Advanced Process Control
Computing Examination III (2 hours)
Question Paper

Problem: Simulate closed loop servo and regulatory response of MIMO LQG controller

- **Plant Simulation** Dynamics of the system / plant under consideration is governed set of ODEs given in the attached pdf file. Measured outputs are related to the states as follows

$$\mathbf{Y}(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{X}(k) + \mathbf{v}(k)$$

  where $\mathbf{v}(k)$ are zero mean Gaussian white noise sequence with

$$Cov\left[\mathbf{v}(k)\right] = \begin{bmatrix} (0.01)^2 & 0 \\ 0 & (0.1)^2 \end{bmatrix}$$

  - **Load data file '*CSTR_para.mat*' to initialize the model parameters in Matlab environment**
  - Initialize the plant dynamics with $\mathbf{X}(0) = \begin{bmatrix} -0.1 & 7 \end{bmatrix}^T + \mathbf{X}_s$

- **Identified Model for State Estimation:** Innovation form of state space model (observer) identified from input - output data of the form

$$\begin{aligned}
\mathbf{z}(k+1) &= [\Phi_{id}]\,\mathbf{z}(k) + [\Gamma_{id}]\,\mathbf{u}(k) + L_\infty \mathbf{e}(k) \\
\mathbf{y}(t) &= [\mathbf{C}_{id}]\,\mathbf{z}(k) + \mathbf{e}(k) \\
\Phi_{id} &= \begin{bmatrix} -0.0566 & -0.0172 & 0.0152 & -0.0024 \\ -0.4700 & 0.4462 & -1.6525 & 1.4489 \\ 0.0701 & 0.2267 & -1.1241 & 0.2364 \\ 0.2757 & 0.4180 & -2.6440 & -0.1962 \end{bmatrix} \\
[\Gamma_{id}] &= \begin{bmatrix} -0.0637 & 1.4410 \\ 0.2292 & -0.7569 \\ -0.0001 & -0.2745 \\ -0.2813 & 0.5369 \end{bmatrix} \; ; \quad L_\infty = \begin{bmatrix} 0.1181 & 0.0078 \\ -2.7534 & 0.2056 \\ -1.5151 & -0.0148 \\ -1.6671 & -0.1771 \end{bmatrix} \\
[\mathbf{C}_{id}] &= \begin{bmatrix} -0.0286 & -0.0710 & 0.2545 & -0.1055 \\ 19.0248 & 0.2022 & 0.2906 & 0.7323 \end{bmatrix}
\end{aligned}$$

is to be used for controller design and implementation. Here, $\{\mathbf{e}(k)\}$ is a zero mean Gaussian white noise sequence with

$$Cov\left[\mathbf{e}(k)\right] = \begin{bmatrix} (0.01)^2 & 0 \\ 0 & (0.2648)^2 \end{bmatrix}$$

- **Load data file** '*idmod.mat*' **to initialize the model matrices** $([\Phi_{id}], [\Gamma_{id}], L_\infty, \mathbf{C}_{id})$ **in Maltab simulation environment.**
- Initialize the state estimator with $\widehat{\mathbf{z}}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

- **LQ Controller Design:** Using the identified model given above, develop a linear quadratic state feedback control law (innovation bias formulation) with the following choice of tuning parameters

$$W_x = [\mathbf{C}_{id}]^T \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} [\mathbf{C}_{id}] \quad \text{and} \quad W_u = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}$$

  Use Matlab function '*dlqr*' for LQR design. Matlab syntax for invoking this function is of the form

$$[\mathbf{G}_\infty, \mathbf{S}_\infty, \mathbf{E}] = dlqr(\Phi_{id}, \Gamma_{id}, W_x, W_u);$$

  Other controller related matrices and tuning parameters are as follows

$$\begin{aligned} \mathbf{\Phi}_e &= 0.9\mathbf{I} \quad \text{and} \quad \mathbf{\Phi}_r = 0.8\mathbf{I} \\ \mathbf{K}_u &= [\mathbf{C}_{id}]\,(\mathbf{I}-\Phi_{id})^{-1}\,[\Gamma_{id}] \\ \mathbf{K}_e &= [\mathbf{C}_{id}]\,(\mathbf{I}-\Phi_{id})^{-1}\,\mathbf{L}_\infty + \mathbf{I} \end{aligned}$$

- **Controller calculations at instant k**

$$\begin{aligned} \widehat{\mathbf{z}}(k) &= [\Phi_{id}]\,\widehat{\mathbf{z}}(k-1) + [\Gamma_{id}]\,\mathbf{u}(k-1) + L_\infty \mathbf{e}(k-1) \\ \mathbf{e}(k) &= \mathbf{y}(t) - [\mathbf{C}_{id}]\,\widehat{\mathbf{z}}(k) \end{aligned}$$

$$\begin{aligned} \mathbf{e}_f(k) &= \mathbf{\Phi}_e\ \mathbf{e}_f(k-1) + [\mathbf{I} - \mathbf{\Phi}_e]\mathbf{e}(k) \\ \mathbf{r}(k) &= \mathbf{\Phi}_r\ \mathbf{r}(k-1) + [\mathbf{I} - \mathbf{\Phi}_r]\mathbf{y}_{sp} \end{aligned}$$

$$\begin{aligned} \mathbf{u}_s(k) &= \mathbf{K}_u^{-1}\,[\mathbf{r}(k) - \mathbf{K}_e\,\mathbf{e}_f(k)] \\ \mathbf{z}_s(k) &= (\mathbf{I}-\Phi_{id})^{-1}\ \ [(\Gamma_{id})\,\mathbf{u}_s(k) + \mathbf{L}_\infty \mathbf{e}_f(k)] \end{aligned}$$

$$\mathbf{u}(k) = \mathbf{u}_s(k) - \mathbf{G}_\infty\ [\widehat{\mathbf{z}}(k) - \mathbf{z}_s(k)]$$

- **Servo and regulatory problem simulation:** Simulate the closed loop system for k = 0,1,2,...300 with the following servo and regulatory changes

  - Setpoint changes to be implemented

$$\begin{aligned} \mathbf{y}_{sp} &= \begin{bmatrix} 0 & 0 \end{bmatrix}^T \qquad \text{for } 0 \le k \le 50 \\ \mathbf{y}_{sp} &= \begin{bmatrix} -0.1 & 0 \end{bmatrix}^T \qquad \text{for } 51 \le k \le 125 \\ \mathbf{y}_{sp} &= \begin{bmatrix} 0.1 & -5 \end{bmatrix}^T \qquad \text{for } k > 125 \end{aligned}$$

  - Unmeasured disturbance changes

$$\mathbf{d}(k) = \mathbf{d}_s + \mathbf{w}(k)$$

    where $\mathbf{w}(k)$ represents zero mean white noise signal with variance $\sigma^2 = 0.1^2$ and $\mathbf{d}_s$ changes as follows

$$\begin{aligned} \mathbf{d}_s &= \mathbf{0} \qquad \text{for } 0 \le k \le 225 \\ \mathbf{d}_s &= -0.15 \quad \text{for } k > 225 \end{aligned}$$

- **Display of simulation results**

  - Controlled Outputs: Compare $\mathbf{y}_i(k)$ v/s k and $\mathbf{r}_i(k)$ v/s k in **same** figure for $i = 1, 2$.
  - Manipulated Inputs: Plot $\mathbf{u}_i(k)$ v/s k for $i = 1, 2$ using *stairs* function in Matlab
  - Plot unmeasured disturbance $\mathbf{d}(k)$ using *stairs* function in Matlab