Chapter 1
Introduction

(Lectures 1 to 9)

Keywords: Importance of study of turbulence; sources of literature on turbulence; definition of turbulence; characterization of turbulent flows; conditions for convergence of time average and true average; references for terminology of turbulence.

Topics

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Introduction - 1

Topics

1.1 Opening remarks

Turbulent flows occur in many situations of practical interest. For example flows (i) past vehicles like airplanes, cars and ships (ii) in pipes, ducts and process equipment (iii) in internal combustion engines, (iv) in atmosphere etc. are turbulent. Hence, study of turbulence is of primary interest in aerospace engineering, chemical engineering, civil engineering, mechanical engineering, metallurgy, meteorology, ocean engineering and other branches of engineering dealing with fluid flow.
Before describing the outline of the course, some aspects of turbulent flows are described in the next few paragraphs.

A detailed definition of turbulence would be given in the next section. At this juncture it may be mentioned that in a flow which is laminar, the flow variables like velocity components, pressure, temperature etc., have either constant values at a point in the flow field or the flow variables show a known variation with time. On the otherhand, when the flow is turbulent, the flow variables display random variations with time and the fluctuations are three dimensional in nature. Further, turbulent flows have higher diffusion and dissipation. Hence, transfer of heat, mass and momentum are faster in turbulent flows. At the same time, the pressure loss and drag are also higher in these flows. The random fluctuations increase the complexity in measurement and computation of turbulent flows as compared to the laminar flow. The instrumentation needed for measurement of fluctuations is more complex than that for the mean flow measurements. As regards the computation it should be noted that (a) the turbulent flows are governed by Navier-Stokes equations, (b) turbulent flows can be considered as consisting of large number of eddies of different sizes and the ratio of the length scale of the largest eddy to the smallest eddy is very large. Keeping in view all these features, it is evident that the computation of turbulent flow in all its details, called Direct Numerical Simulation (DNS), involves solution of three dimensional, unsteady Navier-Stokes equations with a fine grid resolution. Since, grid is fine, the time step in computation would also be very small. Thus, DNS, is computation intensive from computer time and memory points of view. An alternate way is to decompose the flow parameter, say U, as ($\bar{U}$+u') where $\bar{U}$ is the average of U over a time interval 'T' and u' is the fluctuating part (Fig.1.1); mathematical definition of $\bar{U}$ will be given in section 1.2.

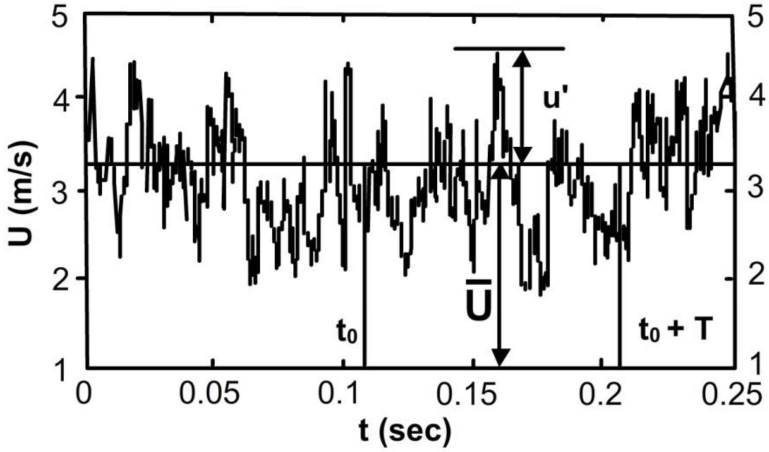


Fig1.1 Typical turbulent signal and time averaging

When ($\bar{U}$+u') is substituted for U in the Navier-Stokes equations and time average is taken, the details of which are given in chapter 3, a set of equations called Reynolds averaged Navier-Stokes (RANS) equations is obtained. These equations involve averages of products of fluctuating quantities. These are additional unknowns. To make the RANS equations a closed set of equations (i.e. number of equations equal to the number of unknowns), these additional terms need to be modelled. This aspect of the study of turbulence is called turbulence modeling.

The RANS equations for incompressible flow are elliptic in nature (books on numerical techniques can be referred to for a description of the nature of partial differential equations). These equations can be simplified to parabolic equations in special types of flows called thin shear flows (see chapters 4 and 6 for details). Boundary layers, wakes, jets come under this category of flows.

The equations governing turbulent flows are non-linear, partial differential equations. Solution of these equations involves use of numerical techniques. However, numerical solutions are subject to errors such as inadequate grid resolutions, inappropriate model of turbulence etc. Hence, the numerical technique and the computer codes need to be validated i.e. the computed results need to be compared with data from carefully conducted experiments. Such data are called bench mark data.

In the above paragraphs the important aspects of the study of turbulent flows have been hinted at. Since, turbulent flows are encountered in many branches of engineering, there is a need for persons who can work in experimental / computational work or in both aspects of turbulent flows. To cater to this need the following topics are covered in this course material.

Section 1.2 and 1.3 deal respectively with the definition of and the ways to characterize turbulent flows. Chapter two presents outline of the experimental techniques like hot-wire anemometry (HWA), Laser Doppler Velocimetry (LDV) and Particle Image Velocimetry (PIV). In chapter three, starting with Navier-Stokes equations, the equations for turbulent quantities are derived. The terms which indicate transfer of energy from mean flow to turbulence and its conversion into heat are highlighted. Chapter 4 presents the ways to describe the uncertainty in measured data and the bench mark data in basic cases like (a) flow behind a grid, (b) boundary layer (c) wake (d) jet. These data also shed light on the behavior of turbulence in different situations and help in formulation of models of turbulence described in chapter 5. Chapter 6 deals with techniques to solve thin shear flows which are governed by parabolic equations.

Topics like historical background, Cartesian tensors, derivation of basic equations, transition from laminar to turbulent flow, derivation of equation for rate of dissipation and the list of term paper topics are presented in appendices A to F. Preparing term paper on some of these topics would give the student the ability to carry out self study and later become a researcher. It is expected that the student has background in basic fluid mechanics, boundary layers and numerical techniques.

This being an introductory course the attention is mainly focused on incompressible, single phase flow. It is hoped that after going through the material, the reader can begin
(i) experimental studies in turbulent flows,
(ii) computation of turbulent flows (iii) use of commercial packages with appreciation of models of turbulence used and the limitations of computed results. A large number of books and articles are cited, in various chapters, as sources of further information.

1.1.1. Literature on turbulence – Some important sources

Treatment of turbulence in a particular book, depends on the background of the author and the branch of engineering he/she belongs to. References 1.1 to 1.46a give a chronological list of books on turbulence known to the present author. Many books on turbulence have been brought out on special occasions. References 1.47 to 1.52 are given as some of the examples. AIAA (American Institute of Aeronautics and Astronautics) periodically brings out books on specialized aspects of turbulence under "Progress in Astronautics and Aeronautics". Reference 1.53 is an example of such a book. For further details see: www.aiaa.org.

A researcher in the area of turbulence would benefit by referring to journals. Some of the important journals which contain papers on turbulent flows are as follows.

1) Aeronautical Journal
2) AIAA Journal
3) Annual review of Fluid Mechanics
4) Experiments In Fluids
5) Flow turbulence and combustion
6) Fluid Dynamics Research
7) International Journal for Numerical Methods in Fluids

8) International Journal for Heat and Mass Transfer

9) Journal of Aircraft

10) Journal of Computational Physics

11) Journal of Fluid Mechanics

12) Journal of Wind Engineering and Industrial Aerodynamics

13) Physics of Fluids

14) Progress in Aerospace Sciences

15) Transaction of ASME – Journal of Fluids Engineering

Some of the important conference proceedings which deal with turbulent flows are :

1) International Conference on Numerical Methods in Fluid Dynamics.
2) International Conference on Numerical Methods in Laminar and Turbulent Flows.
3) International Symposium on Engineering Turbulence Modeling and Measurements.
4) Symposia held periodically by Issac Newton Institute (INI), Cambridge, U.K., Ref.1.54 and 1.55 are typical examples.
5) Symposium on Turbulent Shear Flows which has now been renamed as Symposium on Turbulence and Shear Flow Phenomena.

The reports of following organizations are valuable sources of information.

1) AGARD (Advisory Group for Aerospace Research & Development) Reports
2) ICASE Reports – ICASE is a division of NASA Langley Research Centre
3) Imperial College London Reports
4) Lecture notes of Von Karman Institute (VKI), Belgium
5) NASA Reports
6) Stanford University : CTR and Thermal Sciences Division Reports and Annual Research Briefs of Centre for Turbulence Research. These reports are now available on http://ctr.stanford.edu

Remark: Prof. P. Bradshaw has created the following website which contains bibliographic details of nearly 10,000 articles on various aspects of turbulence from 1980 to 2002 with some earlier and later entries.

http://navier.stanford.edu/bradshaw/pbref/pbref.html

1.2 Definition of turbulence

Defining turbulence in a few sentences is not found to be adequate to describe various features of turbulence. For example O.Reynolds, famous for his early work on turbulence in 1880's & 1890's called turbulence as sinuous motion. This discription only highlighted the irregular nature of turbulent fluctuations. G.I.Taylor and Von Karman defined it in 1937 (as quoted in Ref.1.3, chapter 1 of second edition) as: "Turbulence is an irregular motion which in general makes its appearance in fluids, gas or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another". This definition highlights two features of turbulent flows viz. (i) irregularity or randomness of fluctuations and (ii) need of shear or velocity gradient to sustain turbulence. Two cases, mentioned below illustrate the second feature. When the fluid flows past a stationary solid body a boundary layer is produced, in which the velocity is zero on the surface and reaches the external value in a thin layer (Fig.1.2a).

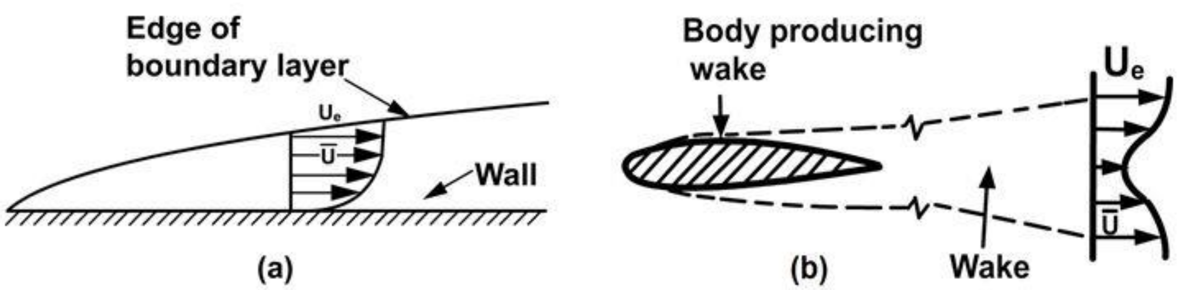


Fig 1.2 Typical flow examples (schematic)
(a) Boundary layer (b) Wake

In this layer the shear i.e. $\left(\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\right)$ is seen to be non-zero. In a flow without a solid boundary, for example a wake as shown in Fig.1.2b, also the shear is non-zero. The production of turbulence due to shear is explained with the help of equations in chapter 3. The turbulent flows have many features in addition to those mentioned above and following Tennekes and Lumley (Ref.1.9, chapter 1), the nine features common to all turbulent flows are described in the following subsections. A few commonly used terms are also defined in the course of the description.

1.2.1 Irregularity

Figure 1.1 shows a typical variation, with time, of a fluid flow parameter in a turbulent flow. The parameter could be, for example, a velocity component, or the pressure, or temperature. It appears from the variation shown in Fig.1.1, that it is not possible to say as to what the value of the parameter would be at a chosen instant of time. The parameter would also show such a variations in spatial directions. Such variations are called random. However, the fluid motion is organized in such a way that statistical averages can be taken. On the other hand, a flow situation is called deterministic, when, the flow parameters, at a point have either (a) constant values with time or (b) are known functions of time. A deterministic flow is not turbulence.

Prior to 1980s, the turbulent motion was regarded as random. Currently, it is regarded as chaotic. Whenever, the motion of a system is fully deterministic in the mathematical sense but is unpredictable in the conventional experimental sense, the motion is termed as “chaotic” (Ref.1.56, page 121). The detailed behaviour of such a system is very sensitive to initial conditions. However, motion is organized in such a way that statistical averages can be taken. On the other hand, a flow, where the flow parameters at a point have either constant values with time or are known functions of time, is deterministic in nature and is not turbulence.

Remarks :

(i) Example of chaotic behaviour.

Systems/phenomenon governed by non-linear equations, display chaotic behaviour after a certain parameter exceeds a value, which is specific to the system. As an example, Ref.1.30 chapter 3, considers the following set of algebraic non-linear equations called Lorenz equations :

$$\frac{dx}{dt} = \sigma(y - x); \frac{dy}{dt} = \rho x - y - xz; \frac{dz}{dt} = \beta z + xy$$

The effect of small changes in initial conditions, on the solution is examined in the following manner.

(a)Take $\beta = 8/3$, $\sigma = 10$ and $\rho = 28$ in the above equations.

(b) Obtain the solutions of the above three equations for 2 sets of initial conditions viz. {x(0), y(0), z(0) = 0.1,0.1,0.1} and {x(0), y(0),z(0)} = {0.100001,0.1,0.1}. It may be noted that the two initial conditions differ from each other only in the 6th place of decimal in the value of x(0).

(c)The solutions for 'x' in the two cases are denoted by x(t) and $\hat{x}(t)$.

If the equations were linear, the difference between x(t) and $\hat{x}(t)$ would be negligibly small. However, in the case of the above non-linear equations the two solutions are nearly same only upto t = 35 s. For t > 35 s, the two solutions differ from each other by large amount and the difference $[\hat{x}(t)-x(t)]$ varies with time. Figure 1.3 shows the difference.

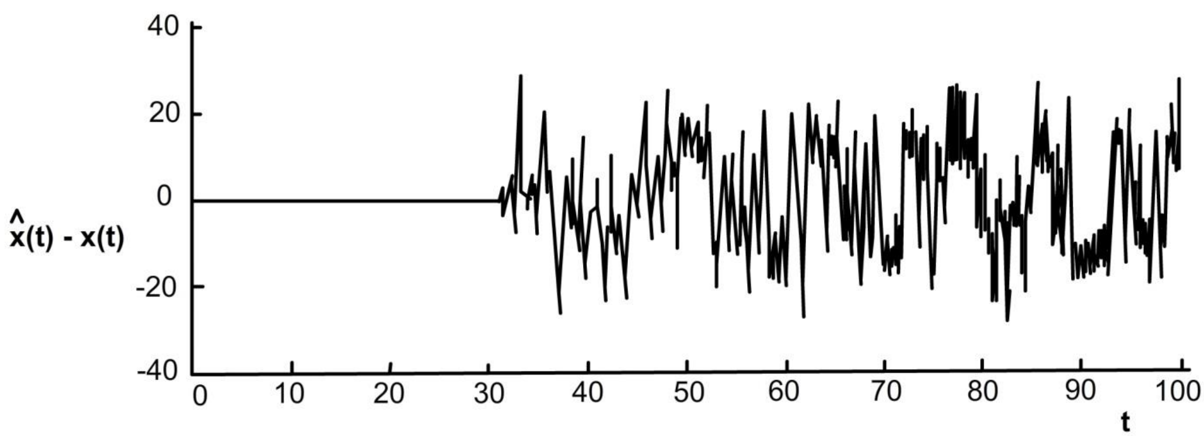


Fig1.3 Chaotic behaviour

[Based on Fig. 3.2 of Ref. 1.30; Reproduced with permission from Prof. S.B. Pope]

Thus, small changes in initial conditions do not cause small changes in the solution. The behaviour for t > 35 appears random but, is predictable in the mathematical sense and lies within certain limits. Such a behaviour is called 'chaotic behaviour'. Chaotic behaviour is not observed when ρ is less than 24.74 for the values of $\beta = 8/3$, $\sigma = 10$.

(ii)The Navier-Stokes equations, which govern the fluid motion, are also non-linear. They display chaotic behaviour when Reynolds number (Vl/ν), exceeds a certain value. Note that V and l are reference velocity and length and ν is kinematic viscosity. It may be pointed out that, small disturbances are always present and in the flow field. At low Reynolds number the effects of disturbances are damped out and the flow remains laminar. At high Reynolds number the flow would display chaotic behaviour. One of the manifestations of this is that the flow variables show random variations with space and time. However, it is found that in flows of engineering interest, the statistical averages taken over a reasonable interval of time would nearly be the same in different realizations of the flow.

Cebeci (Ref.1.46a, chapter 1) carried out flow visualization study in a turbulent boundary layer using Hydrogen bubble technique; information on this technique is briefly given in subsection 4.3.11. He obtained the velocity profiles at the same point but at 17 instants of time. Typical eight velocity profiles are shown in Fig. 1.4. It is observed that the instantaneous velocity profiles display chaotic variation. However, when the 17 profiles were superimposed on each other, the average profile is very similar to the typical boundary layer profile in turbulent flow as shown in Fig. 1.2a. This confirms that in flows of engineering interest the statistical averages, taken over a reasonable instant of time, would nearly be the same in different realizations of the flow.

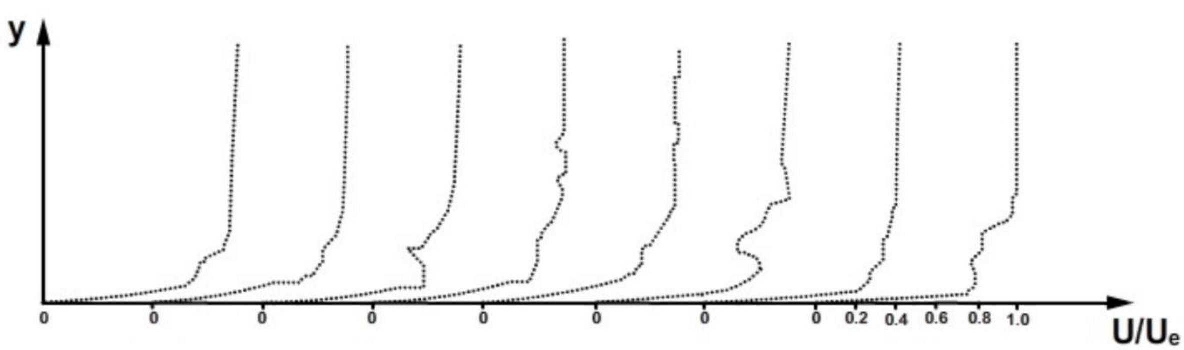


Fig. 1.4 Instantaneous velocity profiles in a boundary layer at a point at 8 instants (schematic); The dots schematically represent the Hydrogen bubbles

At this stage, the averages used in turbulent flows are briefly discussed.

Time average

It is defined as :

$$\bar{U}(\mathbf{X},t_0) = \frac{1}{T}\int_{t_0}^{t_0+T} U(\mathbf{X},t)dt \qquad (1.1)$$

where, U is a flow variable. The averaging is started at time t_0 and taken over a sufficiently long interval of time (T) (Fig.1.1). The measurement is carried out at a specific location denoted by **X**; the quantities in bold are vectors. It (measurement) is done by inserting a suitable sensor like hot wire (see chapter 2 for details). The time average is taken by an instrument called mean value unit or is currently done with the help of software. The duration of the time interval which will give correct average is discussed in section 1.4. In some texts on statistics "limit T $\rightarrow \infty$" is written along with Eq.(1.1). It is ommitted with the understanding that T is sufficiently long.

There are flow situations where the average $\bar{U}$ would depend on the time 't_0' at which is the averaging is started. However, in many laboratory flows, which are steady in the mean, $\bar{U}$ is independent of 't_0'. In such a situation the phenomenon is called 'stationary random phenomenon'. In this case :

$$\bar{U}(\mathbf{X}) = \frac{1}{T}\int_{0}^{T} U(\mathbf{X},t)dt \qquad (1.2)$$

In this case, it does not matter when the measurement, at a chosen point, is started. This is important when the domain of measurement is large. The measurements, each averaged over 'T', could take a long period to complete but, still the averages at various points would bear some definite relationship with each other.

Let, u' = U - $\bar{U}$ (1.3)

Then, for a stationary random phenomenon,

$$\overline{u'} = \frac{1}{T}\int_{0}^{T} (U-\bar{U})dt = \bar{U}-\bar{U} = 0 \qquad (1.4)$$

Note that in this case, the fluctuations are equally distributed around the mean as seen in Fig.1.1.

Space average

It is defined as:

$$\bar{U}(\mathbf{X}_0,t_0) = \frac{1}{x_1}\int_{\mathbf{x}_0}^{\mathbf{x}_0+x_1} U(\mathbf{X}_0,t_0)\,dx_1 \qquad (1.5)$$

where, the average of the values of U, at time t_0, is taken at various points over a distance x_1.

If the average in Eq.(1.5) is independent of the spatial location (X_0), then the turbulence is called homogeneous. Space average is generally used in theoretically work on turbulence.

Phase average

In certain flows e.g. that past a rotating blade, the turbulent fluctuations are superimposed over a periodic motion (Fig.1.5).

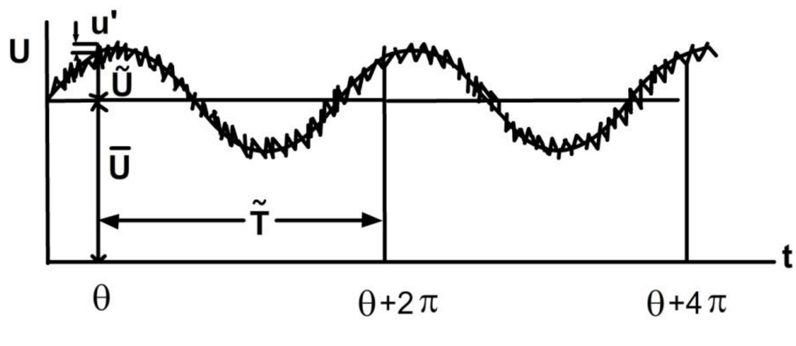


Fig.1.5 Phase average

In this case the average at a phase angle (θ) is defined as :

$$\bar{U}+\tilde{U}(\theta) = \frac{1}{N}\sum_{n=0}^{N-1} U(\mathbf{X},\theta+2n\pi) \qquad (1.6)$$

The turbulent part, in this case is :

$$u' = U-\left(\bar{U}+\tilde{U}\right) \qquad (1.7)$$

Remarks:

(i) The frequency of the periodic component ($\tilde{U}$) is, in general, much lower than the frequencies associated with turbulence (see section 1.2.6).

(ii) Following steps may be followed to measure the signals at chosen values of θ in various cycles i.e. θ, $\theta+2\pi$, $\theta+4\pi$ and so on.

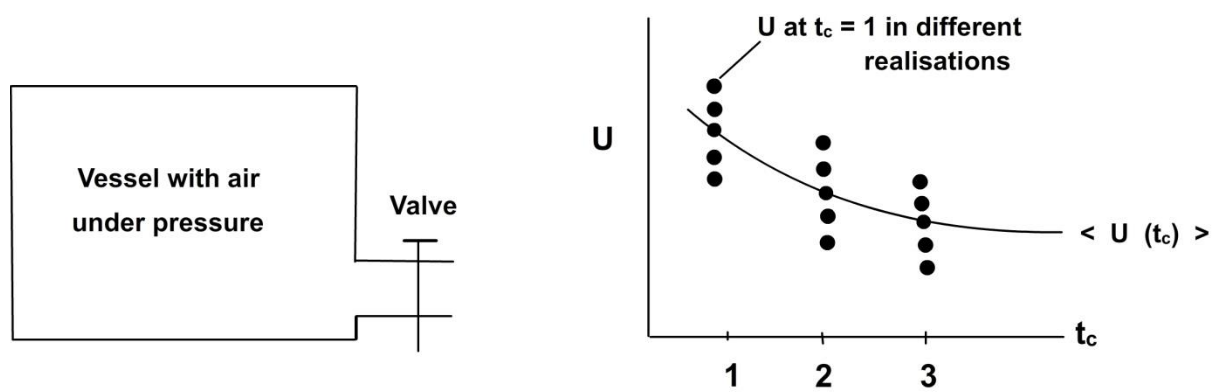
The frequency of the mean motion i.e. of $\tilde{U}$, is either known or can be found out from a spectrum analysis, which would show a large peak in the spectrum at this frequency. Knowing the frequency, the time period of the mean motion (T) can be calculated and the readings are recorded at regular intervals of T.

Ensemble average

In certain situations, the values of flow variables are available at discrete time intervals. The examples are (i) the digital output of a Laser Doppler Anemometer (See chapter 2) or (ii) unsteady aperiodic phenomenon where the mean value changes with time. In such cases the average is taken over the values of the flow parameter in different realizations of the event. Such an average is called 'Ensemble average' and is defined as :

$$<U(X,t_c)> = \frac{1}{N}\sum_{m=1}^{N} U_m(\mathbf{X},t_c) \qquad (1.8)$$

where, U_m is the value of the quantity U in the m^{th} realization of the event at location **X** at a chosen time t_c during the event. Bradshaw (Ref.1.6, chapter 1), gives the example of air coming out of a pressure vessel. Consider the velocity at a point 10 cm from the exit and 1 second after opening the valve. Here, **X** is 10 cm and t_c is 1 s. When several realizations i.e. experiments are carried out, 'U' may be different in each realization (Fig.1.6).



(a) Set - up

(b) Velocity at time t_c in different realization

Fig.1.6 Ensemble average

In this case the fluctuating part is

$$u'(\mathbf{X},t_c) = U_m(\mathbf{X},t_c) - \langle U(\mathbf{X},t_c)\rangle \qquad (1.9)$$

Remark:

In the example shown in Fig.1.6 the flow is not stationary random because, the pressure inside the vessel decreases with time and consequently the average velocity at a point decreases with time. On the other hand in a stationary random phenomenon there is no dependence on t_c and the average is given by:

$$\langle U(\mathbf{x})\rangle = \frac{1}{N}\sum_{m=1}^{N} U_m(\mathbf{X}) \qquad (1.10)$$

Further, in a stationary random phenomenon the ensemble average and the time average are same. Hence, in the study of stationary random flow, the ensemble average is used in theoretical work, whereas the time average is used in experimental work.