1.1 REVIEW OF FLUID MECHANICS

Before we discuss thrust generated from a rocket motor, we review basic assumptions and physical laws that govern fluid motion encountered in rocket propulsion systems.

Continuum hypothesis

Here fluid is viewed at the macroscopic level where it is assumed that fluid is continuous. The molecular level discontinuities in fluid are not considered. This hypothesis holds good for most propulsion systems except certain electric propulsion systems.

Laws Governing Fluid Motion

A. Continuity (Conservation of Mass):

The equation physically signifies the law of conservation of mass.
Consider an arbitrary control volume bounded by imaginary control surface (Figure 1). Consider a small elementary control volume *dV*. Then,

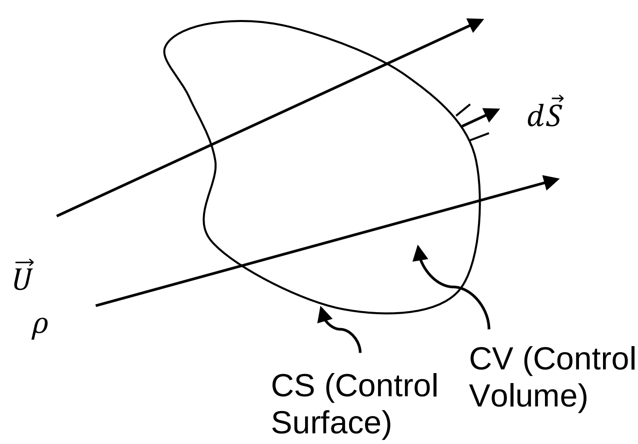

Figure 1

$$\frac{\partial}{\partial t}\iiint_{CV} \rho dV + \oint\!\!\oint_{CS} \rho\vec{U}\cdot d\vec{S} = 0 \;\Rightarrow\; \frac{dM_V}{dt} + \rho_e U_e A_e = 0$$

Rate of change of mass inside control volume V Mass fluxthrough the control surfaces

Let us discuss further with an example.

Example: Consider a bend pipe with 90^0angle. First a control volume is to be chosen.

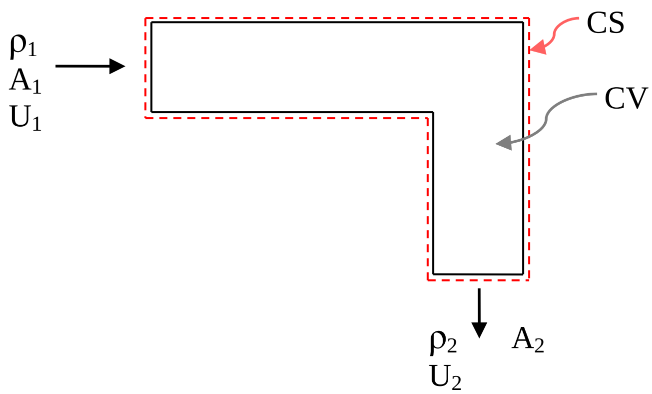

Figure 2

The dotted line represents the control surface. Assume flow is steady and apply continuity equation to this control volume. This results in

$$\oint\!\!\oint_{CS} \rho\vec{U}\cdot d\vec{S} = 0$$

$\vec{U}\cdot d\vec{S}$ is positive for mass flowing out and negative for mass flowing in.

Then,

$$-\rho_1 A_1 U_1 + \rho_2 A_2 U_2 = 0$$

or $\rho_1 A_1 U_1 = \rho_2 A_2 U_2$

or Mass flux going in = Mass flux going out

B. The Newtons Second Law (Momentum Equation)

Momentum equation is based on Newton's second law of motion, which says that the net force acting on a body causes a rate of change in momentum of the body.

Over a control volume, mathematically the equation can be written as

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\vec{U}dV + \iint_{CS} \rho\vec{U}\vec{U}\cdot d\vec{S} = \sum \vec{F} = \frac{d(m\vec{U})}{dt}$$

This is a vector equation and has three components in x,y,z directions and therefore can be expressed as three different equations.

For example the x-momentum equation is given by

$$\frac{\partial}{\partial t}\iiint_{CV} \rho u dV + \iint_{CS} \rho u\vec{U}\cdot d\vec{S} = \sum F_x$$

Rate of change of momentum inside control volume,CV	Momentum flux through the control surfaces, CS	External forces on the system

here *u* is the velocity component in x direction.

Similarly equations can also be written for the y-momentum.

$$\frac{\partial}{\partial t}\iiint_{CV} \rho v dV + \iint_{CS} \rho v\vec{U}\cdot d\vec{S} = \sum F_y$$

Let us discuss further an application using the same example of 90° bend pipe.

Now applying X and Y momentum equations to this control volume for steady flow,

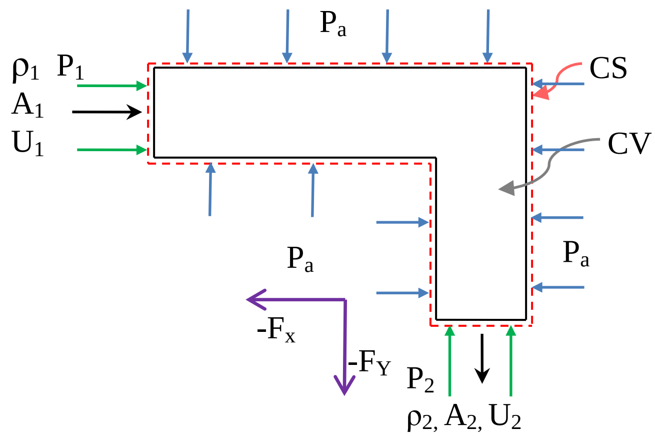

Figure 3

X-momentum equation

$$\oiint_{CS} \rho u \vec{U} \cdot d\vec{S} = \sum F_x$$

Y-momentum equation

$$\oiint_{CS} \rho v \vec{U} \cdot d\vec{S} = \sum F_y$$

Let $\vec{F}$ be the reaction on the pipe due to the fluid. So force acting on the fluid is $-\vec{F}$.

$$\vec{F} = F_x \hat{\imath} + F_y \hat{\jmath}$$

Other forces acting on the control volume are the pressure forces as shown in figure 3.

$$\sum F_x = P_1 A_1 - P_a A_1 - F_x \;\&\; \oiint \rho u \vec{U}.\overrightarrow{ds} = -\rho_1 U_1^2 A_1$$

$$\sum F_y = P_2 A_2 - P_a A_2 - F_y \;\&\; \oiint \rho v \vec{U}.\overrightarrow{ds} = \rho_2 U_2^2 A_2$$

Therefore,

$$F_x = \rho_1 U_1^2 A_1 + (P_1 - P_a)\, A_1$$

$$F_y = (P_2 - P_a) A_2 - \rho_2 U_2^2 A_2$$

First law of thermodynamics or the conservation of energy(Energy Equation)

The energy equation is the statement of the first law of thermodynamics. For an open system or a control volume, this equation takes the following form:

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\left(e+\frac{U^2}{2}+gz\right)dV + \iint_{CS} \rho\left(e+\frac{U^2}{2}+gz\right)\vec{U}\cdot d\vec{S} \quad = \delta\dot{Q}-\delta\dot{W}$$

The total external work can be represented as a sum of shaft/stirring/electrical work and flow work

$$\underbrace{\delta\dot{W}}_{\text{Total work}} = -\left(\underbrace{\delta\dot{W}'}_{\text{External (shaft/stirring/electrical) work}} + \underbrace{\iint_{CS} p\vec{U}.d\vec{S}}_{\text{Flow work}}\right)$$

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\left(e+\frac{U^2}{2}+gz\right)dV + \iint_{CS} \rho\left(e+\frac{U^2}{2}+gz\right)\vec{U}\cdot d\vec{S} + \iint_{CS} p\vec{U}.d\vec{S} \quad = \delta\dot{Q}-\delta\dot{W}'$$

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\left(e+\frac{U^2}{2}+gz\right)dV + \iint_{CS} \rho\left(\frac{p}{\rho}+\frac{U^2}{2}+gz\right)\vec{U}\cdot d\vec{S} \quad = \delta\dot{Q}-\delta\dot{W}'$$

Neglecting changes in potential energy and with given enthalpy (h) as

$$h = e+\frac{p}{\rho}$$

The energy conservation equation becomes

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\left(e+\frac{U^2}{2}\right)dV + \iint_{CS} \rho\left(h+\frac{U^2}{2}\right)\vec{U}\cdot d\vec{S} \quad = \delta\dot{Q}-\delta\dot{W}'$$

An example application of integral form of energy equation will be discussed later.