Advanced Process Control
Mid-semester Examination
(2 hours and 25 Marks)

1. Consider an ARMA process

$$v(k) = \alpha v(k-1) + e(k) + \beta e(k-1) \tag{1}$$

where $\{e(k)\}$ is a zero mean white noise process with variance λ^2. It can be shown that stochastic process $\{v(k)\}$ has zero mean.

   (a) Derive expressions for cross-covariance $r_{ve}(1) = E[v(k)e(k-1)]$ (2 marks)

   (b) Derive expressions for auto-covariance $r_v(1) = E[v(k)v(k-1)]$. (4 marks)

2. Consider Box-Jenkin's model

$$y(k) = \frac{q^{-1} + 0.5q^{-2}}{(1-0.5q^{-1})(1-0.8q^{-1})}u(k) + \frac{1+0.5q^{-1}}{(1-0.8q^{-1})}e(k)$$

Derive one step prediction

$$\begin{aligned} \widehat{y}(k|k-1) &= [H(q)]^{-1} G(q)u(k) + \left[1 - (H(q))^{-1}\right] y(k) \\ y(k) &= \widehat{y}(k|k-1) + e(k) \end{aligned}$$

and express dynamics of $\widehat{y}(k|k-1)$ as a time domain difference equation. (6 marks)

3. Consider a coupled tank system in which dynamics of levels in the two tanks is governed by

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

where x denotes perturbations in level and u denotes perturbations in inlet flow.

   (a) It is desired to control this system (at the setpoint equal to the origin) using a feedback control law of the form

$$u = -\begin{bmatrix} \alpha & \beta \end{bmatrix} \mathbf{x}$$

   Determine the state space model (differential equation) that governs the closed loop dynamics in terms of unknowns (α, β). (2 marks)

   (b) Determine, if it exists, controller gains $\begin{bmatrix} \alpha & \beta \end{bmatrix}$ such that the state transition matrix for the closed loop system has eigenvalues at the roots of the following quadratic equation (4 marks)

$$\lambda^2 + 11\lambda + 30 = 0$$

4. Consider a **continuos time** linear perturbation model

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{A} = \begin{bmatrix} -2 & 1/2 & 1/2 \\ 1 & -3/2 & -1/2 \\ 1 & 1/2 & -5/2 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} \mathbf{u}$$

Eigenvalues of matrix **A** are -1, -2 and -3 and the continuous time system is asymptotically stable. Suppose discretization of the continuous time system is carried out using the Euler's method i.e. $[\Phi]_{Euler} = \mathbf{I} + T\mathbf{A}$. Then, find the range of sampling time T for which the discrete time model will retain the stability characteristics of the continuous time system. (7 marks)

Hint: *If matrix A is diagonalizable, can you relate eigenvalues of* **A** *with eigenvalues of* $[\Phi]_{Euler}$?