Advanced Numerical Analysis for Chemical Engineering
Quiz -1 (1 hrs.)

1. Which of the following subsets of R^3 constitute a sub-space of R^3? Justify your answer in each case. *(4 points)*

   (a) All $\mathbf{x}$ such that $x_1 = x_2$ and $x_3 = 0$

   (b) All $\mathbf{x}$ such that $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

2. Consider space $X \equiv C$ (i.e. the set of complex numbers) with scalar field $F = C$. Show that $\langle z_1, z_2 \rangle = \overline{z}_1 z_2$ defines an inner product on C. *(4 points)*

3. Consider $X = R^2$ with $\langle \mathbf{x}, \mathbf{y} \rangle_W = \mathbf{x}^T W \mathbf{y}$

$$W = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

   Given a set of two linearly independent vectors in R^2

$$\mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} ; \ \mathbf{x}^{(2)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

   it is desired to construct and orthonormal set. Applying Gram Schmidt procedure, find a set of orthonormal vectors $\left\{ \mathbf{e}^{(1)}, \mathbf{e}^{(2)} \right\}$ starting from $\left\{ \mathbf{x}^{(1)}, \mathbf{x}^{(2)} \right\}$. *(3 points)*

4. Show that in R^n the 2-norm (Euclidean norm) and the $1-$norm are equivalent **i.e.** (*4 points*)

$$\sqrt{n} \left\| \mathbf{x} \right\|_1 \leq \left\| \mathbf{x} \right\|_2 \leq \left\| \mathbf{x} \right\|_1$$

Additional Information:

Definition 1 ***(Subspace):*** *A non-empty subset M of a vector space X is called subspace of X if every vector $\alpha \mathbf{x} + \beta \mathbf{y}$ is in M wherever $\mathbf{x}$ and $\mathbf{y}$ are both in M.*

Definition 2 ***(Inner Product Space):*** *An inner product space is a linear vector space X together with an inner product defined on $X \times X$. Corresponding to each pair of vectors $\mathbf{x}, \mathbf{y} \in X$ the inner product $\langle \mathbf{x}, \mathbf{y} \rangle$ of $\mathbf{x}$ and $\mathbf{y}$ is a scalar. The inner product satisfies following axioms.*

1. $\langle \mathbf{x}, \mathbf{y} \rangle = \overline{\langle \mathbf{y}, \mathbf{x} \rangle}$ (complex conjugate)

2. $\langle \mathbf{x} + \mathbf{y}, \mathbf{z} \rangle = \langle \mathbf{x}, \mathbf{z} \rangle + \langle \mathbf{y}, \mathbf{z} \rangle$

3. $\langle \lambda \mathbf{x}, \mathbf{y} \rangle = \overline{\lambda} \langle \mathbf{x}, \mathbf{y} \rangle$ and $\langle \mathbf{x}, \lambda \mathbf{y} \rangle = \lambda \langle \mathbf{x}, \mathbf{y} \rangle$

4. $\langle \mathbf{x}, \mathbf{x} \rangle \geq 0$ and $\langle \mathbf{x}, \mathbf{x} \rangle = 0$ if and only if $\mathbf{x} = \overline{0}$.