder of effective radius r_g and liquid flow in a cylinder of effective radius r_l

$$\alpha = \frac{r_g^{\,2}}{r_o^{\,2}} \quad 1-\alpha = \frac{r_l^{\,2}}{r_o^{\,2}}$$

$$\left(-\frac{dp}{dz}\right) = \left(-\frac{dp}{dz}\right)_g = \frac{f_g}{r_g}\rho_g\left(\frac{j_g}{\alpha}\right)^2 = \frac{f_g \rho_g j_g^{\,2}}{r_o}\frac{1}{\alpha^{5/2}}$$

Where f_g is the constant friction factor for the gas phase

$$\phi_g^{\,2} = \frac{\left(-\frac{dp}{dz}\right)}{\left(-\frac{dp}{dz}\right)_{gasonly}} = \frac{1}{\alpha^{5/2}}$$

or

$$\alpha^{5/2} = \frac{1}{\phi_g^{\,2}}$$

Similarly

$$\phi_l^{\,2} = \frac{1}{(1-\alpha)^{5/2}}$$

or

$$(1-\alpha) = \left(\frac{1}{\phi_l^{\,2}}\right)^{2/5}$$

Thus $\alpha = \left(\frac{1}{\phi_g^{\,2}}\right)^{2/5}$, $\alpha = 1-\left(\frac{1}{\phi_l^{\,2}}\right)^{2/5}$

Therefore, $\left(\frac{1}{\phi_g^{\,2}}\right)^{2/5} = 1-\left(\frac{1}{\phi_l^{\,2}}\right)^{2/5}$

Or $\left(\frac{1}{\phi_g^{\ 2}}\right)^{2/5} + \left(\frac{1}{\phi_l^{\ 2}}\right)^{2/5} = 1$

Problem 4

For flow through a packed bed of spheres with diameter'd' and void fraction ε, deduce the values of and $f_f\left(-f_1\right)$ and $f_s\left(=f_2\right)$ using the Carman-Kozeny equation for the frictional pressure drop during viscous flow through void space between the solid

$$\left(-\frac{\partial p}{\partial z}\right)_F = \frac{180\mu_f j_{fo}}{d^2}\frac{\left(1-\varepsilon\right)^2}{\varepsilon^3}$$

j_{fo} is the fluid flux relative to the particles and ε is the liquid fraction in the3 solid-liquid system. Subscripts f and s refer to the fluid and solid respectively.

Solution:

In one dimensional form the momentum equation per unit volume of individual phase is

$$\rho_1\left[\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial z}\right]=b_1+f_1-\frac{\partial p}{\partial z}$$

$$\rho_2=\left[\frac{\partial u_2}{\partial t}+u_2\frac{\partial u_2}{\partial z}\right]=b_2+f_2-\frac{\partial p}{\partial z}$$

Now, for steady state condition

$$\frac{\partial u_1}{\partial t}=0,\ \frac{\partial u_2}{\partial t}=0,$$

If we neglect the inertial effect then $u_1\frac{\partial u_1}{\partial z}=0\ \ and\ \ u_2\frac{\partial u_2}{\partial z}=0$

If we neglect body force then

$b_1=0$ and $b_2=0$

Then from momentum equation we get,

$$f_1=\left(\frac{\partial p}{\partial z}\right)=\left(\frac{\partial p}{\partial z}\right)_F=f_2 \quad ....(1)$$

The force f_2 on particles is made up of two parts, one due to fluid , f_{21} and the other due to the f_{22} particles .

.

Thus

$$f_2 = f_{21} + f_{22}$$

But force f_1 on fluid is only due to particle

So, $f_1 = f_{12}$

Therefore, from equation (1) we get

$$f_{12} = f_1 = -180\frac{\mu_f j_{fo}}{d^2}\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right] \quad ....(2)$$

$$f_2 = f_{21} + f_{22} = -\frac{180\,\mu_f j_{fo}}{d^2}\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right] \quad ....(3)$$

Now, F_{12} = equivalent of f_{12} per unit volume of whole flow.

$$= (1-\alpha)\, f_{12}$$
$$= \varepsilon\, f_{12}$$

$F_{21} =$ equivalent of f_{21} per unit volume of whole flow

$$= \begin{matrix} = \alpha\, f_{21} \\ = (1-\varepsilon)\, f_{21} \end{matrix}$$

F_{12} and F_{21} forces are entirely due to mutual hydrodynamic drag and since action and reaction are equal and opposite we have.

$$F_{12} = -F_{21}$$
$$or, \varepsilon f_{12} = -(1-\varepsilon) f_{21}$$
$$or, \varepsilon f_1 = -(1-\varepsilon) f_{21}$$
$$or,\ f_{21} = \frac{\varepsilon}{(1-\varepsilon)} . f_1$$
$$= -\frac{\varepsilon}{1-\varepsilon}\left[-\frac{180\,\mu_f\, j_{fo}}{d^2}.\frac{(1-\varepsilon)^2}{\varepsilon^3}\right]$$
$$= \frac{180\,\mu_f\, j_{fo}}{d^2}\frac{(1-\varepsilon)}{\varepsilon^2}$$

Now, from equation (3) we get

$$f_{22} = f_2 - f_{21}$$
$$= -\frac{180\mu_f\, j_{fo}}{d^2}.\frac{(1-\varepsilon)^2}{\varepsilon^3} - \frac{180\mu_f\, j_{fo}}{d^2}\frac{(1-\varepsilon)}{\varepsilon^2}$$
$$= -\frac{180\mu_f\, j_{fo}}{d^2}\left[\frac{1-2\varepsilon+\varepsilon^2+\left(\varepsilon-\varepsilon^2\right)}{\varepsilon^3}\right]$$
$$= -\frac{180\mu_f\, j_{fo}}{d^2}\left[\frac{1-\varepsilon}{\varepsilon^3}\right]$$
$$= -\frac{180\mu_f\, j_{fo}}{d^2}\frac{(1-\varepsilon)}{\varepsilon^3}$$

Problem 4 Using the results of the previous problem, deduce the fluid flux necessary to cause fluidization in a bed with void fraction ε and estimate the pressure gradient through the bed in this case.

Solution:

In a fluidized bed particle are supported by an upward flow of fluid around them and inter particle force are negligible. Mathematically

$$f_{22} = 0$$

For steady state flow, neglecting the inertial term, the momentum equation per unit volume of the individual phase becomes,

$$b_1 + f_1 - \frac{\partial p}{\partial z} = 0 \quad \text{……….(1)}$$

And $$b_2 + f_2 - \frac{\partial p}{\partial z} = 0 \quad \text{…. (2)}$$

where $b_1 = -\rho_1 g, \; b_2 = -\rho_2 g$

$$f_1 = f_{12} = -180\frac{\mu_f j_{fo}}{d^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

$$f_2 = f_{21} = \frac{180\,\mu_f j_{fo}}{d^2}\frac{(1-\varepsilon)}{\varepsilon^2}$$

Therefore eqns (1) and (2) become,

$$-\frac{\partial p}{\partial z} - \rho_1 g - 180\frac{\mu_f j_{fo}}{d^2}.\frac{(1-\varepsilon)^2}{\varepsilon^3} = 0 \quad ....(3)$$

$$-\frac{dp}{dz} - \rho_2 g + 180\frac{\mu_f j_{fo}}{d^2}.\frac{(1-\varepsilon)}{\varepsilon^2} = 0 \quad ....(4)$$

Subtracting eqn (2) from equation (1) we get

$$(\rho_2 - \rho_1)g - \frac{180\,\mu_f j_{fo}}{d^2}.\frac{(1-\varepsilon)}{\varepsilon^2}\left[\frac{(1-\varepsilon)}{\varepsilon} + 1\right]$$

$$or,\; j_{fo} = \frac{d^2 g(\rho_2 - \rho_1)}{180\mu_f}.\frac{\varepsilon^3}{(1-\varepsilon)}$$

And from equation (1) we get,

$$\frac{\partial p}{\partial z} = b_1 + f_1$$

$$= -\rho_1 g - \frac{180\mu_f j_{fo}}{d^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

$$= -\rho_1 g - \frac{180\mu_f}{d^2}.\frac{(1-\varepsilon)^2}{\varepsilon^3}.\frac{d^2 g(\rho_2 - \rho_1)}{180\mu_f}.\frac{\varepsilon^3}{(1-\varepsilon)}$$

$$= -\rho_1 g - (1-\varepsilon)(\rho_2 - \rho_1)$$

$$= g\left[-\rho_1 - (1-\varepsilon)\rho_2 + (1-\varepsilon)\rho_1\right]$$

$$= g\left[-\rho_1 - (1-\varepsilon)\rho_2 + \rho_1 - \varepsilon\rho_1\right]$$

$$= -g\left[\varepsilon\rho_1 + (1-\varepsilon)\rho_2\right]$$

Applying Lockhart Martinelli assumption to annular flow

$$\frac{A_l}{A} = 1-\alpha = \frac{\gamma \frac{\pi}{4} D_l^{\ 2}}{\frac{\pi}{4} D^{\ 2}} = \gamma \left(\frac{D_l}{D} \right)^2 \quad (1)$$

or

$$\gamma = (1-\alpha)\left(\frac{D}{D_l} \right)^2 \quad (2)$$

Substituting the value of γ in eqn (1) we get:

$$\phi_l^2 = (1-\alpha)^{n-2} \left(\frac{D}{D_l} \right)^{2(n-2)} \left(\frac{D_l}{D} \right)^{n-5} \quad (3)$$

Chapter 8

Problem 1 : Air water mixture flows in a 3m long 5cm diameter pipe and discharges at 94.7 psia. Assume bubbly flow under turbulent conditions at 27^0c, calculate inlet pressure for a volumetric flow of J_l=0.15 m/s and J_g=4.5 m/s at atmospheric pressure & temperature.

Solution : $j_{21} = U_{gj}\alpha$

$$\alpha = \frac{j_2}{j_{TP}}\left(1 - \frac{j_{21}}{j_2}\right)$$

$$= \frac{j_2}{j_1 + j_2 + U_{gj}} \qquad --------(1)$$

As per changes down the pipe, j_2 will change

Assuming isothermal expansion, $j_2 = \frac{pa}{p}(j_2)|_{pa} \quad -----(2)$

Check for sonic flow -----> con. Choking

Then $p_{exit} \neq p_{atm}$

Acc. $\left(\frac{-dp}{dz}\right)_{acc} = G_2 \frac{du_2}{dz} + G_1 \frac{du_1}{dz}$

u_2, u_1 from equation (1) in equation $U_1 = \frac{j_1}{1-\alpha} \quad U_2 = \frac{j_2}{\alpha}$

$$U_2 = j_1 + j_2 + U_{gj}$$

$$U_1 = j_1 \quad \frac{j_1 + j_2 + U_{2j}}{j_1 + U_{2j}}$$

Only j_2 changes down duct

$$\therefore \frac{dU_g}{dz} = \frac{dj_2}{dz} \qquad \frac{du_1}{dz} = \frac{dj_2}{dz} \frac{j_1}{j_1 + U_{gj}}$$

$$\therefore \left(\frac{-dp}{dz}\right)_{acc} = \left(G_2 + G_1 \frac{j_1}{j_1 + U_{2j}}\right)\frac{dj_2}{dz}$$

For small intervals, $\frac{dj_2}{dz}$ found by differentiating eqn. (2)

$$\frac{dj_2}{dz} = -(j_2)_{pa} \frac{pa}{p^2} \frac{dp}{dz}$$

$$\therefore \left(\frac{-dp}{dz}\right)_{acc} = \left(G_2 + G_1 \frac{j_1}{j_1 + U_{2j}}\right)(j_2)_{pa} \frac{pa}{p^2} \frac{dp}{dz}$$

Friction

$$\left(\frac{-dp}{dz}\right)_f = \frac{2 f_{TP} G_j}{D}$$

$$f_{TP} = 0.005$$

$$G = \rho_1 j_1 + \rho_2 j_2$$

$$j = j_1 + j_2$$

$$\left(\frac{-dp}{dz}\right)_g = g\left[\alpha\rho_2 + (-\alpha)\rho_1\right]$$

$$\left(\frac{-dp}{dz}\right) = \frac{2 f_{TP} Gj \ / D + g\left[\alpha\rho_2 + (1-\alpha)\rho_1\right]}{1 - \left[G_2 + G_1 j_1 /(j_1 + U_{2j})(j_2)_{pa} \frac{pa}{p^2}\right]}$$

$\swarrow$

$$\mu a^2 \qquad for \quad \mu a^2 > 1 \quad \left(\frac{-dp}{dz}\right)_{acc} - ve$$

Flow supersonic

Not permissible.

$\therefore$ P_{exit} adjust until choking is reached & $\mu_a = 1$

$\therefore$ Condition of choking

$$P_c^2 = Pa(j_2)_{Pa}\left[G_2 + G_1\,\frac{j_1}{j_1 + U_{2j}}\right]$$

$\swarrow$

<u>Problem 2 :</u> For annular flow pattern (Gas Core + Liquid film), deduce $fn(\alpha)$ for $\phi_l^2 = fn\ (\alpha)$

<u>Solution:</u> Annular flow can be analyzed by separated flow model.

A very simple model of separated flow can be developed by assuming that the two phase flow, without interaction, in two horizontal separate cylinders and that the areas of the cross sections of these cylinders add up to the cross-sectional area of the actual pipe. The pressure drop in each of the imagined cylinder is the same as in the actual flow, is due to frictional effects only, and is calculated from single phase flow theory.

Therefore,

$$\left(\frac{-dp}{dz}\right)_{F_{TP}} = \left(\frac{-dp}{dz}\right)_{liquid\ cylinder} = \left(\frac{-dp}{dz}\right)_{gas\,cylinder}$$

$A_l = \gamma\frac{\pi}{4}D_l^2 =$ cross-sectional area of liquid cylinder

$A_g = \delta\frac{\pi}{4}D_g^2 =$ cross-sectional area of gas cylinder

Where $A_l + A_g = A =$ cross-sectional area of main cylinder

γ and δ are the shape factor.

$A = \frac{\pi}{4}D^2, D =$ Diameter of main cylinder

Now

$$\left(\frac{-dp}{dz}\right)_{f,liquid\ cylinder}$$

$$= \frac{2C_{fl}}{D_l}\rho_l U_l^2$$

$$= \frac{2C_{fl}}{D_l}\rho_l\left(\frac{W_l}{A_l\rho_l}\right)^2$$

$$= \frac{2C_{fl}}{D_l}\frac{\rho_l W_l^2}{A_l^2\rho_l^2}$$

$$= \frac{2C_{fl}}{D_l}\frac{W_l^2}{A_l^2\rho_l}$$

$$= \frac{2}{D_l}\left[K\,\mathrm{Re}_l^{-n}\right]\frac{W_l^2}{A_l^2\rho_l}$$

$$Now\ \mathrm{Re}_l = \frac{\rho_l U_l D_l}{\mu_l}$$

$$Now\ W_l = \rho_l U_l A_l$$

$$\therefore \rho_l U_l = \frac{W_l}{A_l}$$

$$\therefore \mathrm{Re}_l = \frac{W_l D_l}{\mu_l A_l}$$

$$\therefore \left(\frac{-dp}{dz}\right)_{liquid\ cylinder} = \frac{2}{D_l}\left[k\left\{\frac{W_l D_l}{\mu_l A_l}\right\}^{-n}\right]\frac{W_l^2}{A_l^2\rho_l}$$

$$= \frac{2}{D_l}.k.\frac{W_l^{2-n}.D_l^{-n}}{\mu_l^{-n}.(A_l)^{2-n}.\rho_l}$$

$$= \frac{2}{D_l}.k.\frac{W_l^{2-n}.D_l^{-n}}{\mu_l^{-n}.\left(\frac{\pi}{4}\right)^{2-n}.\left(D_l^2\right)^{2-n}.\rho_l.\left(\gamma\right)^{2-n}}$$

$$= \left[\frac{2kW_l^{2-n}\mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n}.\rho_l}\right]\gamma^{n-2}.D_l^{-n-1-4+2n}$$

$$= \left[\frac{2kW_l^{2-n}\mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n}.\rho_l}\right]\gamma^{n-2}.D_l^{n-5}$$

$$Now\ \phi_l^2 = \frac{\left[\frac{-dp}{dz}\right]_{F_{T.P}}}{\left[\frac{-dp}{dz}\right]_{F_l}}$$

$$or\left[\frac{-dp}{dz}\right]_{F_l} = \frac{2}{D}.f_l.\frac{G^2(1-x^2)}{\rho_l}$$

$$= \frac{2}{D}\left[k\,R_{e_l}^{-n}\right]\frac{W_l^2}{A^2\rho_l}, \left[\because G(1-x) = \frac{W_l}{A}\right]$$

$$= \frac{2}{D}\left[k\left\{\frac{G(1-x)}{\mu_l}\right\}^{-n}\right].\frac{W_l^2}{A^2\rho_l}\ \left[\because G(1-x) = \frac{W_l}{A}\right]$$

$$= \frac{2}{D}.k.\left[\frac{W_l\ D}{\mu_l A_l}\right]^{-n}.\frac{W_l^2}{A^2\rho_l}$$

$$= \frac{2}{D}.k.\frac{W_l^{2-n}.D^{-n}}{A^{2-n}.\mu_l^{-n}.\rho_l}$$

$$= \frac{2}{D}.k\frac{W_l^{2-n}}{\left(\frac{\pi}{4}D^2\right)^{2-n}}.\frac{\mu_l^n}{\rho_l}.D^{-n}$$

$$= \left[\frac{2k.W_l^{2-n} \mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n} \rho_l} \right] D^{-n-1-4+2n}$$

$$= \left[\frac{2k.W_l^{2-n} \mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n} \rho_l} \right] D^{n-5}$$

$$\phi_l^2 = \frac{\left[\frac{-dp}{dz}\right]_{F_{T.P.}}}{\left[\frac{-dp}{dz}\right]_{F_l}}$$

$$= \frac{\left(\frac{-dp}{dz}\right)_{liquid\,cylinder}}{\left[\frac{-dp}{dz}\right]_{F_l}}$$

$$= \frac{\left[\frac{2\,k\,W_l^{2-n} \mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n} \rho_l} \right] \gamma^{n-2}.D_l^{n-5}}{\left[\frac{2\,k\,W_l^{2-n} \mu_l^n}{\left(\frac{\pi}{4}\right)^{2-n} \rho_l} \right] .D_l^{n-5}}$$

$$\therefore \phi_l^2 = \gamma^{n-2}.\left(\frac{D_l}{D}\right)^{n-5}$$

In the expression of ϕ_l^2 , n is coming from expression of f_l

For laminar flow n=1

If we take γ =shape factor=1, then

$$\phi_l^2 = \left(\frac{D_l}{D}\right)^{1-5} = \left(\frac{D_l}{D}\right)^{-4}$$

$$= \left(\frac{D_l^2}{D^2}\right)^{-2} = \left[\frac{\frac{\pi}{4}D_l^2}{\frac{\pi}{4}D^2}\right]^{-2}$$

$$\therefore \phi_l^2 = \left(\frac{A_l}{A}\right)^{-2}$$

$$= (1-\alpha)^{-2} \quad \left(\because \frac{A_l}{A} = 1-\alpha\right)$$

$$= \frac{1}{(1-\alpha)^2}$$

Problem 3 Estimate the rise velocity of air bubbles in H_2O for equivalent radius of 0.25, 1.5, 0.85 cm given $\rho_l = 1 g/cc,\ \mu_l = 0.01\ poise\ \ \sigma = 70\ dyne/cm\ \ g = 981\ cm/s^2$

Solution: All the bubbles lie in the Stokes region

$$U_\infty = \frac{1}{18} d^2 g\left(\rho_b - \rho_g\right)/\mu_b$$
$$= 611624.72 \times d^2\ m/s$$
$$= 3.823 m/s\left(r = 0.125 cm\right)$$
$$= 15.29\ m/s\left(0.25 cm\right)$$
$$= 550.46 m/s\left(1.5 cm\right)$$
$$= 176.76 m/s\left(0.85 cm\right)$$

Problem 4 Find relation between Q_1 and Q_2 for flooding in a pipe of radius 25 cm assuming bubbly flow with n=2 and $u_\infty = 1.5\ m/s$

Solution:

For Flooding Condition,

$$u_\infty\left[\frac{\alpha}{1-\alpha}.n+1\right](1-\alpha)^n=1$$

$$or,\ 1.5\left[\frac{2\alpha}{1-\alpha}+1\right](1-\alpha)^2=1$$

$$or,\ 3(\alpha+1)(1-\alpha)=2$$

$$or,\ 3(1-\alpha^2)=3$$

$$or, \alpha=\pm\frac{1}{\sqrt{3}}$$

$$j_{21}=u_\infty\alpha(1-\alpha)^n$$

$$=0.0478$$

$$A=\frac{3.14}{4}(0.5)^2=0.19625$$

$$\alpha=\frac{Q_2-Aj_{21}}{C_0(Q_1+Q_2)}$$

$$\Rightarrow Q_1+0.0054-0.4226Q_2=0$$

Problem 5 :

What is the velocity of sound in a hydrogen-water mixture at 6894.7KPa, 210C, and with mean density 640.7 kg/m3? Repeat the problem with the pressure as 3.44KPa and the radius of the bubble as 1 mm.

Solution :

Mean density =640.7 kg/m^3

$$\therefore \frac{\rho_g+\rho_f}{2}=640.7$$

$$\rho_f=1000\,kg/m^3$$

$$\therefore \rho_g=281.4\,kg/m^3$$

Taking $\alpha=0.5$

The velocity of sound in bubbly mixture can be calculated from the following equation.

$$C^2=\frac{1}{\left[\alpha\rho_g+(1-\alpha)\rho_f\right]\left[\alpha/\rho_g C_g{}^2+(1-\alpha)/\rho_f C_f{}^2\right]}$$

$$C_f>C_g;\rho_g<\rho_f$$

The above case be approximated as

$$C \approx \frac{C_g}{\left[\alpha(1-\alpha)^{\rho_f}/_{\rho_g}\right]^{1/2}}$$

Now, $C_g^2 \approx r_g RT$ for a rapid compress

$$R = 8.314\, Jmol^{-1}\, K^{-1}$$
$$T = 294\, K$$
$$r_g = 1.4(\text{for diatomic } H_2 \text{ gas})$$
$$\therefore C_g^2 = 1.4 \times 8.314 \times 294$$
$$C_g = 58.49$$

$$C = \frac{C_g}{\left[\alpha(1-\alpha)^{\rho_f}/_{\rho_g}\right]^{1/2}} = \frac{58.49}{\left[0.5 \times 0.5 \times \frac{1000}{281.4}\right]^{1/2}}$$
$$C = 62.05\, m/s.$$

At low pressure the sonic velocity can be obtained by following equation

$$C = \frac{C_g}{\left[\alpha(1-\alpha)^{\rho_f}/_{\rho_g}\right]^{1/2}} \times \left[1 + \frac{2\sigma}{3PR_0 + 4\sigma}\right]^{-1/2}$$

$$\sigma = 70 \times 10^{-3}\, N/m$$
$$P = 3.44 KPa$$
$$R_b = 10^{-3} m$$
$$C = 61.64 m/s$$

Problem 6 :

In a 10 cms diameter Counter current flow bubble column, it was observed that flooding occurred for the following mass fluxes of carbon dioxide and water in (Kg/hr/m^2)

$-W_f$	**73305**	**48870**	**24435**
W_g	**185.7**	**205.54**	**234.57**

Compare the results with theoretical values at 15 Psia, 21^0 C.

Solution Let us consider bubble rise velocity through infinite medium (u_∞) is independent of bubble size i.e. , in region 4

$$\therefore u_\infty = 1.18\left(\frac{g\sigma}{\rho_f}\right)^{0.25}$$
$$= 0.1909 m/s$$

Now, $j_{21} = u_\infty \propto (1-\alpha)^n$

$$j_{21} = j_2 - \alpha j$$

Again, $j_2 - \alpha j = u_\infty \propto (1-\alpha)^n$

$$j_2 = \alpha j + u_\infty \propto (1-\alpha)^n$$

Now, $\rho_2 = \frac{PM}{RT} = \frac{101.325 \times 44}{8.314 \times 294} = 1.823 kg/m^3$

We have $G_2 = 185.7 Kg/m^2 hr$

$$\therefore j_2 = \frac{185.7}{1.823 \times 3600} \frac{m^3}{m^2.s}$$
$$= 0.0282 m^3/m^2.s$$

Similarly, $j_1 = \frac{73305}{1000 \times 3600} = 0.0203 m^3/m^2.s$

$$\therefore j = 0.0485 m^3/m^2 s$$

Now, $j_2 = \alpha j + (1-\alpha)^n \alpha u_\infty$

$$0.0485 j + \alpha(1-\alpha)^{1.53} \times 0.1909 - 0.0282 = 0$$

After solving this equation we get,

$$\alpha = 0.1412$$

Now, we know that at flooding point,

$$\frac{dj_{21}}{d\alpha} = \frac{j_{21} - j_2}{\alpha}$$

$$j_{21} = u_\infty \alpha (1-\alpha)^n$$

$$\frac{dj_{21}}{d\alpha} = u_\infty \left[-\alpha n (1-\alpha)^{n-1} + (1-\alpha)^n \right]$$

$$= u_\infty (1-\alpha)^n \left[1 - \frac{\alpha n}{1-\alpha} \right]$$

$$= u_\infty (1-\alpha)^{n-1} (1 - \alpha - \alpha n)$$

$$\frac{j_{21} - j_2}{\alpha} = \frac{dj_{21}}{d\alpha}$$

$$j_{21} - j_2 = \alpha \frac{dj_{21}}{d\alpha}$$

$$j_{21} - \alpha \frac{dj_{21}}{d\alpha} = j_2$$

$$j_2 = u_\infty \alpha (1-\alpha)^n - \alpha u_\infty (1-\alpha)^{n-1} (1 - \alpha - \alpha n)$$

$$= u_\alpha \alpha (1-\alpha)^n \left[1 - \frac{(1 - \alpha - \alpha n)}{1-\alpha} \right]$$

$$= u_\infty \alpha (1-\alpha)^n$$

$$j_2 = u_\infty \alpha (1-\alpha)^n \left[\frac{\cancel{1} - \cancel{\alpha} - \cancel{1} + \cancel{\alpha} + \alpha n}{1-\alpha} \right]$$

$$j_2 = u_\infty \alpha^2 n (1-\alpha)^{n-1}$$

$$\therefore j_1 = -j_{21} - (1-\alpha)\frac{dj_{21}}{d\alpha}$$
$$= -u_\infty \alpha (1-\alpha)^n - (1-\alpha) u_\infty (1-\alpha)^{n-1} (1-\alpha-\alpha n)$$
$$= -u_\infty \alpha (1-\alpha)^n - u_\infty (1-\alpha)^n (1-\alpha-\alpha n)$$
$$= -u_\infty (1-\alpha)^n \left[\not{\alpha} + 1 - \not{\alpha} - \alpha n \right]$$
$$= -u_\infty (1-\alpha)^n (1-\alpha n)$$

So, far $\alpha = 0.1412$

$$j_2 = 0.1909 \times (0.1412)^2 \times 1.53 \times (1-0.1412)^{1.53-1}$$
$$= 5.4406 \times 10^{-3} m^3 / m^2.s$$
$$G_2 = 35.70 kg / m^2.hr$$

Similarly,

$$j_1 = -0.11856 m^3 / m^2.s$$
$$G_1 = 426834.35 Kg / m^2.hr$$

So , far other cases theoretical mass fluxes are shown in table

G_1 $\left(kg / m^2.hr\right)$	-426834.35	-389878.22	-338020.6
G_2 $\left(kg / m^2.hr\right)$	35.70	47.429	68.56
α	0.1412	0.1650	0.2007

Problem 7:

A contain silicone fluid has a viscosity of 5000 CP a surface tension of 21 dynes/cm, and a density of 1 g/cm^3

. What is the rise velocity of slug – flow bubble in stationary liquid in vertical pipes with diameters 0.25 cm, 1.2 cm, 12.5 cm, and 24 cm.

Solution :

$\mu_f = 5000CP \quad \sigma = 21 dynes/cm \quad \rho_f = 1gm/cm^3$

Neglecting gas density in comparison to liquid density

Eotvos number

$$N_{EO} = \frac{gD^2(\rho_f - \rho_g)}{\sigma} = \frac{gD^2\rho_f}{\sigma} = 2.916 < 3.37$$

Inverse viscosity

$$N_f = \frac{\left[D^2 g(\rho_f - \rho_g)\rho_f\right]^{1/2}}{\mu_f}$$

$$= \frac{\left(D^3 g \rho_f^{\ 2}\right)^{1/2}}{\mu_f}$$

$$= \frac{\left(0.25^3 \times 980 \times 1\right)^{1/2}}{5000 \times 10^{-2}} = 0.0782 < 2$$

Archimedes No.

$$N_{Ar} = \frac{\sigma^{3/2}\rho_f^{\ 1/2}}{\mu_f^{\ 2} g^{1/2}} = \frac{(21)^{3/2} \times 1}{(50)^2 \times (980)^2} = 1.229 \times 10^{-3}$$

Properly group,

$$Y = \frac{1}{N_{Ar}^{\ 2}} = 661375.66$$

Now,

$$K_1 = 0.345\left[1 - \exp(-0.01Nf/0.345)\right]\left[1 - \exp((3.37 - NE_0)/m)\right]$$

For, 0.25 cm diameter pipe

$$K_1 = 0.345$$

$$u_\infty = K_1 \rho_f^{\ -1/2}\left[gD\rho_f\right]^{1/2} = 5.4cm/s$$

$Diameter$ $d\,(cm)$	0.25	1.2	12.5	24
Eotvos N_{EO}	2.9167	67.2	7.2917×10^{3}	26880
Inverse Viscosity N_f	0.0783	0.823	27.6699	73.6199
K_1	-1.4315×10^{-5}	0.0075	0.1903	0.3042
u_∞ (cm/s)	—	0.2572	21.0624	46.6528
m	25	25	21.58	15.32

Problem 8 :

When a long bubble rises in a tube closed at the bottom, the value of j ahead of the bubble is not zero because of expansion of the gas in the hydrostatic pressure gradient . A bubble 0.00016 m^3 in volume is injected into a column of water 30.5 m high in a 0.0254 m pipe. If the temperature is 21^0 C and the pipes is closed at the bottom and open to the atmosphere at the top, how long does it take after release before the bubble breaks the surface.

Solution :

Bubble volume = $0.000lbm^3$

L_{tube} =30.5 m

D=0.0254m

Now, initially when bubble rises from bottom of the liquid column,

$$\frac{Bubble\,volume}{Tube\,volume}=\left(\frac{D-2\sigma}{D}\right)^2=\frac{4\times0.000lb}{3.14\times(0.0254)^2}$$

Where $\delta\rightarrow$ film thickness

$$\therefore \left(\frac{D-2\delta}{D}\right)^2 = 0.315$$

Now, considering a single bubble rises through a stationary water column from bottom to the upward direction. When it gets a long shape after rising certain height we can write :

$$\frac{Bubble\, volume}{Tube\, volume} = \left(\frac{D-2\sigma}{D}\right)^2 \frac{L_{TB}}{L_{TB}+L_{IS}}$$

For one unit cell,

$\therefore$ film thickness is very small in comparison to D

$$\therefore \frac{L_{TB}}{L} = 0.315$$

$$L_{TB} = 0.315 \times 30.5 = 9.6075m$$

Column

$$\therefore \mu = 1CP, \quad \rho_f = 1gm/cm^3 \quad \sigma = 70 dynes/cm$$

Neglecting gas density.

Eotvos No.

$$N_{EO} = \frac{gD^2\left(\rho_f\right)}{\sigma} = 90.3224$$

$$N_f = \frac{\left(D^3 g \rho_f^{\ 2}\right)^{1/2}}{\mu_f} = 1.267 \times 10^4$$

$$N_{Ar} = \frac{\sigma^{3/2} \rho_f^{\ 1/2}}{\mu_f^{\ 2} g^{1/2}} = 187082.86$$

$$Y = \frac{1}{N_{Ar}^{\ 2}} = 2.857 \times 10^{-11} \qquad m = 10$$

$$K_1 = 0.345\left[1-e^{\left(-0.01N_f/0.345\right)}\right]\left[1-e^{\frac{(3.37-N_{EO})}{m}}\right]$$

$$= 0.3198$$

$$\therefore u_\infty = 15.95 cm/s$$

So, time take for bubble to break

$$= \frac{9.6075 \times 10^2}{15.95} S$$
$$= 60.23$$
$$= 1 \min 23 s$$

Problem 9 :

What is the minimum tube size in which large bubbles of air will rise in stationary water as 30^0 C

a) On earth b) in a spaceship for which 'g'=0.003048cm/s^2

Solution : Bubble will not rise when there is surface tension dominating effect

$$N_{EO} = 3.37$$

i.e. $$\frac{gD^2\left(\rho_f - \rho_g\right)}{\sigma} = 3.37$$

On earth, g=980cm/s^2

Neglecting ρ_g

$$\frac{gD^2\rho_f}{\sigma} = 3.37$$
$$D^2 = \frac{3.37 \times \sigma}{g}$$
$$D^2 = \frac{3.37 \times 70}{980}$$
$$D = 0.49 cm$$

On space, $g = 0.003048 cm/s^2$

$$\frac{gD^2\rho_f}{\sigma} = 3.37$$
$$D^2 = \frac{3.37 \times 70}{0.003048}$$
$$D = 278.19 cm$$

So, minimum tube size on earth = 0.49cm

Minimum tube size on space=278.19cm

Problem 10 :

A liquid metals $\left(\sigma = 300 dynes/cm; \rho_f = 5g/cm^3, \mu = 0.02 poise\right)$ **fills a** 0.0095 **m diameter horizontal pipe. It is derived to blow gas through the pipe to the metal and solidify it as a uniform film** $0.0127cm$ **thick on the walls. What gas flow rate should be used ?**

Solution :

$\sigma = 300 dynes/cm \quad \rho_f = 5g/cm^3 \quad \mu = 0.02 poise \quad D = 0.0095m \quad \delta = 0.0127cm$

The available flow area for gas in the tube

$$A_b = \pi\left(D/2 - \delta\right)^2 = 3.14\left(\frac{0.95}{2} - 0.0127\right)^2 = 0.671cm^2$$

Area of tube $A = \frac{\pi D^2}{4} = \frac{3.14 \times 0.95^2}{4} = 0.708cm^2$

$$\frac{v_b}{j} = \frac{A}{A_b} = \frac{0.708}{0.671} = 1.0558$$

$$j/v_b = 0.947$$

$$\lambda = \frac{\mu_f^{\,2}}{D\rho_f^{\,\sigma}} = \frac{(0.02)^2}{0.95 \times 5 \times 300} = 2.80 \times 10^{-7}$$

From figure

$$\frac{j\mu_f}{\sigma} = 3.2 \times 10^{-3}$$

$$j = 48cm/s$$

$$v_b = 48 \times 1.0558 = 50.64cm/s$$

$$Q_g = 35.90cm^3/s$$

Chapter 10

1. Discuss a commonly used technique for measuring volume average void fraction of a flowing vapor-liquid mixture? What are the drawbacks of the technique?

   Quick closing value technique

   Drawbacks –

   i) Finite Time required to close down the values. This may cause changes in system hydrodynamics.

   ii) Finite Time required to bring system back to steady state. So not possible for a continuous operating plant.

   ii) Not suitable for transient measurements.

2. How can area average and chordal average measurements of void fraction be converted to volume average values?

   Area average measurements of void fraction is the volume average value for infinitesimal length of the test section. So several area average values at different axial lengths gives the volume average value. When void fraction does not vary with length (fuliy developed flows), both are equal.

   Chordal average value is connected to area average value either by mathematical manipulation or by the use of multiple beams.

3. Why does one need to know the voidage profile in addition to the average void fraction of a gas-liquid mixture?

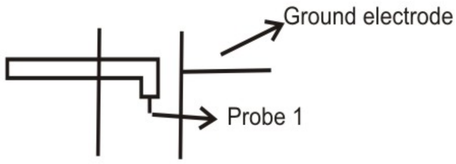
   Voidage profile gives an estimate of the distribution of voids in the flow passage. This adequately describes the structures of the flow field and identifies sites of active transport and reactions.

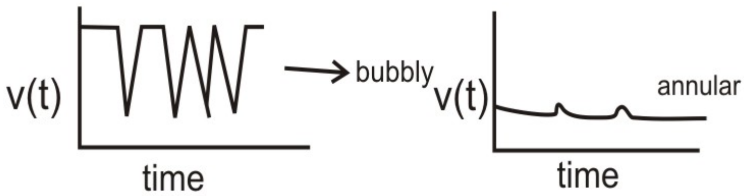
4. What are the drawbacks of photographic methods of flow pattern estimation?

   Refer to section 10.3, part 1

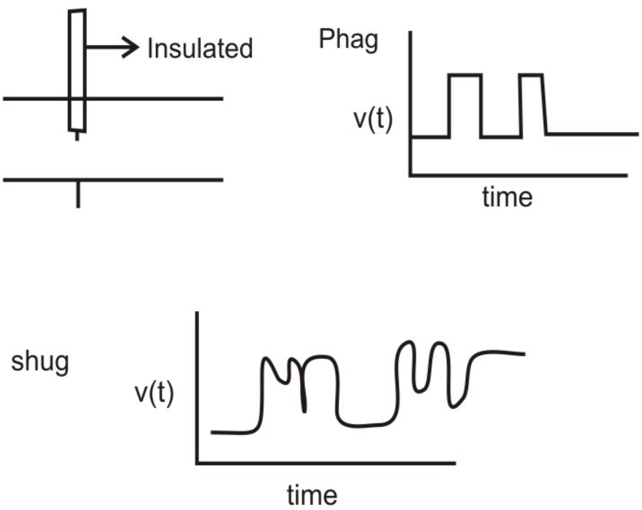
5. Suggest a suitable arrangement (shown diagrammatically) of the conductivity probes for distinguishing between (a)bubbly and annular flow in vertical pipes, (b) plug and slug flow in horizontal pipes, (c) stratified and annular flow in horizontal pipes,

   The probe signal for bubbly and annular flow are as follows a)





b)



6. State any three limitations of the radiation attenuation technique for estimation of void fraction and suggest ways of minimizing them.

   Refer to section 10.2.1

7. State any two limitations of the conductivity probe technique for gas-liquid systems.

   i) Needs a priori knowledgeof flow pattern

ii) Does not work for gas continuous patterns

8. State the principle and the specific application of (a) infra red absorption method (b) Electromagnetic flow metering technique for estimation of void fraction.

   a) IR absorption technique based on the differential amount of absorption of IR ray by the two phases. Specifically suitable for high void fraction flows.
   b) Electromagnetic flow metering based on principle of independent measurement of average liquid velocity (u_L) from which α can be calculated as $(1-\alpha)=\frac{j_L}{u_L}$ for low quality flows.

   Specific application for liquid metal system.

9. For measurement of two phase pressure drop when are gas filled lines preferred to liquid filled ones?

   For low offset value at zero Δp

10. What are the advantages of liquid filled lines in general?

    Less chances of gas ingress since liquid tends to meet the manometer lines and pumping action is less severe since it is incompressible.

11. What are the advantages and disadvantages of differential pressure transducer over absolute ones?
    Refer to text, page
12. Define liquid holdup for gas-liquid systems.

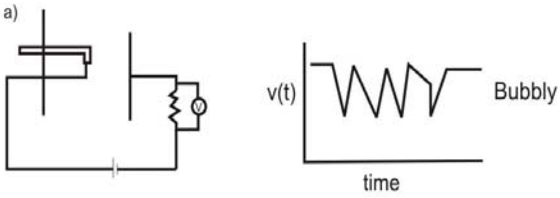
    Refer to chap 4

13. Discuss briefly the different ways of expressing liquid holdup and one conventional technique to measure each of them.

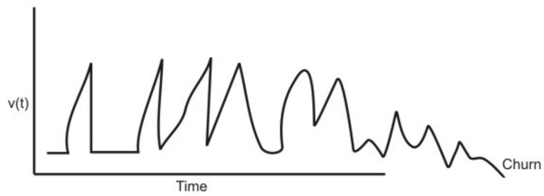
    Refer to chap 4

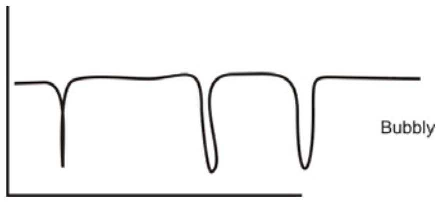
14. With the help of a schematic (i) show the location of the different probes and (ii) mention the characteristics of the probe signals which distinguish between the following gas-liquid flow patterns: wavy annular and bubbly in vertical flow (b) churn and bubbly in vertical flow

a)



(b)





15. How can the following methods be used for flow pattern detectors: Average pressure gradient (b) transient pressure signal

Refer to section 10.3 subsection 3

16. (ii) State a suitable technique to measure the in-situ composition of two phase flow under the following conditions: (a) High quality steam water flow (b) water content of margarine

a) Infra red absorption
b) Microwave absorption

17. Discuss the PSDF analysis for flow pattern identification. What are the different PSDFs obtained for gas-liquid two phase flow in **vertical** tubes.

Refer to section 10.3 subsection 3

18. What are the problems of using differential pressure transducers for measurement of pressure drop in two phase system.

i) All problems associated with ambiguity in tapping line content as discussed for manometers
ii) Slight fluctuations due to rig vibration can alter readings
iii) Pressure smaller than offset cannot be measured

19. How can we measure the void fraction during sodium liquid / vapor flow?

Electromagnetic flow metering

20. Discuss the acoustic method of void fraction measurement and state its problems.

Refer to section 10.2.7